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Analysis of Effect of Phase Dithering on SFDR of DDS

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Abstract— **Direct Digital Synthesizer (DDS) is a versatile programmable signal synthesis device. The important parameters of DDS are frequency resolution, number of frequency spots and frequency step size. These parameters are mainly dependant on the minimum phase increment of accumulator. Any improvement in these parameters requires width of the accumulator register to increase. As size of lookup memory grows exponentially with width of the accumulator output and hardware limitations restrict the size of memory which can be allocated for Lookup Table (LUT). To overcome this, accumulator output is truncated to match LUT size. Truncation, however results in generation of spurious signal which reduces Spurious Free Dynamic Range (SFDR). Phase dithering is employed to reduce truncation induced spurious. There are various phase dithering techniques available, some of the prominent ones are analyzed in this paper for their effect on SFDR.**

Index Terms— **DDS, Frequency Resolution, Frequency Synthesis, Phase dithering**

I. INTRODUCTION

Direct Digital Synthesis is a programmable approach to generate radar waveforms from its digital definition. DDS works on the principal of digital phase wheel and consists of a phase accumulator, a phase-to-amplitude lookup table, and digital-to-analog converter (DAC) to generate signals of desired frequency. This digital approach allows for precise control over the output frequency and phase, enabling rapid frequency switching and fine resolution.

The DDS is characterized by its frequency resolution, which is given by

$$\Delta f = \frac{f_{clk}}{2^N} \qquad (1)$$

Where,

Δf = Frequency Resolution,
f_{clk}= Clock Frequency,
N = No. of bits in accumulator register

As clear from equation (1), increasing the width of the phase accumulator improves the frequency resolution of a DDS, since a wider accumulator allows for finer frequency steps. However, this also leads to an exponential increase in the size of the LUT, which stores waveform samples such as sine values. The LUT size is constrained by hardware resources and power consumption, making it impractical to match the full accumulator width. Therefore, while higher accumulator widths offer better resolution, they are limited by implementation constraints. To address this, the accumulator width is chosen based on the desired frequency resolution, and its output is truncated before being used to address the LUT. This reduces the LUT size while maintaining sufficient resolution, as illustrated in Figure 1.

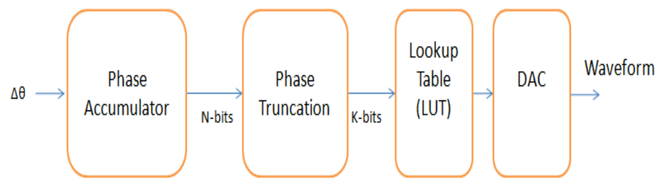

Figure 1: Direct Digital Synthesizer

Due to this truncation of accumulator output, instantaneous phase contain error proportional to truncated bits. As this error is periodic, the spurious frequencies corresponding to this error also get generated in addition to the desired output. The phase truncation, thus considerably reduces the SFDR of DDS output. There are various techniques reported in the literature to reduce this issue of phase truncation and related spurious signal, such as CORDIC, Nicholas Modified Architecture, Sine Approximation, phase dithering etc. In this paper we will be analyzing phase dithering techniques and their effect on SFDR of DDS.

II. PHASE DITHERING IN DDS

Phase dithering is a technique in which, a sequence of randomly generated bits, derived from a pseudo-random number generator (PRNG), is added to the output of the phase accumulator before it undergoes truncation to match the address space of the LUT. Without dithering, truncating the accumulator output introduces a deterministic truncation error that is periodic in nature. This periodicity causes the error to concentrate energy at specific frequencies, generating prominent spurious spectral components. These unwanted components degrade signal purity and limit the SFDR of the output.

By adding random phase dither before truncation, the truncation error becomes randomized rather than periodic. This randomization of the phase truncation

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error breaks up the regular pattern, effectively spreading the spurious energy across a wider frequency range. As a result, the spurious signals become less pronounced, leading to a significant reduction in peak spurious power and an overall improvement in SFDR. However, this improvement comes at a trade-off. The phase dithering helps improve spectral performance by reducing coherent spurious tones through error randomization, enhancing SFDR at the cost of a modest increase in broadband noise.

III. Phase Dithering Techniques

Various phase dither generation techniques, beginning with the baseline case of no dithering, are summarized and analyzed in the following sections with respect to their effectiveness in enhancing SFDR. For the purpose of this analysis, a phase accumulator with a resolution of 14 bits and a lookup table with a depth of 8 bits are considered. The performance evaluation of each technique is conducted through simulations in MATLAB.

3.1. DDS Without Dithering

Simulation of DDS was carried out without adding phase dither to compare performance of various techniques wrt no dithering case. In this case, depending upon the LUT space and phase resolution, some of the least significant bits are truncated. In the current example 6 least significant bits out of 14 bits of accumulator output are truncated before presenting to LUT for conversion of phase to amplitude. The frequency spectrum of DDS output for phase truncation case is shown in figure 2.

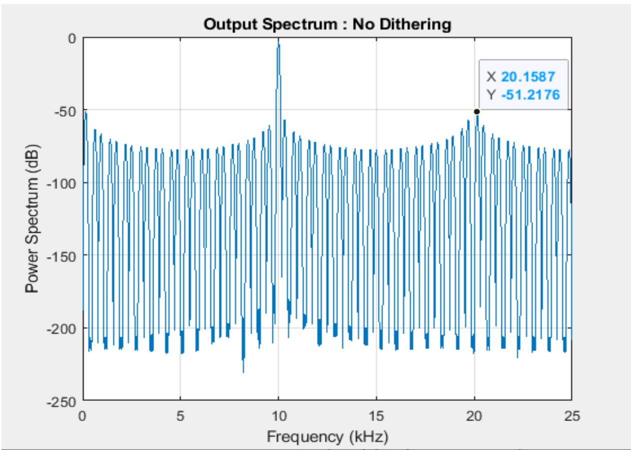

Figure 2: Output Spectrum without Phase Dithering

3.2. Linear Congruential Generator

Linear Congruential Generator (LCG) is one of the oldest methods for generation of pseudorandom numbers. This algorithm calculates randomized numbers with the help of a linear equation, given in equation (2).

$$X_{n+1} = \mathrm{mod}\,(a * X_n + c, m) \qquad (2)$$

In equation (2), X_k is k^{th} number in the sequence and X_0 is starting number or seed, ‘a’ and ‘c’ are algorithm parameters, called multiplier and increment respectively and ‘m’ is the maximum value of a number in the sequence. Simulation was carried out with LCG sequence as phase dither for DDS and frequency spectrum of DDS output with LCG phase dithering is shown in figure 3.

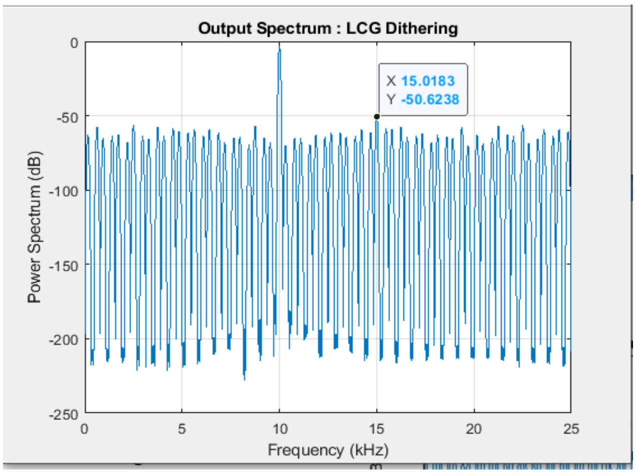

Figure 3: Output Spectrum with LCG Dithering

3.3. Lagged Fibonacci Generator

Lagged Fibonacci Generator (LFG) is a pseudorandom generator based on Fibonacci sequence. LFGs exhibit long periods and good statistical properties. It generates a sequence of numbers from previously generated values using a recurrence relation as given in equation (3).

$$X_{n+2} = \mathrm{mod}\,(X_{n+1} + X_n\,, m) \qquad (3)$$

In equation (3), X_k is k^{th} number in the sequence and X_0 X_1 are starting numbers or seed and ‘m’ is the maximum value of a number in the sequence. Simulation was carried out with LFG sequence as phase dither for DDS and frequency spectrum of DDS output with LFG phase dithering is shown in figure 4.

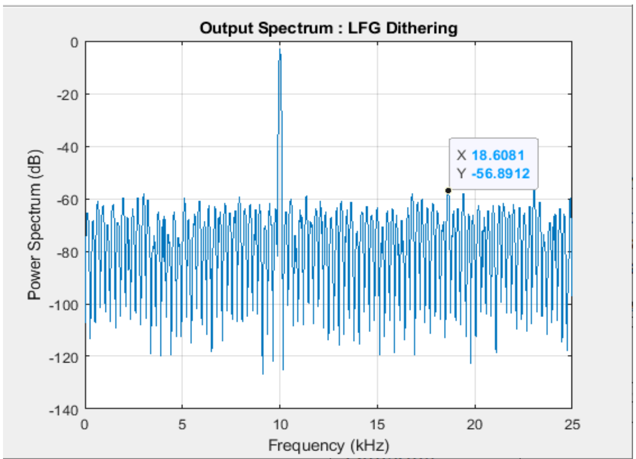

Figure 4: Output Spectrum with LFG Dithering

3.4. Mersenne Twister Generator

Mersenne Twister Generator (MTG) is pseudorandom number generator with good statistical properties and relatively long period. The generation using MTG involve two steps namely, state update and number extraction. These steps can be represented as given in equation (4).

$$X_{n+1} \quad X_n \mid twist(X_n, c)$$

$$X_{n+1} \quad mod(Extract(X_n, c), m) \qquad (4)$$

In equation (2), X_k is k[th] number in the sequence, 'c' is algorithm parameters, and 'm' is the maximum value of a number in the sequence. Simulation was carried out with MTG sequence as phase dither for DDS and frequency spectrum of DDS output with LCG phase dithering is shown in figure 5.

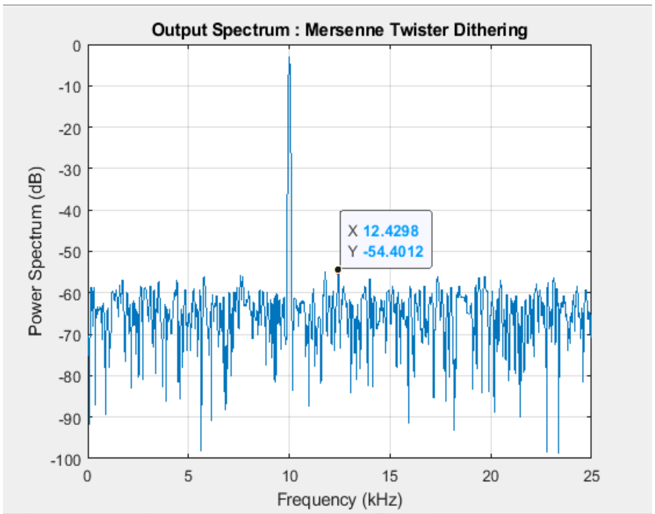

Figure 5: Output Spectrum with MTG Dithering

3.5. XOR Shift Based Generator

XOR Shift based pseudorandom generator is well recognized for its speed and efficiency. It generates pseudorandom sequence for a given seed by applying bitwise XOR and Shift operation, as given equation (5).

$$X_{n+1} \quad mod(X_n \mid (X_n \ll a) \mid (X_n \gg b) \mid (X_n \ll c), m) \qquad (5)$$

In equation (5), X_k is k[th] number in the sequence and X_0 is starting number or seed, 'a', 'b' and 'c' are algorithm parameters and 'm' is the maximum value of a number in the sequence. Simulation was carried out with XOR Shift sequence as phase dither for DDS and frequency spectrum of DDS output with XOR Shift phase dithering is shown in figure 6.

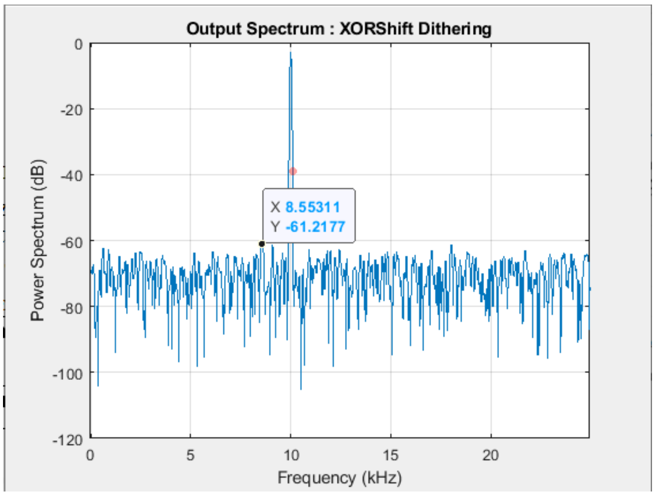

Figure 6: Output Spectrum with XOR Shift Dithering

3.6. Blum Blum Shub Generator

Blum Blum Shub (BBS) Generator based pseudorandom generator is a cryptographic random sequence generator. It generates pseudorandom sequence from a given seed by taking square and modulus, as given equation (6).

$$X_{n+1} = mod(X_n^2, m) \qquad (6)$$

In equation (6), X_k is k[th] number in the sequence and X_0 is starting number or seed, and 'm = p*q' is the maximum value of a number in the sequence, which is product of two prime numbers, p and q. Simulation was carried out with BBS sequence as phase dither for DDS and frequency spectrum of DDS output with BBS phase dithering is shown in figure 7.

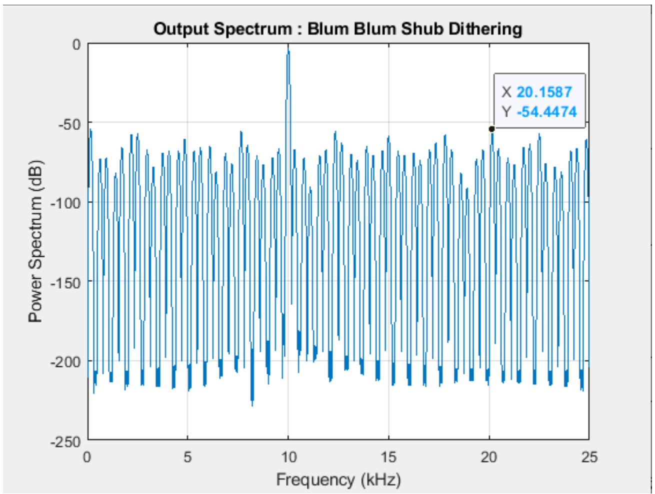

Figure 7: Output Spectrum with BBS Dithering

IV. Comparison and Results

The algorithms are analyzed for ease of implementation and performance. From the implementation point of view, all the algorithms presented in previous section are hardware implementable with minimum complexity. For comparison, simulation was carried and performance of algorithms was analysed for achievable SFDR. For simulation phase resolution of 14 bits, lookup table depth of 8 bit and clock frequency of 50 kHz was assumed. SFDR of -51.2 dB was achieved without any phase dithering. When phase dither generated with the help of LCG was added before truncation, not many changes were observed in the output in terms of spurious spreading or SFDR, however when dither sequence was generated with LFG, spurious were spread over complete band and SFDR was improved to -56.89. Similarly with Mersenne Twister and BBS, spurious reduction was observed wrt non-dithering case with SFDR of -54.40 dB and -54.44 dB respectively. With XOR Shift based generation, spurious were more evenly distributed and SFDR of -61.21 dB was achieved as compared to -51.21 achieved without phase dithering. Comparison of SFDR achieved for these algorithms is tabulated in Table 1.

Table 1: Comparison of Phase Dithering Techniques

Phase Dithering Technique	**SFDR Achieved (dB)**
DDS without Dithering	-51.21
Linear Congruential Generator	-50.62
Lagged Fibonacci Generator	-56.89
Mersenne Twister Generator	-54.40
XOR Shift Based Generator	-61.21
BBS	-54.44

Based on the results presented in Table 1, the XOR-shift-based pseudo-random number generator demonstrates the most effective SFDR improvement for DDS systems. Additionally, its simplicity and ease of implementation make it well-suited for hardware realization.

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