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Weighted Number Operator in Continuous-Time Guichardet-Fock Space

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Abstract—**This paper proves that the two-dimensional weighted number operator S_ω is a densely defined unbounded self adjoint operator in continuous-time Guichardet-Fock space $L^2(\Gamma;\eta)$. But when $\sup_\sigma v_\omega < +\infty$, S_ω is a bounded linear operator. Meanwhile this paper gives two representations of S_ω.**

$$(1)\int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t\,\mathrm{d}s\,\mathrm{d}t,$$

$$(2)\,S_\omega = \sum_{n=1}^{\infty} cn^2 J_n,\ \omega(s,t)\equiv c,$$

where $\omega(s,t)$ is nonnegative function on , J_n is the orthogonal projection operator from $L^2(\Gamma;\eta)$ to its linear subspace $L^2(\Gamma^{(n)};\eta)$. Furthermore some conclusions related to are presented.

***Index Terms*—continuous-time Guichardet-Fock space, weighted number operator, nonnegative real function, point-stae modified stochastic gradient.**

I. INTRODUCTION

In 1984, Husdson and Patharathy proposed the quantum stochastic calculus theory[1], which is a noncommutative extension of stochastic integral theory of classical , and an operator stochastic integral theory[2]. Quantum stochastic integral has a very perfect development in Fock space, which can describe physical systems with properties, such as accretion and annihilation, so quantum stochastic integral has a wide range of applications in physics, engineering and other disciplines [2],[3].

Guichardet-Fock space is a basic concept in the quantum field, which is mainly applied to quantum stochastic integral problems. Guichardet-Fock space is not only isomorphic with Fock of classical Itô stochastic integral theory in linear operations, but also richer than its spatial structure[4]. Therefore, it is very meaningful to study the relevant conclusions of Guichardet-Fock space.

In the literature [5], the author defined the weighted number operator, and studied its properties and applied it in quantum Markov semigroups in the Bernoulli functional space $L^2(\Omega)$. In the literature [6], Attal discussed the continuous-time Guichardet-Fock space $L^2(\Gamma;\eta)$ the generalized operator Mallivin the variational theory, in which the operator has a maximum definition field, thus realizing the composition of the algorithm with the help of exponential vectors. In the literature [7], the author modified the stochastic gradient ∇ and point-stae stochastic gradient ∇_s. The modified operator $\widetilde{\nabla}$, $\widetilde{\nabla}_s$ and its conjugate operator $\widetilde{\nabla}_s^*$ which has the physical meaning of true annihilation and accretion. Thus, it can describe the physical system with accretion and annihilation. Reference [8] extends the modified stochastic gradient and Skorohod integral of [7], and provides the relationship between the modified stochastic gradient and Skorohod integral after extension. In the literature [9], the author discussed the properties and representations of the number operator N in $L^2(\Gamma;\eta)$, and makes the first attempt at the representation of the operators in $L^2(\Gamma;\eta)$. Then, in the literature [10], the author studied the Dirichlet forms with in $L^2(\Gamma;\eta)$. Based on the above analysis, this paper proved that the properties and representations of the weighted number operator S_ω in $L^2(\Gamma;\eta)$.

This paper is organized as follows. In section 2, we fix some necessary notation and recall main notion and facts about the Guichardet-Fock space. In section 3, we state and prove our main results.

II. PRELIMINARIES

Throughout the paper, let $\mathbf{R}_+$ be the set of all nonnegative real number, Γ denotes the finite power set of , namely

$$\Gamma = \{\sigma \subset \mathbf{R}_+ \,|\, \#\sigma < \infty\},$$

where $\#\sigma$ means the cardinality of σ as a set. For $\forall n \geq 1$, let $\Gamma^{(n)}$ be the collection of n elements subsets, namely

$$\Gamma^{(n)} = \{\sigma \in \Gamma \,|\, \#\sigma = n\},$$

and agree that $\Gamma^{(0)} = \{\Phi\}$. Obviously, $\Gamma = \bigcup_{n\geq 0}\Gamma^{(n)}$. For

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convenience, σ, τ denotes the elements in Γ. Usually, we denote by $L^2(\Gamma)$ the space of square integral complex-valued functions space on Γ.

Let η be a complex separable Hilbert space, the inner product and norm denoted by $\langle\cdot\rangle_\eta$ and $|\cdot|_\eta$. The space $L^2(\Gamma;\eta)$ of η-valued square-integral functions defined on Γ, well-known as product

$$L^2(\Gamma;\eta) \cong L^2(\Gamma)\otimes\eta.$$

The norm in $L^2(\Gamma;\eta)$ is $\|\cdot\|$, and the inner product defined as

$$\langle\langle f,g\rangle\rangle = \int_\Gamma \langle f(\sigma), g(\sigma)\rangle_\eta \mathrm{d}\sigma, \forall f,g\in L^2(\Gamma;\eta).$$

Definition 1[4]. For $\forall s,t\in\mathbf{R}_+, \forall\sigma,\tau\in\Gamma$, define

$\vee\sigma := \max\{s : s\in\sigma\}$; $\quad \wedge\sigma := \min\{s : s\in\sigma\}$;

$\sigma_- := \sigma\setminus\{\vee\sigma\}; \sigma\setminus s := \sigma\setminus\{s\}; \sigma\cup s := \sigma\cup\{s\}$;

$\mathbf{1}_\tau(s) = \begin{cases}1, & s\in\tau,\\ 0, & s\notin\tau.\end{cases}$ denotes the indicative function of τ.

Definition 2[7]. For $\forall f\in L^2(\Gamma;\eta)$, the modified stochastic gradient $\widetilde{\nabla} f$ of f be a η-valued process on $\Gamma\times\mathbf{R}_+$ defined as

$$\widetilde{\nabla} f(\tau,s) = (1-\mathbf{1}_\tau(s)) f(\tau\cup s), \forall(\tau,s)\in\Gamma\times\mathbf{R}_+,$$

and

$$\mathrm{Dom}\,\widetilde{\nabla} = \{f\in L^2(\Gamma;\eta) \mid \int_\Gamma (\#\sigma)|f(\sigma)|_\eta^2 \mathrm{d}\sigma < +\infty\}.$$

Definition 3[7]. For $\forall f\in L^2(\Gamma;\eta), s\in\mathbf{R}_+$, the point-state modified stochastic gradient $\widetilde{\nabla} f$ of f be a function on defined as

$$\widetilde{\nabla}_s f(\tau) = \widetilde{\nabla} f(\tau,s) = (1-\mathbf{1}_\tau(s)) f(\tau\cup s), \forall(\tau,s)\in\Gamma\times\mathbf{R}_+$$

and its adjoint operator defined as

$$\widetilde{\nabla}_s^* f(\tau) = \mathbf{1}_\tau(s) f(\tau\setminus s), \forall\tau\in\Gamma.$$

Remark 1 [7]. The point-state modified stochastic gradient $\widetilde{\nabla}_s$ and its adjoint $\widetilde{\nabla}_s^*$ are bounded linear operator on $L^2(\Gamma;\eta)$ and

$$\|\widetilde{\nabla}_s\| = 1, \qquad \|\widetilde{\nabla}_s^*\| = 1.$$

Lemma 1[9]. There's the only telescope

$$J : L^2(\Gamma;\eta) \to \bigoplus_{n=0}^{\infty} L^2(\Gamma^{(n)};\eta).$$

It meets condition $\forall f\in L^2(\Gamma;\eta)$, existing $f_n\in L^2(\Gamma^{(n)};\eta)$, $\forall n\geq 0$ makes $f = \oplus_{n=0}^\infty f_n$ and $\|f\|^2 = \sum_n \|f_n\|^2$. Here, $L^2(\Gamma^{(n)};\eta)$ naturally seen as the subspace of $L^2(\Gamma;\eta)$. Specifically, $L^2(\Gamma^{(0)};\eta) = \eta$.

Definition 4 [10]. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$, two-dimensional weighted number operator S_ω of $L^2(\Gamma;\eta)$ as defined below

$$S_\omega f(\sigma) = \nu_\omega(\sigma) f(\sigma), \quad f\in\mathrm{Dom}\,S_\omega,$$

and

$$\mathrm{Dom}\,S_\omega = \{f\in L^2(\Gamma;\eta) \mid \int_\Gamma (\nu_\omega(\sigma))^2 |f(\sigma)|_\eta^2 \mathrm{d}\sigma < +\infty\}.$$

There $\nu_\omega(\sigma) = \sum_{s\in\sigma}\sum_{s\in\sigma}\omega(s,t)$, which is the number function of $L^2(\Gamma;\eta)$.

Definition 5 [10]. $h(s)$ is nonnegative real function on $\mathbf{R}_+$,

one-dimensional weighted number operator N_h of as defined below

$$N_h f(\sigma) = \#_h(\sigma) f(\sigma), \quad f\in\mathrm{Dom}\,N_h,$$

and

$$\mathrm{Dom}\,N_h = \{f\in L^2(\Gamma;\eta) \mid \int_\Gamma (\#_h(\sigma))^2 |f(\sigma)|_\eta^2 \mathrm{d}\sigma < +\infty\}.$$

There $\#_h(\sigma) = \sum_{s\in\sigma} h(s)$.

Definition 6 [9]. The number operator N in $L^2(\Gamma;\eta)$ defined as

$$Nf(\sigma) = \#\sigma f(\sigma), \quad f\in\mathrm{Dom}\,N,$$

with

$$\mathrm{Dom}\,N = \{f\in L^2(\Gamma;\eta) \mid \int_\Gamma (\#\sigma)^2 |f(\sigma)|_\eta^2 \mathrm{d}\sigma < +\infty\}.$$

Remark 2 [10]. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$.

(1) If

$$\omega(s,t) = \begin{cases} h(s), & s=t,\\ 0, & s\neq t.\end{cases}$$

then $\nu_\omega(\sigma) = \#_h(\sigma)$ and $S_\omega = N_h$.

(2) If

$$\omega(s,t) = \begin{cases} h(s)\equiv 1, & s=t,\\ 0, & s\neq t.\end{cases}$$

then $\nu_\omega(\sigma) = \#\sigma$ and $S_\omega = N$.

III. MAIN RESULTS

In the present section, we will prove S_ω is the linear operator of unbounded densely defined self adjoint in $L^2(\Gamma;\eta)$, but when $\sup_\sigma \nu_\omega(\sigma) < +\infty$, S_ω is a bounded linear operator. Next, we obtains the following two representations of S_ω and related conclusions.

Theorem 1. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$, If $\sup_{s,t\geq 0}\omega(s,t) < +\infty$, then S_ω be a linear densely defined and unbounded operator in $L^2(\Gamma;\eta)$.

Proof: Firstly, we will prove that the S_ω is densely defined operator in $L^2(\Gamma;\eta)$.

$$\alpha = \sup_{s,t\geq 0}\omega(s,t) < +\infty.$$

For $\forall n\geq 0, \forall f\in L^2(\Gamma^{(n)};\eta)$,

$$\begin{aligned} S_\omega f(\sigma) &= S_\omega \mathbf{1}_{\Gamma^{(n)}}(\sigma) f(\sigma) = \mathbf{1}_{\Gamma^{(n)}}(\sigma) S_\omega f(\sigma) \\ &= \mathbf{1}_{\Gamma^{(n)}}(\sigma)\nu_\omega(\sigma) f(\sigma) \leq \alpha n^2 f(\sigma), \end{aligned}$$

and

$$\begin{aligned} \|S_\omega f\|^2 &= \int_\Gamma |S_\omega f(\sigma)|_\eta^2 \mathrm{d}\sigma \leq \int_\Gamma \alpha^2 n^4 |f(\sigma)|_\eta^2 \mathrm{d}\sigma \\ &= \alpha^2 n^4 \|f\|^2 < +\infty, \qquad (1) \end{aligned}$$

which implies that $f\in\mathrm{Dom}\,S_\omega$. And because the algebraic direct sum $\oplus_{n=0}^\infty L^2(\Gamma^{(n)};\eta)$ of $\{L^2(\Gamma^{(n)};\eta); n\geq 0\}$ are densely defined linear subspace of $L^2(\Gamma;\eta)$, hence S_ω is densely defined operator. Next, we prove that the S_ω is unbounded operator. By (1)

$$\|S_\omega f\| \geq \sup_{f\in L^2(\Gamma;\eta),\|f\|=1} \|S_\omega f\| = \alpha n^2 \to \infty (n\to\infty),$$

then we know that the S_ω is unbounded.

Theorem 2. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$, If $\sup_\sigma v_\omega(\sigma) < +\infty$, then S_ω is a bounded operator in $L^2(\Gamma;\eta)$, and

$$\|S_\omega\| = \sup_\sigma v_\omega(\sigma)\cdot$$

Proof: Hypothesis $\beta = \sup_\sigma v_\omega(\sigma)$, for $\forall f\in \mathrm{Dom}\, S_\omega$,

$$\begin{aligned}\|S_\omega f\|^2 &= \int_\Gamma (v_\omega(\sigma))^2 |f(\sigma)|_\eta^2 \,\mathrm{d}\sigma\\ &\leq \beta^2 \int_\Gamma |f(\sigma)|_\eta^2 \,\mathrm{d}\sigma\\ &= \beta^2 \|f\|^2 < \infty.\end{aligned}$$

Hence S_ω is a bounded operator and $\|S_\omega\| \leq \beta$.

On the other hand, for $\forall f\in \mathrm{Dom}\, S_\omega$,

$$v_\omega(\sigma)\|f(\sigma)\| = \|v_\omega(\sigma)f(\sigma)\| = \|S_\omega f(\sigma)\| \leq \|S_\omega\|\|f(\sigma)\|,$$

which implies that $\sup_\sigma v_\omega(\sigma) \leq \|S_\omega\|$. In conclusion

$$\|S_\omega\| = \sup_\sigma v_\omega(\sigma)\cdot$$

Theorem 3. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$. In the weak sense, the two-dimensional weighted number operator S_ω in $L^2(\Gamma;\eta)$ can be expressed as follows

$$S_\omega = \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t \,\mathrm{d}s\,\mathrm{d}t. \tag{2}$$

Proof: Fist of all, we prove that integral

$$\int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\left\langle\left\langle \widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t f, g\right\rangle\right\rangle \mathrm{d}s\,\mathrm{d}t \tag{3}$$

exists. For $\forall f,g\in \mathrm{Dom}\, S_\omega, \forall s\in \mathbf{R}_+$,

$$\int_\Gamma (v_\omega(\sigma))^2 |f(\sigma)|_\eta \,\mathrm{d}\sigma < +\infty, \int_\Gamma (v_\omega(\sigma))^2 |g(\sigma)|_\eta \,\mathrm{d}\sigma < +\infty,$$

$$\begin{aligned}\widetilde{\nabla}_s^*\widetilde{\nabla}_s f(\sigma) &= \mathbf{1}_\sigma(s)\widetilde{\nabla}_s f(\sigma\setminus s)\\ &= \mathbf{1}_\sigma(s)\left(1-\mathbf{1}_{\sigma\setminus s}(s)\right)f(\sigma\setminus s\cup s) = \mathbf{1}_\sigma(s)f(\sigma).\end{aligned}$$

Because of Cauchy-Schwartz inequality,

$$\begin{aligned}&\left|\int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\left\langle\left\langle \widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t f, g\right\rangle\right\rangle \mathrm{d}s\,\mathrm{d}t\right|\\ &\leq \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \left|\omega(s,t)\left\langle\left\langle \widetilde{\nabla}_t^*\widetilde{\nabla}_t f, \widetilde{\nabla}_s^*\widetilde{\nabla}_s g\right\rangle\right\rangle\right| \mathrm{d}s\,\mathrm{d}t\\ &= \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \left|\int_\Gamma \omega(s,t)\mathbf{1}_\sigma(s)\mathbf{1}_\sigma(t)\langle f(\sigma),g(\sigma)\rangle_\eta \,\mathrm{d}\sigma\right| \mathrm{d}s\,\mathrm{d}t\\ &= \int_\Gamma \sum_{s\in\sigma}\sum_{t\in\sigma} \omega(s,t)\left|\langle f(\sigma),g(\sigma)\rangle_\eta\right| \mathrm{d}\sigma\\ &= \int_\Gamma v_\omega(\sigma)\left|\langle f(\sigma),g(\sigma)\rangle_\eta\right| \mathrm{d}\sigma\\ &\leq \int_\Gamma v_\omega(\sigma)|f(\sigma)|_\eta |g(\sigma)|_\eta \,\mathrm{d}\sigma\\ &\leq \left\{\int_\Gamma (v_\omega(\sigma))^2 |f(\sigma)|_\eta^2 \,\mathrm{d}\sigma\right\}^{\frac{1}{2}} \left\{\int_\Gamma (v_\omega(\sigma))^2 |g(\sigma)|_\eta^2 \,\mathrm{d}\sigma\right\}^{\frac{1}{2}}\\ &< +\infty.\end{aligned}$$

Hence, the integral (3) exists.

Next, for $\forall f\in \mathrm{Dom}\, S_\omega, \forall g\in L^2(\Gamma;\eta)$, have

$$\begin{aligned}&\int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\left\langle\left\langle \widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t f, g\right\rangle\right\rangle \mathrm{d}s\,\mathrm{d}t\\ &= \int_{\mathbf{R}_+}\int_{\mathbf{R}_+}\int_\Gamma \omega(s,t)\mathbf{1}_\sigma(s)\mathbf{1}_\sigma(t)\langle f(\sigma),g(\sigma)\rangle_\eta \,\mathrm{d}\sigma\,\mathrm{d}s\,\mathrm{d}t\\ &= \int_\Gamma \left\langle \sum_{s\in\sigma}\sum_{t\in\sigma} \omega(s,t)f(\sigma), g(\sigma)\right\rangle_\eta \mathrm{d}\sigma\\ &= \int_\Gamma \langle v_\omega(\sigma)f(\sigma),g(\sigma)\rangle_\eta \,\mathrm{d}\sigma\\ &= \langle\langle S_\omega f, g\rangle\rangle,\end{aligned}$$

which implies that (2) establishment.

Theorem 4. The two-dimensional weighted number operator S_ω is a self-adjoint operator in $L^2(\Gamma;\eta)$.

Proof: For $\forall f,g\in \mathrm{Dom}\, S_\omega$, ,

$$\begin{aligned}\langle\langle S_\omega f, g\rangle\rangle &= \left\langle\left\langle \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t f \,\mathrm{d}s\,\mathrm{d}t, g\right\rangle\right\rangle\\ &= \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \left\langle\left\langle \omega(s,t)\widetilde{\nabla}_s^*\widetilde{\nabla}_s\widetilde{\nabla}_t^*\widetilde{\nabla}_t f, g\right\rangle\right\rangle \mathrm{d}s\,\mathrm{d}t\\ &= \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \left\langle\left\langle f, \omega(s,t)\widetilde{\nabla}_t^*\widetilde{\nabla}_t\widetilde{\nabla}_s^*\widetilde{\nabla}_s g\right\rangle\right\rangle \mathrm{d}s\,\mathrm{d}t\\ &= \left\langle\left\langle f, \int_{\mathbf{R}_+}\int_{\mathbf{R}_+} \omega(s,t)\widetilde{\nabla}_t^*\widetilde{\nabla}_t\widetilde{\nabla}_s^*\widetilde{\nabla}_s g \,\mathrm{d}s\,\mathrm{d}t\right\rangle\right\rangle\\ &= \langle\langle f, S_\omega g\rangle\rangle.\end{aligned}$$

which implies that $S_\omega^* = S_\omega$, namely S_ω is self-adjoint operator.

Theorem 5. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$. If for $\forall s,t\in\mathbf{R}_+, \omega(s,t)\equiv c$, then two-dimensional weighted number operator S_ω have spectrum decomposition in $L^2(\Gamma;\eta)$, namely

$$S_\omega = \sum_{n=1}^\infty cn^2 J_n.$$

There J_n is the orthogonal projection operator from $L^2(\Gamma;\eta)$ to its linear subspace $L^2(\Gamma^{(n)};\eta)$, namely

$$\begin{aligned}J_n : L^2(\Gamma;\eta) &\to L^2(\Gamma^{(n)};\eta)\\ f &\mapsto \mathbf{1}_{\Gamma^{(n)}}(\cdot)f(\cdot).\end{aligned}$$

Proof: According to Lemma 1 and the proof of Theorem 1, it can be inferred that $\{L^2(\Gamma^{(n)};\eta), n\geq 1\}$ are linear subspace of $L^2(\Gamma;\eta)$ and Algebra in Direct Sums $\oplus_{n=1}^\infty L^2(\Gamma^{(n)};\eta)$ is linear densely defined space, and

$$\oplus_{n=1}^\infty L^2(\Gamma^{(n)};\eta) \subset \mathrm{Dom}\, S_\omega$$

For $f\in L^2(\Gamma^{(n)};\eta)$

$$\begin{aligned}S_\omega f(\sigma) = v_\omega(\sigma)f(\sigma) &= \sum_{s\in\sigma}\sum_{t\in\sigma} cf(\sigma) = c(\#\sigma)^2 f(\sigma)\\ &= c(\#\sigma)^2 \mathbf{1}_{\Gamma^{(n)}}(\sigma)f(\sigma) = cn^2 f(\sigma).\end{aligned}$$

Hence, for $\forall f\in L^2(\Gamma;\eta)$,

$$\int_\Gamma c^2 n^4 |f(\sigma)|_\eta^2 \,\mathrm{d}\sigma = \int_\Gamma c^2 n^4 \sum_{n=1}^\infty \left|\mathbf{1}_{\Gamma^{(n)}}(\sigma)f(\sigma)\right|_\eta^2 \mathrm{d}\sigma$$

Next, by Definition 4, when $\omega(s,t)\equiv c$,

$$= \sum_{n=1}^\infty \int_\Gamma c^2 n^4 |J_n f(\sigma)|_\eta^2 \,\mathrm{d}\sigma = \sum_{n=1}^\infty c^2 n^4 \|J_n f\|.$$

$$\mathrm{Dom}\, S_\omega = \{ f \in L^2(\Gamma;\eta) \mid \sum_{n=1}^{\infty} c^2 n^4 \|J_n f\| < +\infty \},$$

therefore, for $\forall f \in \mathrm{Dom}\, S_\omega$, $\sum_{n=1}^{\infty} c^2 n^4 \|J_n f\|$ is convergent.

For $m > n$, have

$$\left\| \sum_{k=1}^{m} c^2 k^4 J_k f - \sum_{k=1}^{n} c^2 k^4 J_k f \right\|^2 = \sum_{k=n+1}^{m} c^2 k^4 \|J_k f\|^2 \to 0 (m, n \to 0).$$

Namely, $\sum_{n=1}^{\infty} cn^2 J_n f$ is convergent in $L^2(\Gamma;\eta)$.

For $\forall f \in L^2(\Gamma;\eta)$,

$$\left\langle\left\langle \sum_{n=1}^{\infty} cn^2 J_n f, g \right\rangle\right\rangle = \int_\Gamma \sum_{n=1}^{\infty} cn^2 \langle J_n f(\sigma), g(\sigma) \rangle_\eta \, \mathrm{d}\sigma$$

$$= \int_\Gamma \sum_{n=1}^{\infty} cn^2 \mathbf{1}_{\Gamma^{(n)}}(\sigma) \langle f(\sigma), g(\sigma) \rangle_\eta \, \mathrm{d}\sigma$$

$$= \int_\Gamma \sum_{n=1}^{\infty} c(\#\sigma)^2 \langle f(\sigma), g(\sigma) \rangle_\eta \, \mathrm{d}\sigma$$

$$= \int_\Gamma \langle v_\omega(\sigma) f(\sigma), g(\sigma) \rangle_\eta \, \mathrm{d}\sigma$$

$$= \langle\langle S_\omega f, g \rangle\rangle$$

which implies that $S_\omega = \sum_{n=1}^{\infty} cn^2 J_n$.

Theorem 6. $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$. $M = \{L^2(\Gamma^{(n)};\eta), n \geq 1\}$ is a core of two-dimensional weighted number operator S_ω, i.e.

$$M \subset S_\omega \text{ and } S_\omega \mid M = S_\omega.$$

Proof: According to the proof of Theorem 5, it can be inferred that $M \subset \mathrm{Dom}\, S_\omega$.

If $\xi_0 \in \mathrm{Dom}\, S_\omega, \xi_n \in M$, and $\xi_n \to \xi_0 (n \to \infty)$, then

$$\|S_\omega \xi_0 - S_\omega \xi_n\|^2$$

$$= \int_\Gamma \left| v_\omega(\sigma)\xi_0(\sigma) - v_\omega(\sigma) \mathbf{1}_{\Gamma^{(n)}}(\sigma) \xi_n(\sigma) \right|_\eta^2 \mathrm{d}\sigma$$

$$= \int_{\Gamma \setminus \Gamma^{(n)}} (v_\omega(\sigma))^2 |\xi_0(\sigma)|_\eta^2 \, \mathrm{d}\sigma$$

$$= \int_\Gamma (v_\omega(\sigma))^2 |\xi_0(\sigma)|_\eta^2 \, \mathrm{d}\sigma - \int_{\Gamma^{(n)}} (v_\omega(\sigma))^2 |\xi_n(\sigma)|_\eta^2 \, \mathrm{d}\sigma$$

$$\to 0 (n \to \infty),$$

which implies that $S_\omega \mid M = S_\omega$ and the image of $S_\omega \mid M = S_\omega$ is densely, i.e. M is core of two-dimensional weighted number operator S_ω.

□

Theorem 7. If $\omega(s,t)$ is nonnegative real function on $\mathbf{R}_+^2$ and $v_\omega(\sigma)$ is bounded, then $\mathrm{Dom}\, N$ is a core of S_ω. In particular, for bounded nonnegative real function $h(s)$ on $\mathbf{R}_+$, $\mathrm{Dom}\, N$ is a core of N_h.

Proof: For $\alpha = \sup_{s,t \geq 0} \omega(s,t) < +\infty$, have

$$v_\omega(\sigma) = \sum_{s \in \sigma} \sum_{t \in \sigma} \omega(s,t) \leq \alpha (\#(\sigma))^2 < +\infty.$$

For $\forall f \in \mathrm{Dom}\, N$, $\int_\Gamma (\#\sigma)^2 |f(\sigma)|_\eta^2 \, \mathrm{d}\sigma < +\infty$. Hence,

$$\int_\Gamma (v_\omega(\sigma))^2 |f(\sigma)|_\eta^2 \, \mathrm{d}\sigma \leq \int_\Gamma \alpha^2 (\#(\sigma))^4 |f(\sigma)|_\eta^2 \, \mathrm{d}\sigma$$

$$= \int_\Gamma \alpha^2 (\#(\sigma))^2 (\#(\sigma))^2 |f(\sigma)|_\eta^2 \, \mathrm{d}\sigma < +\infty.$$

Namely $f \in \mathrm{Dom}\, S_\omega$. For $\forall n \geq 0, \forall f \in L^2(\Gamma^{(n)};\eta)$, have

$$f(\sigma) = \mathbf{1}_{\Gamma^{(n)}}(\sigma) f(\sigma),$$

$$\|Nf\|^2 = \int_\Gamma |Nf(\sigma)|_\eta^2 \, \mathrm{d}\sigma = \int_\Gamma |nf(\sigma)|_\eta^2 \, \mathrm{d}\sigma$$

$$= n^2 \int_\Gamma |f(\sigma)|_\eta^2 \, \mathrm{d}\sigma \leq n^2 \|f\|^2 < +\infty,$$

which implies that $f \in \mathrm{Dom}\, N$, i.e.

$$\{L^2(\Gamma^{(n)};\eta), n \geq 1\} \subset \mathrm{Dom}\, N.$$

According to Theorem 6, $\mathrm{Dom}\, N$ is a core of S_ω. Similarly, it can be inferred that $\mathrm{Dom}\, N$ is a core of N_h.

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