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Mathematical Model of Air Flow Simulation for Office Working Cabin

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Abstract— **Air flow greatly affects the productivity of office workers. Calculating the design of a working cabin with thermal comfort needs to have input parameters of ambient temperature, air flow rate in the room, percentage of humid air and temperature of surfaces around the room. From the input parameters will directly affect the thermal sensation of the worker, when the input parameter changes, it will lead to a change in the thermal sensation of the worker. Building an air flow simulation model for the office cabin is essential. The article presents the mathematical basis to solve the mathematical simulation model and provides an air flow simulation model for the working cabin in Vietnam.**

Index Terms— **Working cabin, Thermal comfort, Simulation model, Scientific basis**

I. Introduction

Air flow greatly affects the productivity of office workers. In order to calculate the design of a comfortable working cabin, it is necessary to have environmental input parameters. From the input parameters will directly affect the thermal sensation of the worker, when the input parameter changes, it will lead to a change in the thermal sensation of the worker. The simulation model uses numerical methods combined with computer simulation technology to solve problems related to air flow factors of the environment. The physical aspects of any fluid flow are controlled by three basic principles [1]:

- Conservation of mass
- Conservation of momentum F = ma (Newton's 2nd Law)
- Conserve energy

These fundamental principles can be expressed as terms of mathematical equations, the most general of which are partial differential equations. In fluid dynamics calculus replace the dominant partial differential equations for the fluid flow t with numbers and feed these numbers into space or time to obtain a final numerical description of the flow field. sufficient to pay attention [2],[3],[4].

II. RESEARCH METHOD

2.1. Flow Modeling

Consider the motion of an infinitely small liquid particle that moves along with the current as shown in Figure 1.

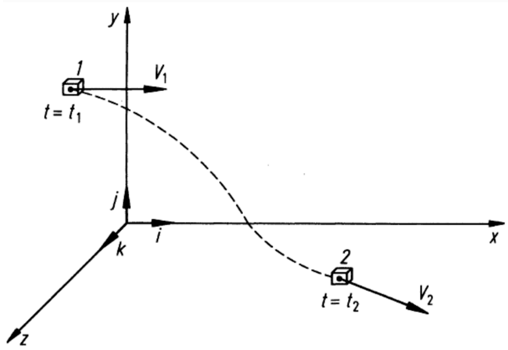

Figure 1. Moving fluid element in a flow field [5]

The fluid element considered moves in Cartesian space. The unit vectors along the x, y and z axes are $\vec{i}$, $\vec{j}$, $\vec{k}$ respectively. The velocity vector field in this Descartes space is equal to:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \qquad (2.1)$$

where the x, y and z components of the given velocity correspond to

$u = u(x, v, z, t)$
$v = v(x, v, z, t)$
$w = w(x, v, z, t)$

where u, v, w are functions of both space and time t.

The scalar density field is expressed by the formula

$$\rho = \rho(x,y,z,t) \qquad (2.2)$$

At time t_1, the liquid element is located at point 1 in figure 1. At this point and this time, the density of the liquid element is:

$$\rho_1 = \rho_1(x_1, y_1, z_1, t_1) \qquad (2.3)$$

At later time t_2, the liquid element has moved to point 2 in figure 1. The density of this liquid element is:

$$\rho_2 = \rho_2(x_2, y_2, z_2, t_2) \qquad (2.4)$$

Expand on Taylor series the function ρ = (x,y,z,t) around point 1:

$$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1 (x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1 (y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1 (z_2 - z_1) + \left(\frac{\partial \rho}{\partial t}\right)_1 (t_2 - t_1) + \ldots \qquad (2.5)$$

Divide by $(t_2 - t_1)$ and discard the higher order terms we get:

$$\frac{p_2 - p_1}{t_2 - t_1} = \left(\frac{\partial \rho}{\partial x}\right)_1 \left(\frac{x_2 - x_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial y}\right)_1 \left(\frac{y_2 - y_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial z}\right)_1 \left(\frac{z_2 - z_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial t}\right)_1 \quad (2.6)$$

Examine the left side of equation (2.6). Physically this is the time-average density variation of the liquid element as it moves from point 1 to point 2 in the limit, as t_2 approaches t_1 this term becomes:

$$\lim_{t_2 - t_1} \left(\frac{p_2 - p_1}{t_2 - t_1}\right) = \frac{D\rho}{Dt} \quad (2.7)$$

Where, ($\frac{D\rho}{Dt}$) is the time density variation of the given liquid element as it moves through space. Unlike ($\frac{D\rho}{Dt}$), ($\frac{\partial \rho}{\partial t}$) is the rate of change over time of the density of the liquid at the fixed point 1.

In Equation (2.6) we see that:

$$\lim_{t_2 - t_1} \left(\frac{x_2 - x_1}{t_2 - t_1}\right) = u \quad (2.8)$$

$$\lim_{t_2 - t_1} \left(\frac{y_2 - y_1}{t_2 - t_1}\right) = v \quad (2.9)$$

$$\lim_{t_2 - t_1} \left(\frac{z_2 - z_1}{t_2 - t_1}\right) = w \quad (2.10)$$

Thus, taking the limit of equation (2.6) when t_2 approaches t_1 is

$$\frac{D\rho}{Dt} = u\left(\frac{\partial p}{\partial x}\right) + v\left(\frac{\partial p}{\partial y}\right) + w\left(\frac{\partial p}{\partial z}\right) + \left(\frac{\partial p}{\partial t}\right) \quad (2.11)$$

Investigating equation (2.10) we can get the expression for the physical derivative in Descartes coordinates:

$$\frac{D}{Dt} = u\left(\frac{\partial}{\partial x}\right) + v\left(\frac{\partial}{\partial y}\right) + w\left(\frac{\partial}{\partial z}\right) + \left(\frac{\partial}{\partial t}\right) \quad (2.12)$$

In there:

$\frac{D}{Dt}$: Physical derivative is the rate of change over time of a moving fluid element.

$\frac{\partial}{\partial t}$: The partial derivative is the time-varying rate of the liquid at a fixed time.

$\vec{V}\ \nabla$: The convection derivative is the rate of change over time due to the motion of the fluid element from one position to another in the flow line. With the vector operator ∇ is calculated as follows:

$$\nabla = \vec{i}\left(\frac{\partial}{\partial x}\right) + \vec{j}\left(\frac{\partial}{\partial y}\right) + \vec{k}\left(\frac{\partial}{\partial z}\right) \quad (2.14)$$

The physical derivative applies to any stream field variable, for example $\frac{D\rho}{Dt}$, $\frac{DT}{Dt}$, $\frac{Du}{Dt}$, … where ρ and T are the hydrostatic pressure and heat corresponding degree.

III. RESEARCH RESULTS

Given that the air flow in the room is determined to be below minus M < 0.3, the assumption that the gas is incompressible can be accepted. Then, the aerodynamic problem is left with only two equations: the equation of continuity and the equation of conservation of momentum.

Then the equation of conservation of mass is written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.15)$$

We can write it in short form:

$$\partial_i u_i = 0 \quad (2.16)$$

The equation for conservation of momentum in the (x, y, z) coordinate system is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = F_x - \frac{1}{\rho}\frac{\partial \rho}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (2.17)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = F_y - \frac{1}{\rho}\frac{\partial \rho}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \quad (2.18)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = F_z - \frac{1}{\rho}\frac{\partial \rho}{\partial z} + v\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (2.19)$$

Where: F_x, F_y, F_z are the components of the volumetric force.

Neglecting the volumetric force (due to the very small density of air), we can write the equation in tensor form as follows:

$$\partial_i u_i + u_j \partial_i u_i = \frac{1}{\rho}\partial_i p + v\partial_j \partial_j u_i \quad (2.20)$$

To simulate the air flow in the working cabin area, we have the system of equations

$$\partial_i u_i = 0 \quad (2.21)$$

$$\partial_i u_i + u_j \partial_i u_i = \frac{1}{\rho}\partial_i p + v\partial_j \partial_j u_i \quad (2.22)$$

It can be seen that the above system of equations consists of 4 partial differential equations with 4 unknowns: pressure p and 3 velocity components in 3 directions (u, v, w). In principle, this is a closed double problem because the system has 4 equations with 4 unknowns. We use approximate methods to calculate the solution of the above system of equations.

IV. CONCLUSION

To build a mathematical model to simulate the air flow for the working cabin, it is necessary to have given input and boundary conditions. Based on the proven conservation laws, we can determine the system of equations representing the correlation between the parameters of the air flow. After building a system of algebraic equations to simulate the comfortable conditions of the working cabin, we will use the differential methods of partial differential equations and the finite volume method to determine the parameters of changes in the environment under different conditions.

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