Effect of Vibration Mode on Material Damping

Kunihiko Ishihara

Abstract— **In turbo-machineries such as a turbine and a compressor, the vibrations of blades which are central parts, have been become problem. The causes are the resonance of blade due to an inlet distortion, a wake of preceding blade row and a potential interference of succeeding blade row, and a self-excited vibration like a flutter. In general, it is said that the material damping can only be expected in the high centrifugal force field. Fortunately, this time we have a chance to carry out the rotating test of the long blade and we obtained the logarithmic decrement of the first to the sixth modes. According to the results, it was clarified that the mode order gave effects to the logarithmic decrement little. Then in this study, it will be presented that the experimental fact is reasonable by using the concept of specific damping.**

***Index Terms*— Logarithmic decrement, Damping ratio, Material damping, Vibration mode**

I. INTRODUCTION

In turbo-machineries such as a turbine and a compressor, the vibrations of blades which are central parts, have been become problem. The causes are the resonance of blade due to an inlet distortion, a wake of preceding blade row and a potential interference of succeeding stator blade, and a self-excited vibration like a flutter. Especially in compressor blades with the deceleration blade row, a rotating stall has been become problem, which often occurs in a low flow rate region.

In any case, the adding of damping is an effective countermeasure for the low vibration possible but the material damping can be expected only in the high centrifugal force field. Therefore, it becomes important to evaluate the material damping for confirming the safety of the blade.

The author proposed the evaluation method of the logarithmic decrement of the blade under the centrifugal force field based on a concept of specific damping [1]. And it showed that the analytical result was in comparatively agreement with the experimental result. However, only the damping of the first bending mode of the blade was evaluated and it did not refer the those of higher modes. And other investigations refer only the first bending mode.

Therefore, we had used the logarithmic damping of the first bending mode for the those of higher modes in conventional.

Fortunately, this time we have a chance to carry out the rotating test of the long blade and we obtained the logarithmic decrement of the first to the sixth modes although the number of data was small.

Kunihiko Ishihara, Department of Health and Welfare, Tokushima Bunri University, Shido, Sanuki-city, Kagawa, Japan, +81878997247.

According to the results, it was clarified that the mode order gave effects to the logarithmic decrement little. Then in this study, it will be presented that the experimental fact is reasonable by using the concept of specific damping.

II. EXPERIMENTAL EQUIPMENT AND METHOD

Figure1(a) and figure2 show the outlines of the rotation tester and the test equipment for obtaining the logarithmic decrement of test blades, respectively. The long blade is shown in Fig.1 (b) and it has 33inch length.

The vibration measurement was performed by using the strain gauge attached the surface of blade root. The measurement data were recorded to the data recorder through the slip ring. The rotor blade is excited by jet air from nozzles put in a vacuum tank.

Here planting blades are not done for all around, the only five test blades are planted to the rotor and they are rotated and measured vibrations. Of course the balancing is done by attaching the counter weight at the opposite side.

The rotor is driven by a motor through the accelerator and the blade tips are excited by jet air from twelve nozzles put at all around. This exciting force is periodic. The vibration was measured by the strain gauge attached at the blade surface.

Initially, the damping ratio was planning to be obtained from a free damped vibration which occurs when the exciting force is removed at the resonance state. However, the policy was changed because the vacuum holding of the test rig had been difficult. The resonance curve was obtained by measuring the vibration data in decent of rotation and tracking process was performed. It will be shown in Appendix A how the sampling parameter was determined.

Figure3 is the Campbell diagram obtained in this experiment. The real line in this figure shows the natural frequency of the blade. The measuring result coincided with the analytical results by FEM. The size of the circle shows the magnitude of vibration due to the rotation tester.

III. MEASUREMENT RESULT AND CONSIDERATION

The measuring results of the logarithmic decrement is shown in figure 4.

A. Effect of Rotating Speed

Figure 4 shows the damping ratios (ceta1～ceta6) of the first mode to the sixth mode to the rotating speed. They are represented by %.

From this figure, the damping in the low rotating speed is large and the tendency can be seen that the damping ratio calm down constantly over 1500rpm of rotation as shown in the red ellipse.

It can be considered that as centrifugal force is small when the rotating speed is small the degree of sticking is small and the structural damping is easy to appear.

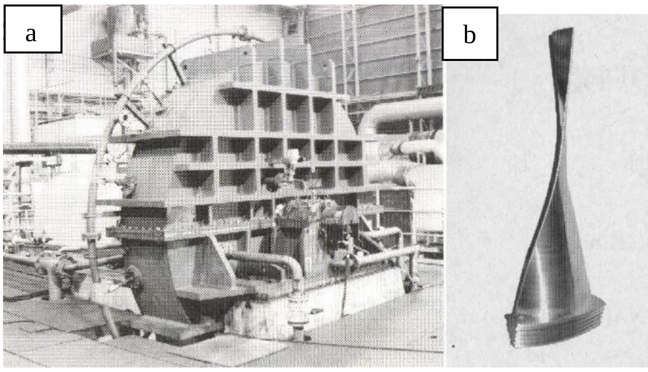


Fig.1 Overview of test rig and long blade

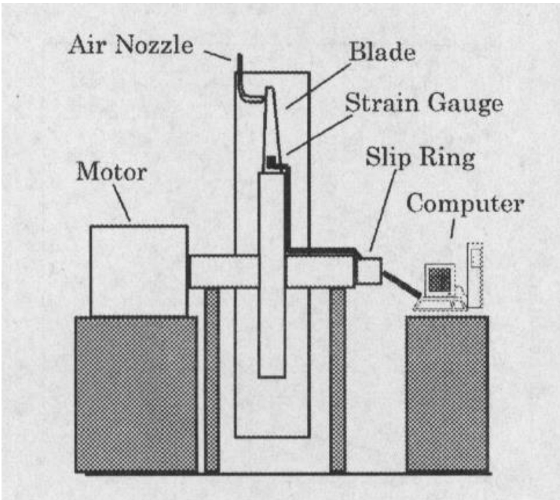


Fig.2 Schematic of test rig

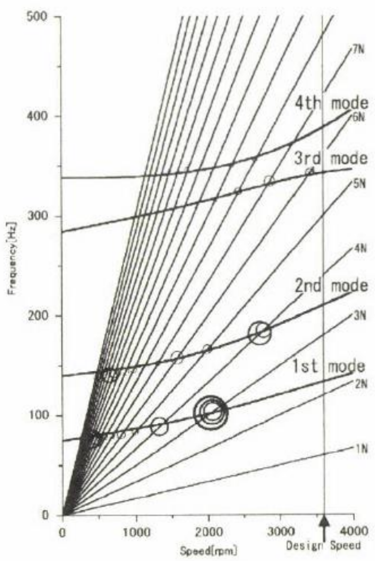


Fig.3 Campbell diagram of this experiment

On the other hand, the centrifugal force becomes large when the rotating speed is higher and the disk and the blade root becomes one, the only material damping appears.

This tendency is the same as the figure 8 which is the result of the material damping of the test blades obtained by using the test rig as shown in the figure 5.

Figure 5 shows the test rig to obtain the damping ratio of the test blades. By using this test rig we can obtain the logarithmic decrement of the blade. At this time the blade root is fixed by the pressing force equivalent of the centrifugal force.

When it is removed after giving 0.6mm displacement forcibly at the blade tip we can obtain the free damped vibration as shown in the figure 6. Figure 7 stretches the time axis of figure 6. The natural frequency can be obtained by using the figure 7. The natural frequency of this blade is 412Hz.

We got the logarithmic decrement of the mean value 0.001165 (See Appendix B) and 0.001174 due to the approximate curve. The both are almost the same value. Then it can be said that this value is considered to be correct.

Figure 8 shows the damping ratio of five test blades for the various pressing forces. The damping ratio is large when the pressing force is small and becomes smaller and calm down constantly when the pressing force becomes larger.

The result of damping in the present rotating test as shown in figure 4 also coincides with the tendency of the figure 8. Therefore, this result is said to be reasonable.

Figure 9 represents the logarithmic decrement for the vibration stress. Actually, the material damping is dependent on the vibrational stress and the centrifugal force. In general, the damping ratio becomes larger when the vibrational stress is larger and the centrifugal force is smaller. However, I was not able to obtain the many data as you can see the event in this test. From this measuring result, the damping ratio is about 0.1% and the logarithmic decrement is about 0.006. This value is the same as one of the figure 8 (a) in the reference [1].

This value can be applied for evaluation of the resonant stress.

B. Effect of Mode Order

Figure 4 and Figure9 show the damping ratio of the first to sixth modes of the test long blade. As far as I see this it can be said that the damping ratio is independent of the mode order.

In previous experiment obtaining the damping ratio, the first mode damping ratio was only obtained. This time experiment obtaining the damping ratio of the first to sixth mode was the first time. Therefore, I thought that very important data were obtained.

It is very useful finding that the damping ratio is independent of the natural vibrational mode. Therefore, I will try to examine the theoretical consideration in the next chapter.

VI THEORETICAL STUDY

For the sake of simplicity, the blade is considered to be the cantilever with the uniform cross section. The logarithmic decrement for vibrational stress can be expressed by use of the specific damping as follows.

$$\delta(\sigma_{Vm}) = \frac{\int_V SD(\sigma_V)dV}{2\int_V \frac{\sigma_V^2}{2E}dV} \quad (1)$$

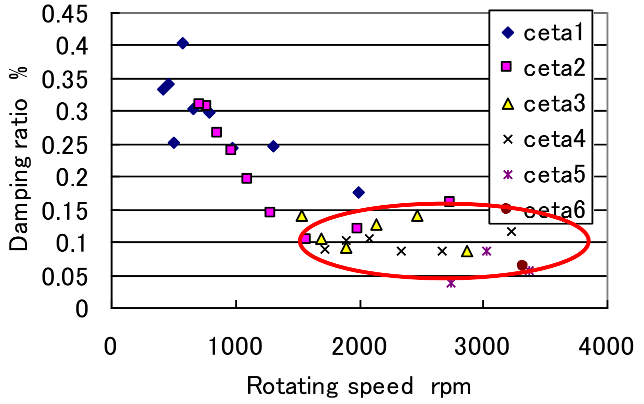


Fig.4 Logarithmic decrement due to rotating test

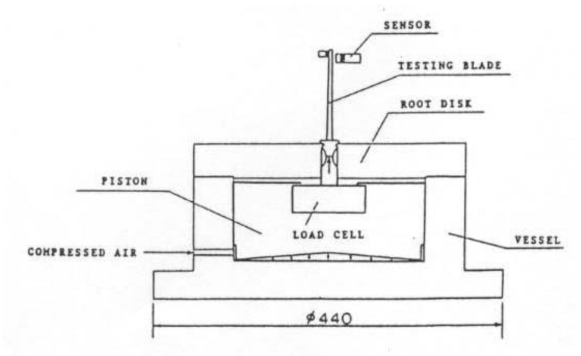


Fig.5 Test rig for obtaining logarithmic damping

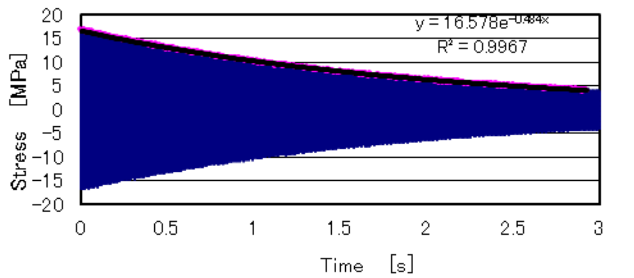


Fig.6 Free damped vibration of test blade

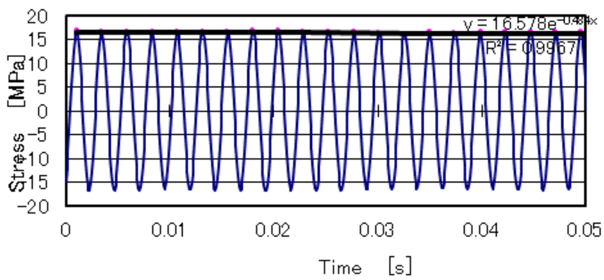


Fig.7 Free damped vibration of test blade enlarged time axis

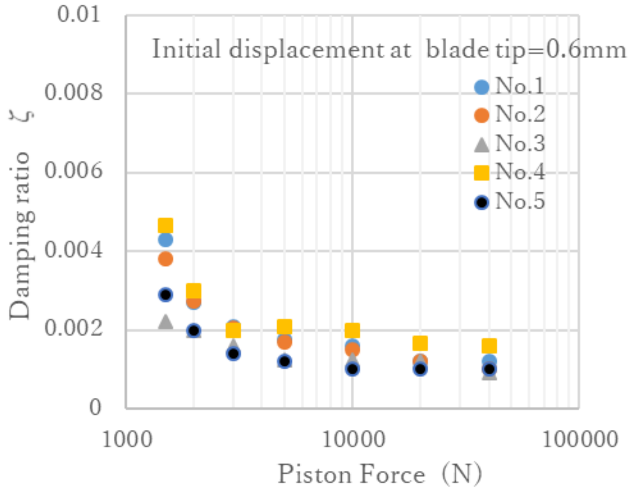


Fig.8 Result of logarithmic damping of test blades

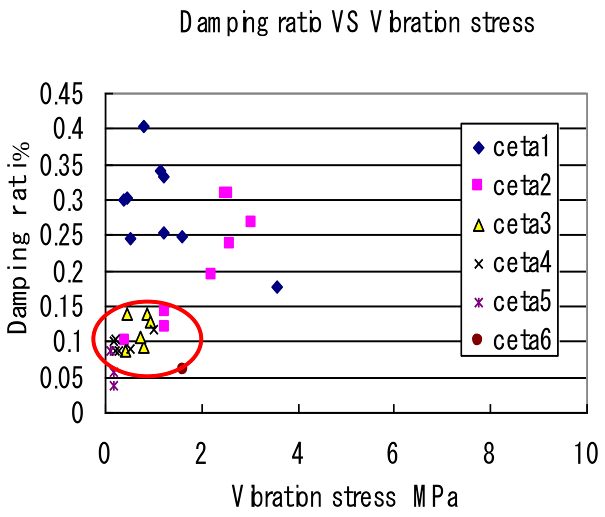


Fig.9 Logarithmic damping for vibration stress

Where σ_{Vm} is the maximum value of the vibrational stress and it indicates the stress of the surface of the blade root. And the specific damping of the material for structure in the comparative low stress level can be approximated as follows.

$$SD(\sigma_V) = J\sigma_V^n \quad (2)$$

Where J and n are constants corresponding to the materials.

In the reference [1], the following equation is given as the *SD*.

$$SD(\sigma_V) = 12150(\sigma_V / \sigma_e)^{2.4} \quad (3)$$

Where σ_e is the fatigue strength at the number of the iteration 2×10^7.

The vibrational stress of the present beam is given by the following equation. Where X is the non-dimensional coordinate of longitudinal direction and y is the distance from the neutral axis and T is the thickness of the beam.

$$\sigma_V = \sigma_{Vm}\cdot\frac{1}{T/2}\cdot|f(X)|\cdot|y| \quad (4)$$

Where $f(X)$ is a stress mode function and given as follows.

$$f(X) = \frac{1}{2}\left\{\cos\beta X + \cosh\beta X - \frac{\cos\beta + \cosh\beta}{\sin\beta + \sinh\beta}\cdot(\sin\beta X + \sinh\beta X)\right\} \quad (5)$$

Where β is eigen values and the those of the first to the fifth mode are as follows.

1st mode：1.875

2nd mode：4.694

3rd mode：7.855

4th mode：10.996

5th mode：14.137

Substituting Eq.(3) and Eq.(4) to Eq.(1) , the logarithmic decrement becomes as follows (See Appendix C).

$$\delta(\sigma_{Vm}) = 12150 \times E \times \sigma_e^{-2.4} \times \sigma_{Vm}^{0.4} \times \frac{f_{2.4}}{f_2} \quad (6)$$

Where

$$f_\alpha = \frac{1}{\alpha+1} \int_0^1 \left| f(X) \right|^\alpha dX \quad (7)$$

As σ_{Vm} is the vibrational stress we will examine the value of $f_{2.4}/f_2$ of Eq.(6) to know the relation between the vibrational stress and the logarithmic decrement. As a result, Table 1 was obtained.

Table 1 Values of $f_{2.4}/f_2$ for each mode

	$f_{2.4} / f_2$
1st mode	0.878165
2nd mode	0.836037
3rd mode	0.828808
4th mode	0.824568
5th mode	0.822325

Table 1 shows that the values of $f_{2.4}/f_2$ is considered to be constant regardless of the mode order. That is to say, it can be seen that the effect of the mode order on the logarithmic decrement is very small.

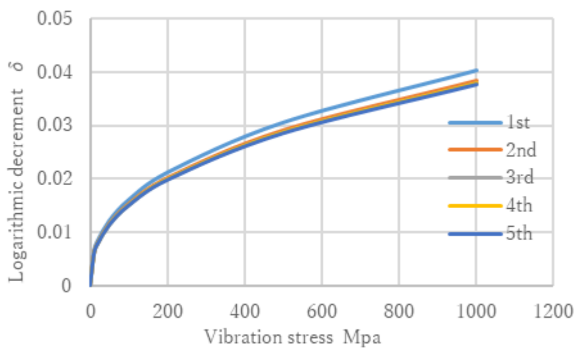


Fig.10 Logarithmic decrement based on specific damping theory

Predicting the logarithmic decrement by using this value Figure 10 was obtained. Therefore, it can be said that the material damping is unaffected by the mode order. It can be considered to be reasonable that the damping ratio is independent of the mode order.

VI CONCLUSIONS

In the rotating test of the long blade, the measurement of the damping ratio and the analysis by using the concept of specific damping were performed and discussed. As a result, the following findings could be obtained.

(1) The damping ratio at around rated rotation is about 0.1% and saturated over 1500rpm.
(2) The damping ratio is independent of the mode order. The fact is a new finding and very significant.
(3) The finding could be verified by using the specific damping theory.

However, I experienced that the logarithmic decrement was dependent on the mode order in the another experiment. Therefore, the concluded remark (2) is needed to confirm as true or false.

APPENDIX

A. Identification of analytical parameters

Denoting the rotating speed by N(rpm) and the order by n, the exciting frequency f_e is given as follows.

$$f_e = nN/60 \quad \text{(A-1)}$$

Denoting the natural frequency of each mode by f_k (k: mode order), the resonance rotating speed is $N = 60 f_k / n$ because of $f_e = f_k$. Therefore,

$$\Delta N = 60 \Delta f_k / n \quad \text{(A-2)}$$

The relation among the logarithmic decrement δ , the frequency width Δf_k and the rotation width ΔN_{rk} is

$$\Delta f_k = \frac{\delta}{\pi} f_k \quad \text{(A-3)}$$

Then, $\Delta N = \frac{\delta}{\pi} \frac{60}{n} f_k \quad rpm$ (A-4)

Therefore, it is desirable to sample every 1/10 of the rotation width. This is supposed that the resonance curve is painted by ten points

Example1) The first natural frequency 100Hz coincides with the third order component(n=3) of rotation at 2000rpm.

Assuming $\delta = 0.01$,

$$\Delta N = \frac{0.01}{\pi} \times \frac{60}{3} \times 100 = 6.37 rpm \quad \text{(A-5)}$$

Therefore, the increment of the rotation becomes 0.637rpm. In the case of the fifth order component(n=5) of rotation it becomes 0.383rpm.

Example2) The fifth natural frequency 560Hz coincides with the tenth order component(n=10) of rotation at 3360rpm.

Assuming $\delta = 0.01$,

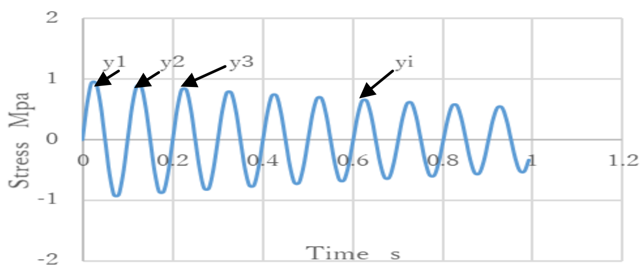
$$\Delta N = \frac{0.01}{\pi} \times \frac{60}{10} \times 560 = 10.7 rpm \quad \text{(A-6)}$$

Therefore, the increment of rotation becomes 1.07rpm.

Summing up above examination, we will obtain the following issues

In the resonance of the high natural frequency and the low rotation order, we obtain the wide rotation range in obtaining the half width and easy to process data. On the other hand, In the resonance of the low natural frequency and the high rotation order, we obtain the narrow rotation range in obtaining the half width and hard to process data.

B. How to obtain the logarithmic decrement



Above figure shows the free damped vibration. The peak values are denoted by $y_1, y_2, y_3, \cdots, y_i, \cdots$, respectively. The logarithmic decrement between adjacent peaks can be given as follows.

$$\delta_i = \ln \frac{y_i}{y_{i+1}} \qquad \text{(B-1)}$$

Therefore, we obtain the mean value as follows.

$$\bar{\delta} = \frac{1}{N}\sum_{i=1}^{N}\delta_i = \frac{1}{N}\left\{\ln\frac{y_1}{y_2} + \ln\frac{y_2}{y_3} + \ln\frac{y_3}{y_4} + \cdots \ \ln\frac{y_N}{y_{N+1}}\right\}$$
$$= \frac{1}{N}\ln\frac{y_1}{y_{N+1}} \qquad \text{(B-2)}$$

Appendix C

Derivation of Eq.(6)

$$\sigma_V = \sigma_{Vm}\cdot\frac{1}{T/2}\cdot|f(X)|\cdot|y|$$

$$SD(\sigma_V) = 12150(\sigma_V/\sigma_e)^{2.4}$$

Putting $X = x/l$ and $dV = b\,dy\,dx$, I will calculate each term of Eq.(1).

$$\int_V SD(\sigma_V)\,dV = \int_V 12150\left(\frac{\sigma_V}{\sigma_e}\right)^{2.4} b\,dy\,dx$$
$$= 12150\, b\sigma_e^{-2.4}\int_V \sigma_V^{2.4}\,dy\,dx$$
$$= 12150\, b\sigma_e^{-2.4}(\sigma_{Vm})^{2.4}\cdot\frac{1}{\left(\frac{T}{2}\right)^{2.4}}\cdot$$
$$2\int_0^{T/2}|y|^{2.4}dy\cdot l\int_0^1|f(X)|^{2.4}\,dX \qquad \text{(C-1)}$$

$$2\int_V \frac{\sigma_V^2}{2E}\,dV = \frac{\sigma_{Vm}^2}{E}\cdot\frac{1}{(T/2)^2}\,2b\int_0^{T/2}|y|^2 dy\cdot l\int_0^1|f(X)|^2\,dX \qquad \text{(C-2)}$$

The logarithmic decrement is represented as follows.

$$\delta(\sigma_{Vm}) = \frac{\int_V SD(\sigma_V)\,dV}{2\int_V \frac{\sigma_V^2}{2E}\,dV} \qquad \text{(C-3)}$$

Substituting Eq.(C-1) and Eq.(C-2) into Eq.(C-3)

$$\delta(\sigma_{Vm}) = \frac{12150\sigma_e^{-2.4}(\sigma_{Vm})^{2.4}\cdot\frac{1}{(T/2)^{2.4}}\cdot 2b\int_0^{T/2}|y|^{2.4}dy\cdot l\int_0^1|f(X)|^{2.4}\,dX}{\frac{\sigma_{Vm}^2}{E}\cdot\frac{1}{(T/2)^2}\,2b\int_0^{T/2}|y|^2dy\cdot l\int_0^1|f(X)|^2\,dX}$$

$$= \frac{12150\sigma_e^{-2.4}(\sigma_{Vm})^{2.4}\cdot\frac{1}{(T/2)^{2.4}}\cdot 2b\cdot\frac{1}{3.4}(T/2)^{3.4}\cdot l\int_0^1|f(X)|^{2.4}\,dX}{\frac{\sigma_{Vm}^2}{E}\cdot\frac{1}{(T/2)^2}\cdot 2b\cdot\frac{1}{3}(T/2)^3\cdot l\int_0^1|f(X)|^2\,dX}$$

$$= \frac{12150\sigma_e^{-2.4}(\sigma_{Vm})^{2.4}\cdot\frac{1}{(T/2)}\cdot 2b\cdot l\cdot\frac{1}{3.4}\int_0^1|f(X)|^{2.4}\,dX}{\frac{\sigma_{Vm}^2}{E}\cdot\frac{1}{(T/2)}\,2b\cdot l\cdot\frac{1}{3}\int_0^1|f(X)|^2\,dX}$$

Therefore Eq.(6) could be derived as follows.

$$\delta(\sigma_{Vm}) = 12150\cdot E\cdot\sigma_e^{-2.4}\cdot\sigma_{Vm}^{0.4}\cdot\frac{f_{2.4}}{f_2}$$

Where

$$f_\alpha = \frac{1}{\alpha+1}\int_0^1|f(X)|^\alpha\,dX$$

ACKNOWLEGEMENT

I would deeply appreciate Dr Naoto Sakai who belongs The Kawasaki Heavy Industries, LTD. for me to write this paper. He measured the vibration stress of the long blade in the rotating test and discussed the results.

REFERENCES

[1] K. Ishihara, H. Koike and H. Yano, “Estimation of Logarithmic Decrement of Rotor Blades under Centrifugal Force (Estimation Method by Using Specific Damping)”, *JSME*, Vol.64, No.624(1998), pp.2908-2914.
[2] K. Shiraki and N. Umemura, “Measuring Method of Vibration of Turbine Blade”, *Journal of the M. E. S. J.*, Vol.9, No.8(1974), pp.724-731.
[3] M. Nakamura, K. Kawakami and Y. Takano, “Study on Measurement Method of Internal Friction under acting Static Stress”, *Mitsubishi Juko Giho*, Vol.10, No.6(1973), pp.875-880.
[4] JSME, Report of Subcommittee of Research on Unsteady Problems of Cascade, JSME (1988), pp.57-64.
[5] M. Tanaka, A. Takehira, K. Kuwata, A. Hoshino, H. Koike, K. Ishihara, T. Isozaki and H. Yamasaki, “Study on Aerodynamic Force Exciting Vibration of Turbine Rotor Blade”, *JSME, IFToMM, Int. Conf. Rotordynamics* (1986), pp.307-312.
[6] B. J. Lazan, “Effect of Damping Constants and Stress Distribution on the Resonanse”, *Trans. ASME, J. Appl. Mech.*, Vol.20, No.2(1953), pp.201-209.
[7] W.C.Hagel and J.W.Clark, “The Specific Damping Energy of Fixed-Fixed Beam Specimens”, *Trans. ASME, J. Appl. Mech.*, Vol.24, No.3(1957), pp.426-430.
[8] E. R. Jerome, “Structural Damping”, (1960), 150, Pergamon Press.

Kunihiko Ishihara was born in 1947 in Kurashiki City, Okayama Prefecture Japan. He received the B.S. degree from Kobe University in 1969. He got a master’s degree in Kobe University in 1971 and earned the Ph.D. degree in Engineering from The Osaka University in 1986.

He worked in Kawasaki Heavy Industry Co. Ltd. as a Mechanical Engineer for 33 years. After that he became a Professor of The University of Tokushima in 2004. He had been studying the vibration and noise control, above all he studied the flow induced vibration and noise problems. He has authored or co-authored over 120 technical journal and over 50 conference papers. He is a fellow of JSME (Japan Society of Mechanical Engineers) now. He is a Professor of Tokushima Bunri University. He teaches a mechanical field subjects for students.