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View-Factor Calculation for Radiation Heat Transfer in Steam Boiler Furnaces

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Abstract— **The heat radiated by flames and absorbed by water in steam boiler furnaces strongly depends on the view-factor between the flames and water tubes. In the present study an analytical method is conducted and a simple expression of the view-factor for the case of direct radiation is given. The values of view-factor compared with the values of other methods found in the literature coincide very well, which means that the proposed solution proves to be a very useful tool for estimating the thermal energy transferred by radiation to water tubes.**

***Index Terms*—Boiler furnace, radiative heat, view-factor, water tube.**

I. Introduction

Modern power plant boilers utilize effectively the radiation released by the flames in the combustion chamber to heat the steam generating tubes. A high temperature in the combustion chamber is usually preferable in order to achieve better fuel combustion and to produce a greater amount of thermal energy. However, elevated temperatures carry risks and sometimes cause damage to water tubes, because the radiation heat flux entering the tubes is a critical factor for the safety of the tube metal. The radiation exchange in the combustion chamber depends on a geometric feature known as "*view-factor*" or "*configuration-factor*" or "*shape-factor*". The objective of this work is to investigate the view-factor that exists between the radiative surfaces in steam boiler furnaces. The surface that releases radiation is the flames and the surface that receives radiation is the tube outer surface and any refractory walls of the furnace. Therefore, the problem of direct radiation to the tubes focuses on determining the existing view-factor between the flames and tubes.

In the past, many scientists investigated the problem of radiation exchange between surfaces, which is related to the notion of view-factors. Hamilton and Morgan [1] published in NACA Technical Note the geometric configuration-factors between surfaces separated by a non-absorbing medium. They had included tables and graphs of configuration-factors for many shapes of surfaces, such as rectangles, triangles, cylinders etc. Later, Wiebelt and Ruo [2] derived a solution for the configuration-factors for right circular cylinders and rectangular planes using numerical methods. They applied contour integration procedure and presented the results in graphs. Also, Feingold [3] calculated view-factors for radiant-interchange between two rectangles that have a common edge and form an arbitrary angle. He showed how the view-factors, for two parallel, convex polygons that form the bases of a right prism, can be derived from the results obtained for the rectangles. The values of view-factors for a number of regular polygons were tabulated.

Sowell and O'Brien [4] presented an efficient computation method for configuration-factors within an enclosure. They applied a straightforward method using the reciprocity law and flux conservation law to obtain results. Gross et al. [5] calculated the view-factors between two plane rectangular surfaces of arbitrary position and size with parallel boundaries. They derived equations for three different cases of surfaces, such as parallel, inclined and perpendicular. Rao and Sastri [6] developed a numerical method for evaluating diffuse view-factors for radiation between planar surfaces. They extended their method to surfaces with curved boundaries using the Gaussian quadrature to perform the contour integration. They also derived the view-factors between two elliptic surfaces. Narayana [7] presented closed-form equations to compute view-factors for nine cases of parallel rectangular plates. The derived equations are suitable for use in computer calculations.

Carvalho and Farias [8] presented a review of the most commonly used methods for predicting radiative heat transfer in combustion chambers with reference to the relevant view-factors. They also referred to methods for solving the problems that arise in the field of radiation modeling and are related to the radiative properties of gases and particulate matter, which are commonly found in the combustion products. Guo and Maruyama [9] investigated the radiative heat transfer in three-dimensional inhomogeneous, nongray and anisotropically scattering participating media using the REM method. They verified the accuracy of the method by benchmark comparisons against the solutions of Monte Carlo method. They applied a mathematical model of radiative heat transfer in a boiler furnace and obtained the distribution of heat flux at the walls. The absorption view-factor and the diffuse scattering view-factor were introduced in the model to obtain the heat transfer rate. The effect of particle volume fraction was also discussed. Mohammed [10] developed a computer program for calculating view-factors that can be used to evaluate the exchange of radiation in a rectangular non-absorbing furnace enclosure. He compared the results with those obtained by the traditional graphical method and proved to be favorably well. Ebrahimi et al. [11] presented a zonal analysis for estimating the radiative heat transfer in industrial furnaces. They determined the direct exchange areas, which are the most general forms of well-known view-factors, by simplified numerical integration in three dimensions for surface-surface, surface-gas and gas-gas zones for absorbing and emitting media. The proposed zone method can be used as an effective tool for modeling the three-dimensional thermal performance of gas-filled enclosures.

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Bopche and Sridharan [12] studied the configuration-factors for a diffuse interchange of radiant energy between two parallel cylindrical rods using the contour integration technique. Their solution is applicable for the analysis of fin-tube radiators, boiler tubes, nuclear fuel rod bundles etc. Also, Minning [13] provided the solution of shape-factors between coaxial annular disks with different radii separated by a solid cylinder. He used a discrete-element thermal model to analyze the problem encountered in thermal control calculations for annular fin-tube radiators. Cabeza et al. [14] calculated configuration-factors between an emitting circle and a free point at a random position. Many complex surfaces can be analyzed in a similar way, provided that they allow for some decomposition into clusters of tangent circular elements. The findings may be useful for the radiative heat transfer calculations or for the solution of engineering and architectural lighting problems. Yang et al. [15] proposed two different radiation heat transfer models to calculate the heat transfer view-factors among fuel rod bundles that are used in nuclear reactors. The first model was the discrete transfer model (DTRM) and the second was the discrete ordinates model (DO). Both models are based on the CFD method. The results showed that the DTRM model can accurately and reliably calculate the view-factors among the rods in square bundles of a pressurized water reactor (PWR). Obando et al. [16] examined the thermal radiation heat transfer in a drying furnace. They developed an algorithm to calculate the energy rate transferred to the load by radiation and obtained the values of view-factors in a three-dimensional way by solving a double integral.

Zekri et al. [17] presented analytical solutions for the radiation view-factor between either a planar emitting surface or a plane-based fractal emitting surface and an arbitrarily-positioned and arbitrarily-oriented receptive element. They showed that the view-factor exhibits a non-monotonic behavior with distance when the receptor is staggered with respect to the emitting panels. Gupta et al. [18] performed a review of the methods used to evaluate the radiation view-factors. They presented results with the matrix method, Howell's method and the cross-string method. They compared the findings and showed that the matrix method was the most suitable method among other methods, due to its simple approach and the best results for any type of closed complex geometries. Muneer and Ivanova [19] presented efficient routines for computing view-factors between a non-uniform emitter field and a receiving surface using a finite element procedure. They examined plane surfaces as the ground-reflected solar radiation to thermal or photovoltaic collectors. Jiang et al. [20] proposed an alternative method to calculate the view-factor between finite and infinite length cylinders in an arbitrary array using a modified numerical integration approach. Their analysis reveals that the length and arrangements have a strong influence on the view-factor. Vinokurov [21] developed a generalized algorithm for calculating diffuse radiation configuration-factors. The algorithm was based on the division of the existing computational algorithms into two groups. He presented the generalized algorithm in the form of a directed graph that is actually a generalized block diagram containing practically all existing and possible algorithms for calculating the configuration-factors. The results can be used as a basis for software packages for a wide range of problems.

II. METHOD OF ANALYSIS

A. Configuration of a steam boiler furnace

A typical furnace or a combustion chamber of a steam boiler is shown in Fig. 1. The water tubes are placed in rows near the refractory walls and water circulates into the tubes to produce steam.

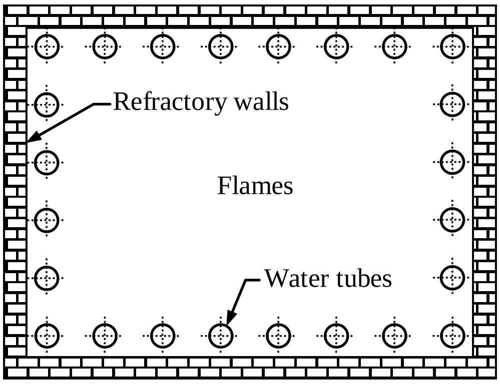

Fig. 1: Boiler furnace and water tubes

The flames in the combustion chamber radiate heat to any direction. We are interested in the determination of view-factor that affects the direct radiation of the flames to a series of water tubes, as shown in Fig. 2. The tubes have a diameter *d* and are arranged in rows with a pitch *t*.

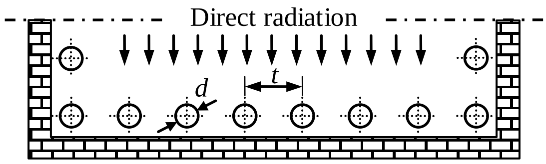

Fig. 2: Direct radiation to water tubes

In order to proceed to the analysis we define the following parameters:
– The ratio R as:

$$R = \frac{d}{t} \tag{1}$$

– The ratio S, which is the reciprocal of R, as:

$$S = \frac{t}{d} \tag{2}$$

We assume that the flames cover all the combustion chamber, so that the part of the circumference of the tube that receives radiation will be more than the half of the circle. The objective is to investigate the view-factor that exists between the radiating surface of flames and outer surface of water tubes. Because this is enough complicated approach, we may start the solution of the problem in a more convenient way. Specifically, we look first for the view-factor between the surface of a single tube and flames. Having this solution we can invert the result to obtain the proper view-factor from the flame surface to tube surface.

B. Geometry of tubes arranged in a single row

Two adjacent tubes of a row have the geometry shown in Fig. 3. We assume an infinitesimal area dA at a point M of the outer surface of tube 1. The point M stands at an angle θ measured from the vertical axis y. We intend to look for the view-factor between point M and flames, which means how point M sees the flames as it moves along the arc from the position A to B and then to C.

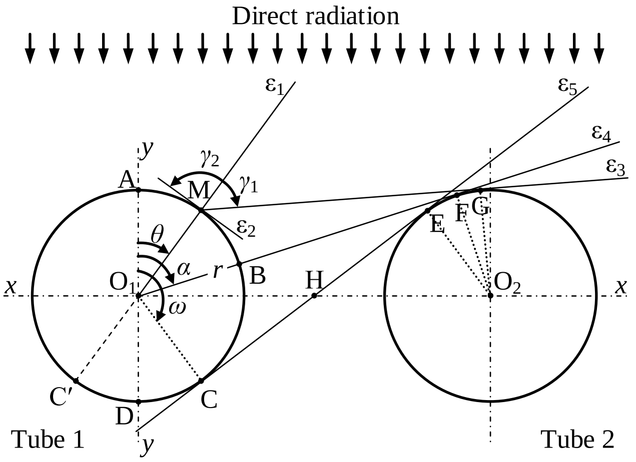

Fig. 3: Geometry of two adjacent tubes

The view of flames from point M is characterized through the angles γ_1 and γ_2. These angles are defined with respect to normal line ε_1 on dA and two other lines ε_2 and ε_3 oriented at the ends of view. Point A is at the top of the periphery of the tube 1, point B is on the line ε_4 which is tangential to the periphery of tube 2, and point C is on the line ε_5 which is tangential to both peripheries of the tubes 1 and 2. At the position A, point M sees the whole flame surface ($\gamma_1 = \gamma_2 = 90°$), but from other positions along the arc AC it sees less. It is obvious that the view angle from point M is limited by the sum of angles $\gamma_1 + \gamma_2$, when it moves from A to B, or it is the difference $\gamma_2 - \gamma_1$ when it moves from B to C. At the right-hand side of line ε_1 point M is partially shadowed by tube 2 ($\gamma_1 \leq 90°$), while at the left-hand side this point has less view of flame surface. There is no view of flames from points along the arc CD, where D is a point at the bottom of the periphery. Consequently, the view-factor varies with the position of point M, otherwise with the value of angle θ.

C. View-factor for point M on the circumference of tube 1

We examine the view-factor for point M when it moves along the arcs AB and BC separately, which means that the variation of angle θ is: $0 \leq \theta \leq \alpha$ and $\alpha < \theta \leq \omega$, respectively. The values of angles α and θ refer to the position of points B and C, and according to the geometry of Fig. 3 the following expressions are valid:

$$\alpha = \frac{\pi}{2} - sin^{-1}\left(\frac{R}{2}\right)$$
$$\omega = \frac{\pi}{2} + cos^{-1}(R) \tag{3}$$

Along the first arc AB the view-factor may be defined through the angles γ_1 and γ_2 as [22]:

$$\psi_t = \frac{sin(\gamma_1) + sin(\gamma_2)}{2} \tag{4}$$

where the subscript t denotes that there is a view from the tube. Along the second arc BC the following expression is valid:

$$\psi_t = \frac{sin(\gamma_2) - sin(\gamma_1)}{2} \tag{5}$$

Using the geometry of Fig. 3 we may find expressions for the angles γ_1 and γ_2 with respect to θ and then applying (4) and (5) we may reach to a solution of the view-factor ψ_t valid along the whole arc AC, as:

$$\psi_t = \frac{1}{2} + \frac{1}{2}\cdot cos\left[\theta + sin^{-1}\left(\frac{R}{\sqrt{R^2 + 4(1 - Rsin\theta)}}\right) - sin^{-1}\left(\frac{Rcos\theta}{\sqrt{R^2 + 4(1 - Rsin\theta)}}\right)\right] \tag{6}$$

where $0 \leq \theta \leq \omega$ and $0 < R \leq 1$. Performing some manipulations we may simplify the above equation to take the form:

$$\psi_t = \frac{1}{2} + \frac{1}{2}\cdot\frac{R^2 - 2Rsin\theta + 4cos\theta\sqrt{1 - Rsin\theta}}{R^2 + 4(1 - Rsin\theta)} \tag{7}$$

Due to symmetry with respect to y axis, as Fig. 3 shows, we may notice that if point M moves counterclockwise along the arc AC′ the same values of the view-factor exist. For this case the expression (7) may be transformed properly if we change θ by $-\theta$ and use negative values for θ along the arc AC′.

D. Inverse problem approach for the view-factor

We proceed with the inverse problem using the reciprocity relation of view-factors to find the view-factor between the flame surface and tube surface. The widely known expression of reciprocity may be written as [23]:

$$\psi_f \cdot A_f = \psi_t \cdot A_t \tag{8}$$

where ψ_f is the view-factor from flame surface to tube surface, A_f is the flame surface. In this expression the view-factor ψ_t refers to the whole number of tubes in the row and A_t is the total radiated tube surface. Therefore, we must proceed to a transformation of (8), as follows:

$$\psi_f \cdot A_f = 2\int_0^{\omega} \psi_t \cdot dA \tag{9}$$

where ψ_t is the view-factor from (7), dA is an infinitesimal area on the tube periphery and ω is the maximum angle for which there is a view of flames. The multiplier 2 is introduced to take account for the whole arc CAC′ of the circumference of the tube, which receives radiation from the flames. The area A_f can be expressed as:

$$A_f = L \cdot h = n \cdot t \cdot h \tag{10}$$

where L, h are the length and height of the combustion chamber, respectively, n is the number of tubes in a row and t is the pitch of the tubes. Taking into account that the flames radiate to a whole row of n tubes then the area dA in (9) can be written as:

$$dA = n \cdot h \cdot ds = n \cdot h \cdot r \cdot d\theta = n \cdot h \cdot \frac{d}{2} \cdot d\theta \tag{11}$$

where ds is the infinitesimal arc at point M, h is the length of the tubes which is equal to the height of the combustion chamber, r is the radius of the tube and $d\theta$ is an infinitesimal part of angle θ. Introducing the expressions (10) and (11) in (9) we have the following equation:

$$\psi_f = R \int_0^{\omega} \psi_t \cdot d\theta \tag{12}$$

The definite integral of the above equation can be calculated analytically. This integral is a function of parameter R only, because the angle ω depends on R, as we can see in (3). We replace ψ_t from (7) in (12) and after performing the integration we obtain the following expression for the view-factor:

$$\begin{aligned}\psi_f &= 1 + \frac{3\omega R}{4} - \sqrt{1 - R\sin\omega} \\ &+ \frac{R}{2}\left[\tan^{-1}\left(\frac{4R - (4 + R^2)\tan\frac{\omega}{2}}{4 - R^2}\right) - \tan^{-1}\left(\frac{4R}{4 - R^2}\right)\right] \\ &+ \frac{R}{2}\left[\tan^{-1}\left(\frac{2}{R}\sqrt{1 - R\sin\omega}\right) - \tan^{-1}\left(\frac{2}{R}\right)\right]\end{aligned} \tag{13}$$

It is possible to further simplify this solution if we use the expressions for the involved trigonometric functions of angle ω, which are:

$$\sin\omega = R \qquad \tan\frac{\omega}{2} = \frac{1 + \sqrt{1 - R^2}}{R} \tag{14}$$

After performing the necessary manipulations we reach to the final solution of the view-factor ψ_f, which refers to the direct radiation on a tube row of any number of tubes, as:

$$\begin{aligned}\psi_f &= 1 - \frac{\pi R}{8} - \sqrt{1 - R^2} \\ &+ \frac{R}{4}\left[3\cos^{-1}R + 2\tan^{-1}\left(\frac{1 + \sqrt{1 - R^2}}{R}\right)\right]\end{aligned} \tag{15}$$

III. Results and Discussion

The presented method is completely analytical and can be applied to obtain results of view-factor for a row of tubes in a steam boiler furnace. In several scientific books we may find an expression for the view-factor between an infinite plane and a row of an infinite number of cylinders [23], which has the form:

$$\psi_f = 1 - \sqrt{1 - R^2} + R \cdot \tan^{-1}\left(\frac{\sqrt{1 - R^2}}{R}\right) \tag{16}$$

If we assume the infinite plane as the flame surface and the cylinders as the boiler tubes, we may use above equation to compare our results. For this scope we show in a graphical representation both equations, (15) and (16), as in Fig. 4.

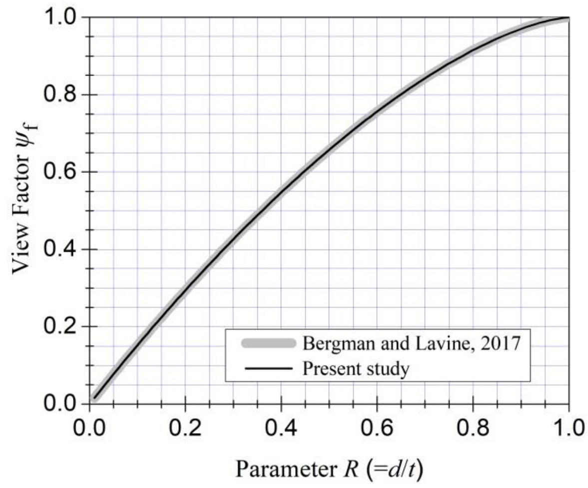

Fig. 4: Variation of view-factor ψ_f with respect to R

We observe that both curves are very close to each other. This fact verifies the accuracy of our solution. It is obvious that when R=1 the value of view-factor is ψ_f =1. This happens when t=d or the tubes are in contact without any free space among them in the row. In this case all the radiative heat in the boiler furnace is received by the tubes.

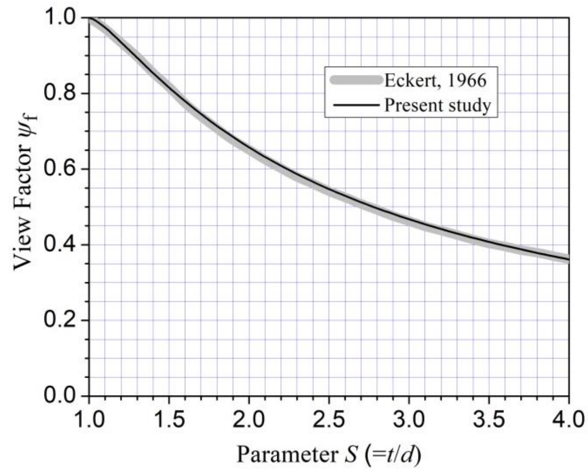

Fig. 5: Variation of view-factor ψ_f with respect to S

Some other scientific books have a reference to the view-factor for direct radiation by including a graph [24], [25], and show the variation of ψ_f as a function of the parameter S given in (2). We can make a comparison with the values of such a graph if we introduce the parameter S (=1/R)

in (15) and after this replacement we may draw the appropriate graph. This graph is presented in Fig. 5, in which we have also redrawn the curve of the graph by Eckert [25], which refers to direct radiation (curve d, p. 257).

It seems that both curves coincide very well, which means that the proposed solution is accurate and simple enough for the determination of radiative heat in boiler furnaces.

IV. Conclusion

The solution of the view-factor that exists between flames and tubes in the combustion chamber of a steam boiler plant was given explicitly in an analytical form. This seems to be simple enough for calculations and therefore, one can use it to solve the radiation heat transfer problems in a convenient way. The comparison of the results, which were obtained from the extracted equation of the view-factor, showed that there is a complete agreement with other found in the literature.

Nomenclature

A_f = flame surface
A_t = tube outer surface
d = tube outer diameter
h = combustion chamber height or tube length
L = combustion chamber length
n = number of tubes
r = tube radius
R = ratio ($=d/t$)
s = arc length
S = ratio ($=t/d$)
t = tube pitch
x, y = orthogonal axes

Greek Symbols

α = angle of arc AB
γ_1, γ_2 = view angles from point M
ε_1, ε_2, ε_3, ε_4, ε_5 = straight lines
θ = angle of arc AM (variable)
ψ_f = view-factor from the flame surface
ψ_t = view-factor from the tube outer surface
ω = angle of arc AC

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