On The Laws of Planetary Motions by Aryabhatta I and Kepler

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ABSTRACT - This paper examines the laws on planetary motions in the solar system proposed by Aryabhatta I who lived in Pataliputra I in the 5th century AD (476-550) and Kepler (1571-1630). The history of astronomy is reviewed where this contribution by Aryabhatta was not commonly known to public or among the scientific community. Kepler's laws are very widely known throughout the world.

The findings of this work is that the Aryabhatta's laws more accurately represent the planetary motions than those of Kepler even though they were written about 1000 years earlier.

1. INTRODUCTION TO THE MOTION OF PLANETS

Figs 1 to 4 show orbits of planets which move along either along a circular or elliptical orbits. Table 1 shows various details of the planetary motions along their orbits. In this table, the motions of planet Mercury look inconsistent with the rest because of the reletivitistic effect [1, 2].

The early history of astronomy starts from Egypt and Samaria. However, not much details are available to us about the knowledge of astronomers in those days. It is mainly Greeks and Indians who had systematically carried out such studies. To review early history one should go through references 3 to 8.

Starting from Ptolemy, the idea of geocentricity arose and continued from ancient times to the time of Copernicus in the 16th century. It was Copernicus who showed that it was the Sun around which all planets moved. After Copernicus, other researchers such as Tyche Brahe made accurate observations of the stars and planets. He lived between 1546 to 1601. Kepler worked in his observatory and the planetary motion data from early astronomers were available with him as he went through the data of Islamic scholars and that of Copernicus.

Based on the earlier data and his own minute observations, Kepler came up with the three laws of planetary motion.

The Islamic astronomers had access to the Indian and Greek data on planetary motion and they also made further contributions in this field.

2. THEORETICAL CONSIDERATIONS

Fig. 3 shows the solar system with the Sun at its center and various planetary system moving in their respective orbits around the Sun. Kepler came up with his findings that the orbits of the planets are ellipses and their motion takes place on these orbits in 16th century AD. He came up with three laws governing their motion. These laws are:

1) Every planet's orbit is an ellipse with the Sun at a focus;

2) A line joining the Sun and a planet sweeps out equal areas in equal times; and

3) The square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit. In other words, his third law states that

$$r^3/ T^2 = K1, \text{ a constant} \quad (1)$$

Here, r is the radius and T is the time period. Aryabhatta I was a mathematician and an astronomer who lived in Pataliputra (modern Patna) in India between 476 AD to 550 AD during the time of Magadh king Buddhagupta. He wrote a book named Aryabhatiya in 499 AD at the age of 23 years. The name of the city at that time was Kusumpura, and it was the capital of the Magadh Empire. He wrote this book in Sanskrit which was the language the scientists and learned used this language for writing. India went through many changes since that time and it was ruled subsequently from the 12th century by Islamic kingdoms and by British from the 18th century until 1947 AD. During the British rule in the year 1930, it (Aryabhatiya) was translated into English by W. E. Clark, a professor in the Oxford University. During the British rule, many others had published their works on this book in journals also. Many Indian astronomers had also written articles and commentaries on this book.

Aryabhatta came up with 3 laws regarding the planetary motions and they are, as mentioned in this book, are:

1. The planets move along circular orbits.
2. The planets really all move at the same speed. The nearer ones seem to move more rapidly than the more distant ones because their orbits are smaller.
3. The earth spins on its own axis.

One can consider the first two laws of Aryabhatta encompassing all the three laws of Kepler. Kepler did not write anything about the motion about the earth's axis.

According to the Kepler's first law, the planets move along an elliptical path whereas Aryabhatta says that they are circular based on his observations. Let us look at some equations for an ellipse which are:

$$(x^2/a^2) + (y^2/b^2) = 1 \quad (2)$$

$$c = (a^2 - b^2)^{1/2} \quad (3)$$

$$e = c/a = (1-(b/a)^2)^{1/2} \quad (4)$$

One can see from Fig. 1 that when c approaches zero, an approaches b and then the ellipse becomes a circle. In that case the ratio b/a will approach 1. Aryabhatta only considered planets Saturn and all others inner planets with the locations of the Sun and the earth interchanged i.e. he considered a geocentric model similar to Ptolemy and later on by various Islamic astronomers who came much after Aryabhatta. There were two Aryabhatta living in India at different times at Pataliputra who were astronomers also. The second one lived in the 10^{th} century AD. Just to remind the readers that in this paper, the author is referring by Aryabbhatta to be the same as Aryabhatta I. to be the same. There was another Aryabhatta II who lived in the same city between 920 AD –1000 AD.

If we look at the Table 2 column 5, b/a for all planets except Mercury – this ratio is almost equal to 1. It should be reminded that the planet Mercury does not follow Newton's law of gravitation and it follows Einstein's General Theory of Relativity being that close to the Sun [1,2]. So, it is an exception to the rule and it does not follow Kepler's laws also.

In this table, column 7 shows the calculations for Kepler's third law (K1), and number 8 shows the velocity variation (K2). The maximum difference or variations in numbers within K1 and K2 are shown in Table 3. In K2, the orbital speed decreases as the planets are away farther from the Sun which means Mercury has the highest orbital speed. However, in K1, there is no such uniformly decreasing trend. In both cases, the maximum difference in each of the columns are calculated and reported in Table 3.

In the Table 3, the ratio of the maximum variation is shown. Since this number is extremely small, therefore the Aryabhatta's model is better that of Kepler.

3. CONCLUDIONS

1. In this work, at first, the history of astronomy of the solar planetary system was looked at. It was seen that the Greek planetary studies were most influential and were followed over a long time starting from Ptolemy (second century AD) to 16^{th} century. It was followed by and large by Greek, and Islamic astronomers. The church in Europe followed this astronomy.
2. Aryabhatta did not follow Ptolemy and he came up with his planetary laws of motion but these laws remained in obscurity.
3. The results show that Aryabhatta's model is better.

4. REFERENCES

1. Red Shift, https://en.wikipedia.org/wiki/Gravitational_redshift
2. Tests of General Relativity, https://en.wikipedia.org/wiki/Tests_of_general_relativity#Perihelion_precession_of_Mercury
3. Windows to the Universe, https://www.windows2universe.org/our_solar_system/planets_orbits_table.html
4. Kepler's Three Laws Explained, https://www.youtube.com/watch?v=kyR6EO_RMKE
5. History of Astronomy of Different Civilizations Parts I and II, https://www.youtube.com/watch?v=aovSXG4X5RM&t=800s
6. David Edwin Pingree, https://en.wikipedia.org/wiki/David_Pingree
7. Ancient astronomy in Babylon and China I | Math History | NJ Wildberger, https://www.youtube.com/watch?v=QB6vm6rHFMY
8. Science and Islam - Medieval Islam Influences, https://www.youtube.com/watch?v=4-lHyUYaAIU
9. Clark, W. E. , 1980, "The Aryabhatiya of Aryabhata", The University of Chicago Press, Chicago, USA, pp 50-79

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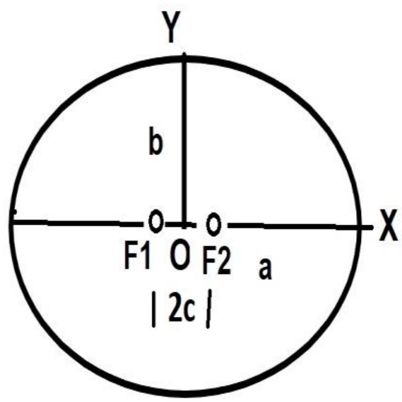

FIG 1 AN ELLIPSE WITH DETAILS

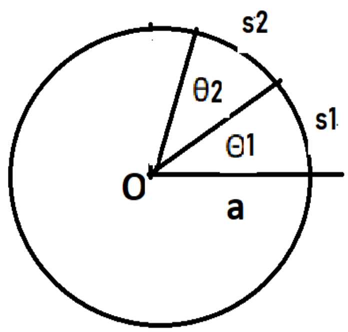

FIG. 2 A CIRCLE WITH DIFFERENT SECTORS

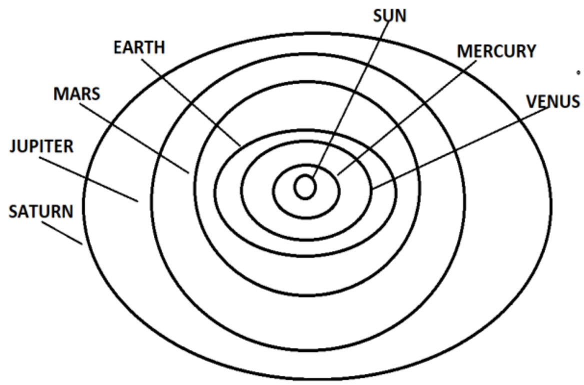

FIG. 3 THE SUN AND THE PLANETARY SYSTEM

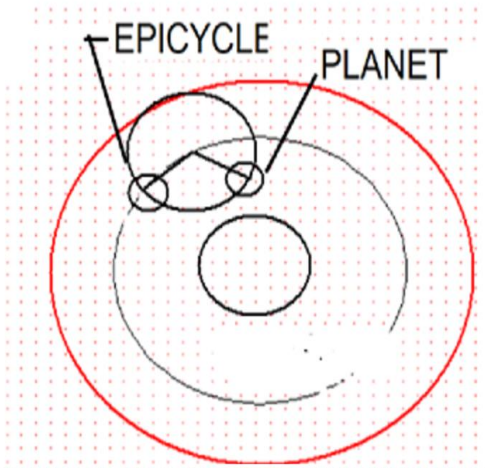

FIG 4 ISLAMIC MODEL OF SOLAR SYSTEM WITH EARTH AT THE CENTER

TABLE 1 VARIOUS DETAILS OF LANETARY MOTIONS IN THE SOLAR SYSTEM [3]

Planet	Semimajor Axis (AU)	Orbital Period (yr)	Orbital Speed (km/s)	Orbital Eccentricity (e)	Inclination of Orbit to Ecliptic (°)	Rotation Period (days)	Inclination of Equator to Orbit (°)
Mercury	0.3871	0.2408	47.9	0.206	7.00	58.65	0
Venus	0.7233	0.6152	35.0	0.007	3.39	-243.01*	177.3
Earth	1.000	1	29.8	0.017	0.00	0.997	23.4
Mars	1.5273	1.8809	24.1	0.093	1.85	1.026	25.2
Jupiter	5.2028	11.862	13.1	0.048	1.31	0.410	3.1
Saturn	9.5388	29.458	9.6	0.056	2.49	0.426	26.7
Uranus	19.1914	84.01	6.8	0.046	0.77	-0.746*	97.9
Neptune	30.0611	164.79	5.4	0.010	1.77	0.718	29.6

TABLE 2: VARIOUS PARAMETERS IN SI UNITS

1	**PLANETS**	**A (m)**	**e (m)**	**b (m)**	**b/a**	**T (seconds)**	**V (Orbital Speed) (m/s) K2**	**Kepler (a^3/T^2) K1**
2	1	2	3	4	5	6	7	8
3		(1.00E+10)	(1.00E+08)	(1.00E+11)	(1.00E+00)	(1.00E+06)	(1.00E+03)	1
4	Mercury	5.8	308.2	0.6	1.0	7.6	47.9	3.36759E+18

5	Venus	10.8	10.5	1.1	1.0	19.4	35.0	3.36578E+18
6	Earth	15.0	25.4	1.5	1.0	31.5	29.8	3.36638E+18
7	Mars	22.8	139.1	2.3	1.0	59.3	24.1	3.39004E+18
8	Jupiter	77.8	71.8	7.8	1.0	374.1	13.1	3.36945E+18
9	Saturn	142.7	83.8	14.2	1.0	929.0	9.6	3.36696E+18

TABLE 3 MAXIMUM VARIATION IN K1 and K2

	K1	**K2**	**RATIO k2/k1**
MAXIMUM VARIATION	$6.3x10^{14}$	$3.8x10^{4}$	$6.0x10^{-11}$