Determination of Area of General Quadrilateral with Four Sides and No Angles Specified

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ABSTRACT- In this work, the area of a quadrilateral is determined where the lengths of all the four sides are given but none of the angles are given. Such problems occur in the area of mechanism design, and surveying, a part of civil engineering.

This quadrilateral cannot be plotted to any scale and poses a challenging problem. The problem has been solved by setting up of an objective function by assuming two angles of the quadrilateral as design variables. By minimizing the square of the difference between calculated length of one side minus the given length - the solution is obtained.

KEYWORDS: Area of a quadrilateral, Minimization of residue of calculated and given length of quadrilateral, Heron's formula of a triangle.

1. INTRODUCTION AND LITERATURE SURVEY

Solving quadrilateral problems are needed in the field of mechanism design by mechanical engineers [1,2]. Similarly, surveyors in civil engineering also encounter different types of geometrical shaped areas. They carry out their surveys by measuring distances and angels. This way they can calculate areas. However, in many deeds of properties, all the information is not available. In such cases, if they happen to be quadrilaterals – the surveyors have to come up with the area and thereby the value of the property is determined. In such cases, a literature search was made about the existing knowledge in this field.

Large number of papers are available for quadrilaterals having specific features [3-12]. Unfortunately, if only the sides are given then it is not possible to plot the quadrilateral thereby determine the area.

The present work has been undertaken to solve such a problem.

2. METHOD OF AREA DETERMINATION

Fig. 1 shows a sketch of one such area that exists in the city of Patna, in the state of Bihar in India where a sketch of an actual property showing four sides was found. It was not drawn to a scale. Since no angle was given, it would not be possible to construct the area to a scale thereby measure parameters such as angles and thereby determine the area within the quadrilateral.

The method to solve the problem is as follows:

1. Draw angles Th1 and Th2 as shown in Fig. 1 and locate the origin at A

2. The x, and y coordinates of point B will be

Bx = S1 cos (Th1) and By = S1 sin(Th1).

Similarly for the point C it will be

Cx= S3 cos (Th3) and Cy will be equal to S3 sin(Th3).

3. Then the calculated length of BC will be equal to

$L= ((Cx-Bx)^2 + (Cy-By)^2)^{0.5}$

4. Let $U =(L-S2)^2$

The minimum of U =0 will yield correct values of Th1 and Th2 and the corresponding coordinates of B and C will be determined.

An optimization routine was used in Maple software and the value of U, Th1 and Th3 were found to be equal to

$U = 1.403x10^{(-16)}$

Th1 = 94.485 degrees and,

Th3 = 104.806 degrees.

The co-ordinates of point D were

Dx = 171.5, and Dy= 0.

Now, we can calculate the length of BD knowing the co-ordinates of points B and D. This comes out as

S5 = 181.189.

Using Heron's formula [13] for areas ABD and BDC, the result came out to be:

ABD= 5016.375 square feet,

BDC = 4163.732 square feet. and

Total area = 9180.107 square feet

Fig. 2 shows details of the result.

3.REFERENCES

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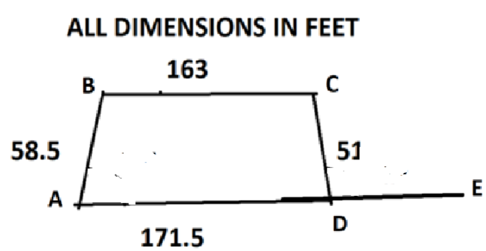

FIG. 1 LENGTHS OF ALL FOUR SIDES

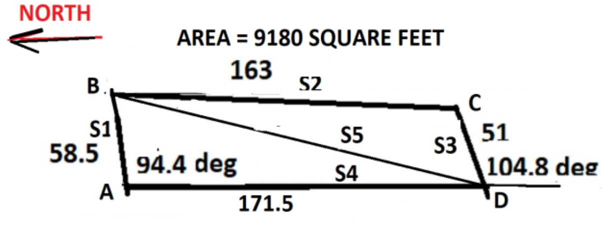

FIG. 2 CALCULATED AREA OF THE QUADRILATERAL