Consideration of Natural Acoustic Frequency of One Dimensional Sound Field Partitioned with Perforated Plate
(Comparing Melling's Eq. with Dah-You Maa's Eq.)

Kunihiko Ishihara

Abstract— **Natural acoustic frequency of one dimensional duct partitioned with a perforated plate was clarified to come down with decreasing an aperture ratio experimentally and analytically. In order to clarify the reason, the sound propagation experiment was conducted. As a result, it was clarified that the smaller the aperture ratio became the longer the sound arrival time became. On the other hand, the impedance of the perforated plate was studied by Melling and it was referred by many researchers. The same analysis was also done by Dah-You Maa for a micro-perforated panel. In this paper, the relationship among the present analysis, Melling's equation and Dah-You maa's analysis are discussed. And the applicability of the present method will be confirmed.**

Index Terms— **Noise control, Sound and acoustics, Natural acoustic frequency, Perforated plate, Transfer matrix method**

I. INTRODUCTION

In an acoustic system like a duct with a perforated plate in the middle, I was pretty sure that a natural acoustic frequency increased with decreasing aperture ratio of the perforated plate. Because the one dimensional acoustic field like the duct is divided into two ducts due to the perforated plate [1].

However according to a result of the analysis by the Transfer Matrix Method with the acoustic impedance derived from Melling and the experiment as previously reported, it was clarified that the natural acoustic frequency decreased with decreasing the aperture ratio of the perforated plate [1].

To make it clear the acoustic mode was obtained by the Transfer Matrix Method and the frequency was calculated by the equation $f=c/\lambda$ after getting the wave length λ from the mode shape. These results of the frequencies were in good agreement with the experimental ones [2]. Then we understood that this phenomenon was due to the decreasing of the apparent sound speed based on the time delay when the wave passed through the holes of the perforated plate [2]-[4].

The experiment as shown in Figure 4 was carried out to make this fact clearer and the apparent sound speed was confirmed to be decreasing [5].

On the other hand, the impedance of the perforated plate was studied by Melling [6] and was cited by many researchers. And Dah-You Maa's has also studied the same analysis as Melling for the micro-perforated panel [7]. Then in this study, I will make it clear why the analytical result by Transfer Matrix Method is in agreement with the experimental one in an integrated manner by comparing Melling's equation and Dah-you Maa's equation and the relationship between them will be discussed.

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II. EXPERIMENTAL APPARATUS AND METHOD

Figure 1 shows the experimental setup for obtaining the natural acoustic frequency of the duct. The perforated plate is inserted at 100 mm from the right end of the one dimensional duct which has 434 mm in total length. The experimental parameters are the aperture ratio of the perforated plate. The aperture ratios are 1 %, 2 %. 4 %, 8 %, 16 % and 32 %. The perforated plate is made of the steel with thickness 2.3 mm and has many holes with the diameter of 3 mm. The sound source is a speaker and the sound pressure level is measured by the microphone set at 30mm from the left end of the duct. The pressure signal is frequency analyzed by the FFT analyzer.

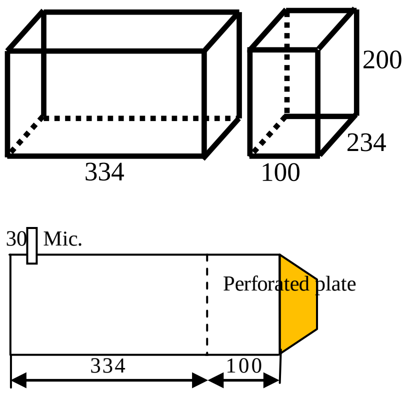

Fig.1 Experimental setup

III. ANALYTICAL MODEL AND METHOD

A. *Analytical Model and Method*

Figure 2 shows the analytical model. The numbering is performed as shown in the figure 2 and the state vector at each position is described as $[P_i, U_i]^T$. The relation between two state vectors of both ends can be written by Eq. (1). Where $Z_i = \rho c/S_i$, S_i is the cross sectional area of each duct element. And *A*ij are the results of multiplication of three matrixes. The aperture ratio φ is defined by S_3/S_1. Where S_3 is calculated by $\pi d^2/4 \cdot$ N. Where *d* is a diameter of hole and N is a number of holes.

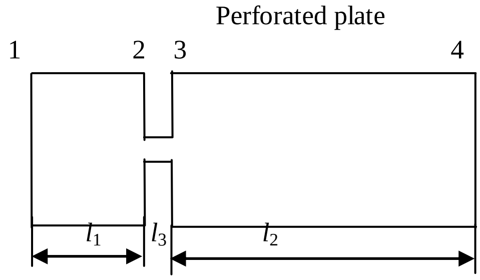

Fig.2 Analytical model

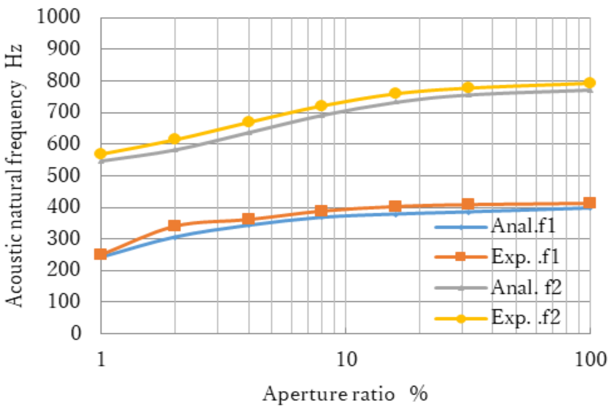

Fig.3 Comparison between analytical and experimental results of natural acoustic frequency

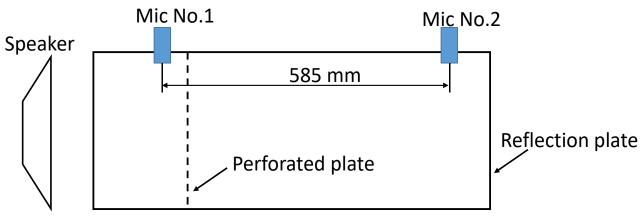

Fig.4 Experiment of obtaining apparent sound speed

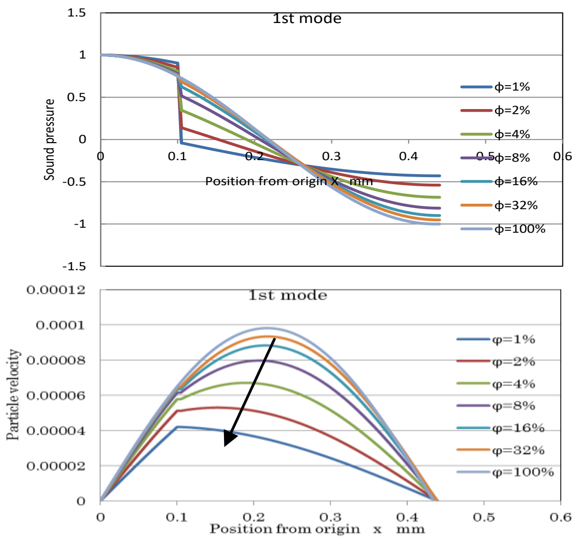

Fig.5 Pressure and particle velocity modes of duct (1st mode)

$$\begin{bmatrix} P_4 \\ U_4 \end{bmatrix} = \begin{bmatrix} \cos kl_2 & jZ_2 \sin kl_2 \\ j\frac{1}{Z_2}\sin kl_2 & \cos kl_2 \end{bmatrix} \begin{bmatrix} \cos kl_3 & jZ_3 \sin kl_3 \\ j\frac{1}{Z_3}\sin kl_3 & \cos kl_3 \end{bmatrix} \times$$

$$\begin{bmatrix} \cos kl_1 & jZ_1 \sin kl_1 \\ j\frac{1}{Z_1}\sin kl_1 & \cos kl_1 \end{bmatrix} \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \quad (1)$$

$$A_{11} = \cos kl_2 (\cos kl_3 \cos kl_1 - \frac{Z_3}{Z_1} \sin kl_3 \sin kl_1)$$

$$-Z_2 \sin kl_2 \left(\frac{1}{Z_3} \sin kl_3 \cos kl_1 + \frac{1}{Z_1} \cos kl_3 \sin kl_1 \right)$$

$$A_{12} = j \cos kl_2 (Z_1 \cos kl_3 \sin kl_1 + Z_3 \sin kl_3 \cos kl_1)$$

$$+ jZ_2 \sin kl_2 \left(-\frac{Z_1}{Z_3} \sin kl_3 \sin kl_1 + \cos kl_3 \cos kl_1 \right)$$

$$A_{21} = j \frac{1}{Z_2} \sin kl_2 (\cos kl_3 \cos kl_1 - \frac{Z_3}{Z_1} \sin kl_3 \sin kl_1)$$

$$+ j \cos kl_2 \left(\frac{1}{Z_3} \sin kl_3 \cos kl_1 + \frac{1}{Z_1} \cos kl_3 \sin kl_1 \right)$$

$$A_{22} = \frac{-1}{Z_2} \sin kl_2 (Z_1 \cos kl_3 \sin kl_1 + Z_3 \sin kl_3 \cos kl_1)$$

$$+ \cos kl_2 \left(-\frac{Z_1}{Z_3} \sin kl_3 \sin kl_1 + \cos kl_3 \cos kl_1 \right)$$

$$\begin{bmatrix} P_4 \\ U_4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \quad (2)$$

As the both ends of the duct are closed the boundary condition is given as follows.

$$U_1 = 0, \quad U_4 = 0 \quad (3)$$

Therefore

$$0 = A_{21} P_1 \quad (4)$$

The characteristic equation becomes as follows.

$$|A_{21}| = 0 \quad (5)$$

Indicating this equation in concrete finally,

$$\sin kl_2 (\cos kl_3 \cos kl_1 - \frac{1}{\phi} \sin kl_3 \sin kl_1)$$
$$+ \cos kl_2 (\phi \sin kl_3 \cos kl_1 + \cos kl_3 \sin kl_1) = 0 \quad (6)$$

B. Analytical Results and Comparing with Experiment

Figure 3 shows the comparison between analytical results and experimental results of the natural acoustic frequency of the 1st and the 2nd modes. “Anal” and “Exp.” in this figure mean the analytical value and the experimental value, respectively. The analytical values are in good agreement with the experimental ones. As can be seen in figure 3 the natural acoustic frequency decreases with decreasing the aperture ratio.

IV. REASON WHY NATURAL ACOUSTIC FREQUENCY DECREASES WITH DECREASING APERTURE RATIO

I have presumed that the sound wave reached the end of the duct in retard due to the time delay when the sound wave passed through the hole of the perforated plate. To confirm the presumption the experiment was conducted as shown in the figure 4. The time difference Δt(s) between two microphone’s positions can be measured. The distance of two sound measuring points is 585mm. So the apparent sound speed can be calculated by 0.585/Δt. The natural acoustic frequency can be calculated by $f_n = c_a/2L$. Where c_a is the apparent sound speed and L is the duct length. The natural acoustic frequency coincides with that of the experiment. The presumption mentioned above has therefore been made clear.

Moreover, I examined the fact by using acoustical mode of the duct. Figure 5 shows the results of the modal analysis. Upper and lower of figure5 are the sound pressure and the particle velocity modes of the 1st mode, respectively. As can be seen from the lower of figure5 the wave length increases with the aperture ratio being small. This means the natural acoustic frequency decreases with decreasing the aperture ratio.

V. COMPARISON BETWEEN ANALYSES BY MELLING AND BY DAH-YOU MAA

When I have been studying the impedance of the perforated plate two references can be attracted attention. One is a paper by Melling who wrote it in 1973 and the other is a paper by Dah-You Maa in 1987. These papers are closely similar. So I will compare these papers below.

A. *Melling's Analysis*

Figure 6 shows the theoretical model of the hole of the perforated plate and the figure 7 shows the detail of the figure 6.

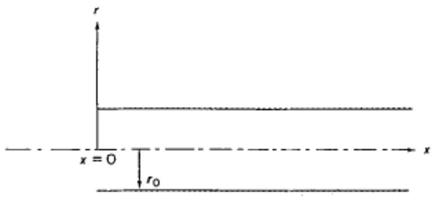

Fig.6 Coordinate system of theoretical model for viscous effects in tube

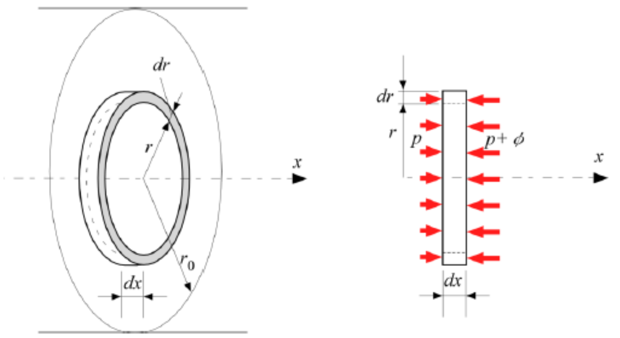

Fig.7 Air in hole

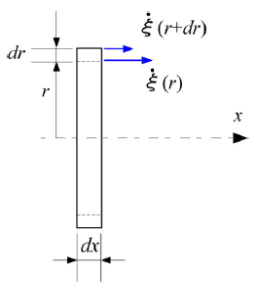

Fig.8 Viscous force

The equation of motion of the air in the hole is given by Melling as follows. This equation was derived based on the Crandall's equation [9].

$$\left[j\omega\rho - \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r} \right) \right]\dot{\xi} = \phi \tag{7}$$

Where ϕ is the pressure gradient along with the tube axis. If the tube length is l, the pressure gradient ϕ is given by $\phi = \Delta p / l$. The particle velocity $\dot{\xi}$ is the function of only r and this equation is rewritten as follows (See APPENDIX A).

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + k_s^2 \right]\dot{\xi} = -\frac{\phi}{\mu} \tag{8}$$

Where $k_s^2 = -j\rho\omega/\mu$ and ρ, ω, μ are air density , angular frequency , viscosity, respectively. The solution of this equation is as follows.

$$\dot{\xi}(r) = -\frac{\phi}{\mu k_s^2} + AJ_0(k_s r) \tag{9}$$

(See APPENDIX B)

J_0 is the zero order Bessel function of the first kind.

The coefficient A is determined under the boundary condition of the velocity being 0 at $r=r_0$

$$A = \frac{\phi}{\mu k_s^2} / J_0(k_s r_0) \quad \text{from} \quad 0 = -\frac{\phi}{\mu k_s^2} + AJ_0(k_s r_0)$$

The equation (9) therefore becomes

$$\dot{\xi}(r) = -\frac{\phi}{\mu k_s^2}\left[1 - \frac{J_0(k_s r)}{J_0(k_s r_0)} \right] \tag{10}$$

Finally the mean value is obtained by integrating $\dot{\xi}(r)$

$$\dot{\xi} = \frac{1}{\pi r_0^2}\int_0^{r_0} \dot{\xi}(r)\cdot 2\pi r dr = \frac{2}{r_0^2}\int_0^{r_0} \dot{\xi}(r) r dr = -\frac{\phi}{\mu k_s^2}\left[1 - \frac{2J_1(k_s r_0)}{k_s r_0 J_0(k_s r_0)} \right] \tag{11}$$

(See APPENDIX C)

In this regard, the formula of the Bessel function : $\int xJ_0(x)dx = xJ_1(x)$ is used in deriving the equation (11)[8]. The amount of [ ] in the equation (11) is the function of the velocity profile and J_0 and J_1 are the zero order and one order Bessel functions of the first kind, respectively. The pressure difference at both ends of the tube becomes as follows.

$$p = \int_0^l \phi dx \tag{12}$$

The impedance per unit cross sectional area of the tube can be obtained by use of this equation with the equation (11) which means the mean velocity at the same time.

$$z' = \frac{p}{\dot{\xi}} = -\mu k_s^2 l / \left[1 - \frac{2J_1(k_s r_0)}{k_s r_0 J_0(k_s r_0)} \right] \tag{13}$$

or

$$z' = j\omega\rho l / \left[1 - \frac{2J_1(k_s r_0)}{k_s r_0 J_0(k_s r_0)} \right] \tag{14}$$

$k_s^2 = -j\rho\omega/\mu$, k_s is the wave number of the viscous Stokes wave and λ_s is the associated wave length which can be written as follows.

$\lambda_s = 2\pi/\beta$, where $\beta = (\rho\omega/2\mu)^{1/2}$

In the case of air, λ_s =0.04(0.044)cm at 1000Hz as μ =2×10^{-4} poise(1.83 × 10^{-5} Pa ・ s). For a wall of high thermal conductivity, the effective value of the viscosity coefficient is μ =4×10^{-4} poise (3.66×10^{-5} Pa・ s). Thus the Stokes wave length in this case is greater by a factor of $\sqrt{2}$.

(a) $|k_s r_0| < 2 \quad (r_0 < \lambda_s / \pi) \quad r_0^2 f < 1.0 \quad (r_0 : cm)$ in the case of air. This is written in the reference [6]. It is however better to describe $|k_s r_0|<1$ and $r_0^2 f <0.025$ (r_0:cm), when μ =1.83×10^{-5} Pa・ s or $|k_s r_0|<1$ and $r_0^2 f<0.05$ (r_0:cm), when μ =3.66×10^{-5} Pa・ s

By using first two terms of the series expansion of the Bessel function the following equation can be obtained.

$$z' = \frac{8\mu l}{r_0^2} + j\frac{4}{3}\rho\omega l = R + j\chi \tag{15}$$

(See APPENDIX D)

Equation $8\mu l / r_0^2$ is known as the Poiseuille' law which means the resistance to the laminar flow of the viscos flow run in the small tube.

The imaginary part of the impedance $\chi = 4\rho\omega l / 3$ has the total effective mass $4\rho l / 3$ and this value is larger than the real mass of the tube part. This added mass is the direct result of the effect of the viscosity on the velocity profile while the viscos modulus being independent explicitly.

(b) $|k_s r_0| > 10 \quad r_0^2 f > 5.0 \quad (r_0 : cm)$ in the case of air.

This is written in the reference [6]. It is however better to describe $|k_s r_0|>10$ and $r_0^2 f >2.5$ (r_0:cm), when μ =1.83×10^{-5} Pa・ s or $|k_s r_0|>10$ and $r_0^2 f >5$ (r_0:cm), when μ =3.66×10^{-5} Pa ・ s.

This time the suitable approximation of the ratio of the Bessel functions is

$$J_1(x\sqrt{-j}) / J_0(x\sqrt{-j}) = -j \tag{16}$$

Then, Equation (14) becomes as follows.

$$z' = \frac{l}{r_0}\sqrt{2\rho\mu\omega} + j\omega\rho l\left[1 + \frac{1}{r_0}\sqrt{\left(\frac{2\mu}{\rho\omega}\right)}\right] \tag{17}$$

(See APPENDIX E)

The real part of the equation (17) is depend on the frequency. This equation to the resistance was first determined by Helmholtz [6]. This additional attached mass, which is resulting from combined the viscosity and the inertia in the tube, is also dependent on the frequency.

Equation (15) and equation (17) mentioned above are the impedances for one hole, the impedance for total holes is described as follows under the assumption of no interference.

$$z = z'\frac{S}{\sigma} = \frac{z'}{P} \tag{18}$$

Where σ, S are the area of one hole and the area of total holes, respectively. P is the porosity.

B. *Dah You Maa's Analysis*

Next, I will explain the result of Dah-You Maa. He derived the impedance for the micro-perforated panel by using the Crandall's equation for the orifice [9]. This is the same as the Melling's result. I will show the derivation of the equation of the impedance of the orifice by Dah-You Maa.

The equation of motion is given as follows.

$$\rho\dot{u} - \frac{\mu}{r_1}\frac{\partial}{\partial r_1}\left(r_1\frac{\partial u}{\partial r_1}\right) = \frac{\Delta p}{t} \tag{19}$$

Where ρ is the air density, μ is the viscosity, u is the particle velocity, r_1 is the radius vector. The solution of u is assumed to be the sin function of the time, thus $\dot{u} = j\omega u$. To take into account $l = t$, the equation (19) coincides with the equation (7) which was derived by Melling. And imposing the boundary condition that the velocity is 0 at the tube wall, u is obtained as follows as the function of r_1.

$$u(r_1) = -\frac{\Delta p}{\mu k^2 t}\left[1 - \frac{J_0(kr_1)}{J_0(kr_0)}\right] \quad , \quad k = \sqrt{-j\omega\rho/\mu} \tag{20}$$

Where r_0 is the radius of the tube, J_0 is the zero order Bessel function of the first kind. This equation coincides with the equation (10). The mean velocity of the tube cross sectional area is known from the equation (20) and the characteristic impedance of the hole becomes as follows.

$$Z_1 = \frac{\Delta p}{u} = j\omega\rho t\left[1 - \frac{2}{x\sqrt{-j}}\frac{J_1(x\sqrt{-j})}{J_0(x\sqrt{-j})}\right]^{-1} \tag{21}$$

Where $x = r_0\sqrt{\omega\rho/\mu}$, J_1 is the first order Bessel function of the first kind. x is the ratio of the hole radius to the boundary layer thickness.

Calculating the impedance from the equation (21), The following equation can be obtained.

$$Z_1 = \frac{8\mu t}{r_0^2} + j\frac{4}{3}\rho\omega t = R + j\chi \quad for \quad x < 1 \tag{22}$$

$$Z_1 = (2\mu t / r_0)\sqrt{\omega\rho/2\mu}(1 + j) + j\rho\omega t = R + j\chi \quad for \quad x > 10 \tag{23}$$

Rewriting the equation (23)

$$Z_1 = (2\mu t / r_0)\sqrt{\omega\rho/2\mu} + j((2\mu t / r_0)\sqrt{\omega\rho/2\mu} + \rho\omega t)$$
$$= \frac{t}{r_0}\sqrt{2\rho\mu\omega} + j\omega\rho t\left[1 + \frac{1}{r_0}\sqrt{\left(\frac{2\mu}{\rho\omega}\right)}\right] = R + j\chi \quad for \quad x > 10 \tag{24}$$

This equation coincides with the equation (17) by Melling.

Obtaining the ratio of the resistance R to the reactance $\chi = \omega M$

$$\frac{R}{\omega M} \to \frac{6}{x^2} \quad (x < 1) \qquad \frac{R}{\omega M} \to \frac{1}{(1 + x/\sqrt{2})} \quad (x > 10) \tag{25}$$

(See APPENDIX F)

That is to say, the value of the ratio is very large for x<1 and small for x>10. This fact is needed for the perforated plate with micro holes.

VI CONCLUSION

In order to clarify why the analytical result of the natural acoustic frequency for the duct with the perforated plate in the middle by Transfer Matrix Method with Melling's impedance is in good agreement with the experimental one in an integrated manner, Melling's equation and Dah-you Maa's equation are compared and discussed. As a result, the following conclusions could be obtained.

1) It was clarified that the theory by Dah-You Maa was the same as one by Melling even though Dah-You Maa had investigated 15 years later than Melling. Because both theories were constructed based on the Crandall theory.
2) As both theories reached the same result, I thought that these results were confirmed to be correct and useful. I will recommend well these theories not only to apply for the natural acoustic frequency but also for acoustic damping in investigations of the perforated plate.

3) The applicability of Melling's theory should be modified that Equation (15) for $|k_s r_0|<1$ and $r_0^2 f<0.025$, Equation (17) for $|k_s r_0|>10$ and $r_0^2 f>2.5$, when $\mu=1.83\times10^{-5}$ Pa・s. On the other hand Equation (15) for $|k_s r_0|<1$ and $r_0^2 f<0.05$, Equation (17) for $|k_s r_0|>10$ and $r_0^2 f>5$, when $\mu=3.66\times10^{-5}$ Pa・s.

APPENDIX A

Deriving Equation of Motion [Equation (8)]

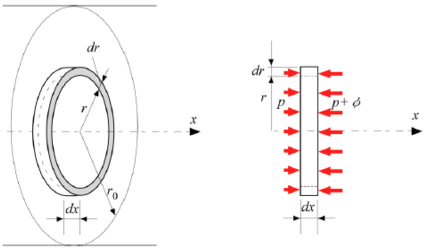

Figure A1 Analytical model of ring shaped

Considering air of ring shaped as shown in the figure A1 at a constant distance from the center axis of the cylinder. Air volume of ring part can be written as follows.

$$\rho dx 2\pi r dr \tag{A-1}$$

Where ρ is the air density, r is the radius of the ring, dr is the thickness of the radial direction, dx is the thickness of the axial direction.

The force acting on the air of the ring shaped can be given as follows as the pressure gradient is ϕ.

$$\phi dx 2\pi r dr \tag{A-2}$$

The shearing force due to the viscosity can be written by the next equation by multiplying the area.

$$F_{in} = -\mu \frac{\partial \dot{\xi}(r)}{\partial r} 2\pi r dx \tag{A-3}$$

And the force acting on the outer surface is given as follows.

$$F_{out} = F_{in} + \frac{\partial F_{in}}{\partial r} dr$$

$$F_{out} = F_{in} - 2\pi dx \mu \frac{\partial}{\partial r}\left(r\frac{\partial \dot{\xi}}{\partial r}\right) dr \tag{A-4}$$

Then the force due to the viscosity acting on the ring part becomes

$$\Delta F = F_{out} - F_{in} = -2\pi dx \mu \frac{\partial}{\partial r}\left(r\frac{\partial \dot{\xi}}{\partial r}\right) dr \tag{A-5}$$

From above equations, the equation of motion of the air of the ring shaped becomes

$$\rho dx 2\pi r dr j\omega\dot{\xi} + \left\{-2\pi dx \mu \frac{\partial}{\partial r}\left(r\frac{\partial \dot{\xi}}{\partial r}\right) dr\right\} = \phi dx 2\pi r dr$$

$$\rho dx 2\pi r dr j\omega\dot{\xi} + \left\{-2\pi r dx \frac{\mu}{r} \frac{\partial}{\partial r}\left(r\frac{\partial \dot{\xi}}{\partial r}\right) dr\right\} = \phi dx 2\pi r dr \tag{A-6}$$

Dividing both sides by $2\pi r dr dx$

Copyediting this equation,

$$j\omega\rho\dot{\xi} - \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \dot{\xi}}{\partial r}\right) = \phi$$

$$\left[j\omega\rho - \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\dot{\xi} = \phi$$

$$\left[j\omega\rho - \frac{\mu}{r}\left(\frac{\partial}{\partial r} + r\frac{\partial^2}{\partial r^2}\right)\right]\dot{\xi} = \phi$$

$$\left[j\omega\rho - \frac{\mu}{r}\frac{\partial}{\partial r} - \mu\frac{\partial^2}{\partial r^2}\right]\dot{\xi} = \phi$$

$$\left[j\frac{\omega\rho}{\mu} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2}\right]\dot{\xi} = \frac{\phi}{\mu} \tag{A-7}$$

Putting

$k_s^2 = -j\omega\rho/\mu$

The equation (A-7) becomes the next equation.

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + k_s^2\right]\dot{\xi} = -\frac{\phi}{\mu} \tag{A-8}$$

APPENDIX B

Solution of Equation (8)

Equation (8) is rewritten here.

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + k_s^2\right]\dot{\xi} = -\frac{\phi}{\mu} \tag{A-9}$$

If the right hand term of Equation (A-9) is put zero, this equation is the differential equation of Bessel. So the general solution can be described as follows.

$$\dot{\xi} = AJ_0 + BY_0$$

Where J_0 and Y_0 are Bessel functions of the first and the second kind, respectively. A and B are constant. As $Y_0(0)$ is infinity and $\dot{\xi}(0)$ is finite B must be zero in this case. And Equation (A-9) has $\dot{\xi}=-\phi/(k_s^2\mu)$ as the special solution the general solution of the equation (A-8) can be written as follows.

$$\dot{\xi}(r) = -\frac{\phi}{\mu k_s^2} + AJ_0(k_s r) \tag{A-10}$$

APPENDIX C

Calculation of mean value

$$\dot{\xi} = \frac{1}{\pi r_0^2}\int_0^{r_0}\dot{\xi}(r) 2\pi r dr$$

$$= -\frac{2}{r_0^2}\int_0^{r_0} r\left\{\frac{\phi}{\mu k_s^2} - \frac{\phi}{J_0(k_s r_0)}\frac{1}{\mu k_s^2}J_0(k_s r)\right\}dr$$

$$= -\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0} r dr + \frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0}\frac{J_0(k_s r)}{J_0(k_s r_0)} r dr \tag{A-11}$$

First term becomes as follows after calculation

$$-\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0} r dr = -\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\left[\frac{r^2}{2}\right]_0^{r_0} = -\frac{\phi}{\mu k_s^2} \tag{A-12}$$

Second term becomes as follows by using the relation between J_0 and J_1.

$$\frac{\partial}{\partial r}(rJ_1(ar)) = arJ_0(r) \tag{A-13}$$

$$\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0}\frac{J_0(k_s r)}{J_0(k_s r_0)}rdr=\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0}\frac{1}{k_s}k_s r\frac{J_0(k_s r)}{J_0(k_s r_0)}dr$$
$$=\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\int_0^{r_0}\frac{1}{k_s}k_s\frac{\partial}{\partial r}\left(r\frac{J_1(k_s r)}{J_0(k_s r_0)}\right)dr$$
$$=\frac{2}{r_0^2}\frac{\phi}{\mu k_s^2}\frac{1}{k_s}\left[r\frac{J_1(k_s r)}{J_0(k_s r_0)}\right]_0^{r_0}$$
$$=\frac{2}{r_0}\frac{\phi}{\mu k_s^2}\frac{1}{k_s}\frac{J_1(k_s r_0)}{J_0(k_s r_0)}$$
$$=\frac{\phi}{\mu k_s^2}\frac{2}{k_s r_0}\frac{J_1(k_s r_0)}{J_0(k_s r_0)} \quad \text{(A-14)}$$

Finally the mean velocity becomes

$$\dot{\xi}=\frac{\phi}{\mu k_s^2}\left[1-\frac{2}{k_s r_0}\frac{J_1(k_s r_0)}{J_0(k_s r_0)}\right] \quad \text{(A-15)}$$

APPENDIX D

Deriving of Impedance z'

Consider the impedance of hole

$$z'=\frac{p}{\dot{\xi}}=-\frac{\phi l}{1-\frac{2}{k_s r_0}\frac{J_1(k_s r_0)}{J_0(k_s r_0)}}\frac{\mu k_s^2}{\phi} \quad \text{(A-16)}$$

Rewriting Eq.(A-16) by using $k_s^2=-j\rho\omega/\mu$

$$z'=\frac{j\omega\rho l}{1-\frac{2}{k_s r_0}\frac{J_1(k_s r_0)}{J_0(k_s r_0)}} \quad \text{(A-17)}$$

Consider the approximation of Eq.(A-17)

Describing the Bessel function of the first kind by using the series.

$$J_n(x)=\sum_{k=0}^{\infty}(-1)^k\frac{x^{n+2k}}{2^{n+2k}k!(n+k)!} \quad \text{(A-18)}$$

The zero order and the first order Bessel functions of the first kind can therefore be written as follows, respectively.

$$J_1(x)=\sum_{k=0}^{\infty}(-1)^k\frac{x^{1+2k}}{2^{1+2k}k!(1+k)!} \quad \text{(A-19)}$$

$$J_0(x)=\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!} \quad \text{(A-20)}$$

Here consider the next equation.

$$f(x)=\frac{1}{1-\frac{2}{x}\frac{J_1(x)}{J_0(x)}} \quad \text{(A-21)}$$

Substituting Eq.(A-19) and (A-20) into Eq.(A-21) and organizing

$$f(x)=\frac{1}{1-\frac{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!(1+k)!}}{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!}}}$$
$$=\frac{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!}}{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!}-\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!(1+k)!}}$$
$$=\frac{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!}}{\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!k!}\left(1-\frac{k!}{(1+k)!}\right)} \quad \text{(A-22)}$$

Here consider from zero order to second order terms in both the numerator and the denominator.

$$f(x)=\frac{1-\frac{x^2}{4}+\frac{x^4}{64}\cdots}{-\frac{x^2}{8}+\frac{x^4}{64}\frac{2}{3}\cdots}$$
$$=\frac{8-2x^2+\frac{x^4}{8}\cdots}{x^2\left(-1+\frac{x^2}{12}\right)\cdots} \quad \text{(A-23)}$$

Now describing this equation the next equation by using the constant *A*, *B* and *C*

$$f(x)=\frac{8-2x^2+\frac{x^4}{8}}{x^2\left(-1+\frac{x^2}{12}\right)}=\frac{A}{x^2}+\frac{B}{-1+\frac{x^2}{12}}+C \quad \text{(A-24)}$$

Determining *A* and *B* from Equation (A-24)

$$f(x)=-\frac{8}{x^2}+\frac{\frac{4}{3}}{1-\frac{x^2}{12}}+C \quad \text{(A-25)}$$

Consequently in the case of $|x|<1$

$$f(x)=-\frac{8}{x^2}+\frac{4}{3}+C \quad \text{(A-26)}$$

The next equation can be obtained in the case of $k_s r_0$ being very small.

$$z'=j\omega\rho l\, f(k_s r_0)$$
$$=j\omega\rho l\left(-\frac{8}{k_s^2 r_0^2}+\frac{4}{3}\right)$$
$$=\frac{8\mu l}{r_0^2}+j\frac{4}{3}\omega\rho l \quad \text{(A-26)}$$

APPENDIX E

Next consider the case of $k_s r_0$ being large.

We can rewrite Eq.(A-17) by deforming it

$$z'=\frac{j\omega\rho l}{1-\frac{2}{k_s r_0}\frac{J_1(k_s r_0)}{J_0(k_s r_0)}}$$
$$=\frac{j\omega\rho l}{1-\frac{2}{k_s r_0}\frac{J_1\left(\sqrt{-j}\sqrt{\frac{\omega\rho}{\mu}}r_0\right)}{J_0\left(\sqrt{-j}\sqrt{\frac{\omega\rho}{\mu}}r_0\right)}} \quad \text{(A-27)}$$

From the characteristics of Bessel function the next formula holds in the case of $x\to\infty$

$$\frac{J_1\left(\sqrt{-j}x\right)}{J_0\left(\sqrt{-j}x\right)}\to -j \quad \text{(A-28)}$$

Therefore Eq.(A-27) becomes as follows in the case of $k_s r_0$ being large

$$z' = \frac{j\omega\rho l}{1 - \frac{2}{k_s r_0}(-j)} = j\omega\rho l\left\{1 + \left(-j\frac{2}{k_s r_0}\right) + \left(-j\frac{2}{k_s r_0}\right)^2 + \cdots\right\} \tag{A-29}$$

Here neglecting more second order terms the following equation can be obtained.

$$\begin{aligned} z' &= j\omega\rho l\left\{1 + \left(-j\frac{2}{k_s r_0}\right)\right\} \\ &= j\omega\rho l + \omega\rho l\frac{2}{r_0}\frac{1}{\sqrt{-j}}\sqrt{\frac{\mu}{\omega\rho}} = j\omega\rho l + \frac{2}{r_0}\frac{\sqrt{2}}{1-j}l\sqrt{\omega\rho\mu} \\ &= j\omega\rho l + \frac{1+j}{r_0}l\sqrt{2\omega\rho\mu} \\ &= \frac{l}{r_0}\sqrt{2\omega\rho\mu} + j\omega\rho l\left(1 + \frac{1}{r_0}\sqrt{\frac{2\mu}{\omega\rho}}\right) \end{aligned} \tag{A-30}$$

APPENDIX F

Deriving Eq.(25)

In the case of x<1

$$Z_1 = \frac{8\mu t}{r_0^2} + j\frac{4}{3}\rho\omega t = R + j\chi \quad for \quad x < 1$$

$$\frac{R}{\chi} = \frac{8\mu t}{r_0^2}\frac{3}{j4\rho\omega t} = \frac{6}{jr_0^2\rho\omega/\mu} = \frac{6}{jx^2} \tag{A-31}$$

In the case of x>10

$$Z_1 = (2\mu t/r_0)\sqrt{\omega\rho/2\mu} + j((2\mu t/r_0)\sqrt{\omega\rho/2\mu} + \rho\omega t)$$

$$= \frac{t}{r_0}\sqrt{2\rho\mu\omega} + j\omega\rho t\left[1 + \frac{1}{r_0}\sqrt{\left(\frac{2\mu}{\rho\omega}\right)}\right] = R + j\chi \quad for \quad x > 10$$

$$\frac{R}{\chi} = \frac{\frac{t}{r_0}\sqrt{2\rho\mu\omega}}{\omega\rho t\left[1 + \frac{1}{r_0}\sqrt{\frac{2\mu}{\rho\omega}}\right]} = \frac{1}{\frac{r_0\sqrt{\omega\rho}}{\sqrt{2\mu}}\left[1 + \frac{\sqrt{2}}{x}\right]} = \frac{\sqrt{2}}{x\left[1 + \frac{\sqrt{2}}{x}\right]} = \frac{1}{(1 + x/\sqrt{2})} \tag{A-32}$$

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