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The Exponential Attractor for Kirchhoff type Suspension Bridge Equations with Linear Memory and Polynomial Damping

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Abstract—**In the article, by using so-called enhanced flattening property, we investigated the existence of exponential attractors for Kirchhoff type suspension equations with linear memory and polynomial damping. Some known results are improved and extended.**

***Index Terms*—Suspension Bridge Equations; Enhanced Flattening Property; Exponential Attractor.**

I. INTRODUCTION

Let Ω is a bounded domain in $\mathbb{R}$ with smooth boundary$\partial\Omega$, we are concerned with the the exponential attractor to the following Kirchhoff type suspension bridge equations

$$\begin{cases} u_{tt}+a\Delta^2u-(\alpha+\beta\|\nabla u\|^2)\Delta u+\gamma\|u_t\|^r u_t+k^2u^+ \\ \quad -\int_0^\infty \mu(s)\Delta^2u(t-s)ds+f(u)=f(x), & x\in\Omega, t\geq 0, \\ u=\Delta u=0(\ or\ \ u=\nabla u=0), & x\in\partial\Omega, \\ u(x,0)=u_0(x), u_t(x,0)=u_1(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where $\delta>0, \beta>0, \gamma>0$ and $r\geq 0$ are given constants, the real constant α accounts for the axial force acting at the end of the road bed of the bridge in the reference configuration, k^2 is the spring constant and $u^+=\max\{u,0\}$, the memory kernel μ and the nonlinearity $f(u)$ satisfy the following assumptions respectively:

(I_1) $\mu\in C^1(\mathbb{R}^+)\cap L^1(\mathbb{R}^+),\ \mu'(s)\leq 0\leq\mu(s),\ \forall\ s\in\mathbb{R}^+.$

(I_2) $\int_0^\infty \mu(s)ds=\mu_0>0,\ \forall\ s\in\mathbb{R}^+.$

(I_3) $\mu'(s)+\delta\mu(s)\leq 0,\ \forall\ s\in\mathbb{R}^+$, $\delta>0$.

(I_4) $\liminf\limits_{|s|\to\infty}\frac{f(s)}{s}\geq\lambda_1,\ \forall\ s\in\mathbb{R}$, where λ_1 is the first eigenvalue of $\triangle^2$ with boundary condition $u|_{\partial\Omega}=\Delta u|_{\partial\Omega}=0$ or $(u|_{\partial\Omega}=\nabla u|_{\partial\Omega}=0)$.

(I_5) $|f'(s)|\leq k_0(1+|s|^p)$, where $k_0>0$ and $p\geq 1$.

Assumption (I_4) and the definition of $F(u)=\int_0^u f(s)ds$ imply that

$$F(s)+\eta s^2+K_1>0,\ \forall\ s\in\mathbb{R},\ K_1>0,\ \eta>0, \tag{1.2}$$

$$-M_0\leq F(u)\leq\frac{1}{2}f(u)u+M_1,\ M_0,M_1>0. \tag{1.3}$$

For the model (1.1), it is originally in suspension bridge equations which were posed as a new problem in the field of nonlinear analysis [1] by Lazer and McKenna in 1990. In the suspension bridge system, suspension bridge can be regarded as an elastic beam with hinged ends (or both fixed points). Lately, similar models have been studied by many authors, see [2-10]. Zhong et al.[2] investigated the existence of the strong solutions and strong global attractors for the single suspension bridge equations utilizing the condition (C) introduced [10] and the technique of energy estimates. In [3], the authors obtained exponential attractors of suspension bridge equation. Kang [4] considered the asymptotic behavior of the thermoelastic suspension bridge equation with past history. Besides, the problem of longtime behavior of the solutions to the suspension bridge equations has been studied by plently of authors [5-7]. In this article, under a weaker condition of the nonlinearity, we take advantage of the enhanced flattening property proposed by Li and Wu [8] to show the existence of exponential attractor for Kirchhoff type suspension equations with linear memory and polynomial damping.

II. PRELIMINARIES

We consider $H=L^2(\Omega)$ with norm $\|\cdot\|$ and inner product $(\cdot,\cdot)$. We also define the space $V_1=H_0^1(\Omega)$, and

$$V_2=D(A^{\frac{1}{2}})=\begin{cases} H_0^2(\Omega), & \text{for } u|_{\partial\Omega}=\nabla u|_{\partial\Omega}=0, \\ H^2(\Omega)\cap H_0^1(\Omega), & \text{for } u|_{\partial\Omega}=\Delta u|_{\partial\Omega}=0, \end{cases}$$

whose norms and scalar products are given by

$$\|u\|_{V_1}=\|\nabla u\|,\ (u,v)_{V_1}=(\nabla u,\nabla v),$$

$$\|u\|_{V_2}=\|\Delta u\|,\ (u,v)_{V_2}=(\Delta u,\Delta v),$$

respectively, where $A=\triangle^2$. And using the Poincaré inequality, for any $\forall\ u\in V_2$, we have

$$\|A^{\frac{1}{2}}u\|^2\geq\lambda_1\|u\|^2, \|A^{\frac{1}{2}}u\|^2\geq\lambda_1^{\frac{1}{2}}\|A^{\frac{1}{4}}u\|^2. \tag{2.1}$$

Then we introduce the Hilbert space

$$L_\mu^2(\mathbb{R}^+,V_i)=\{\xi:\mathbb{R}^+\to V_i|\int_0^\infty \mu(s)\|\xi(s)\|_{V_i}^2ds<\infty\},$$

with the norms and the scalar products

$$(u,v)_{\mu,V_i}=\int_0^\infty \mu(s)(u(s),v(s))_{V_i}ds,$$

$$\|u\|_{\mu,V_i}^2=(u,u)_{\mu,V_i}=\int_0^\infty \mu(s)\|u(s)\|_{V_i}^2ds<\infty, i=1,2.$$

Finally, we define the space $\mathcal{W}=V_2\times H\times L_\mu^2(\mathbb{R}^+,V_2)$, endowed with the norm

$$\|(u,u_t,\xi^t)\|_{\mathcal{W}}^2=\|\Delta u\|^2+\|u_t\|^2+\|\xi^t\|_{\mu,V_2}^2.$$

In order to accomplish our main results, we need to convert (1.1) into a deterministic autonomous dynamical system. Therefore, motivated by [1], we introduce the displacement variable

$$\xi=\xi^t(x,s)=u(x,t)-u(x,t-s),\ (x,s)\in\Omega\times\mathbb{R}^+,\ t\geq 0, \tag{2.2}$$

and thus

$$\xi_t^t(x,s)+\xi_s^t(x,s)=u_t(x,t),\ (x,s)\in\Omega\times\mathbb{R}^+,\ t\geq 0. \tag{2.3}$$

Let $a=1+\int_0^\infty \mu(s)ds$ and $\mu\in L^1(\mathbb{R}^+)$, then the equation (1.1) is equivalent to the equation

$$\begin{cases} u_{tt}+\Delta^2u-(\alpha+\beta\|\nabla u\|^2)\Delta u+\gamma\|u_t\|^r u_t+k^2u^+ \\ \quad +\int_0^\infty \mu(s)\Delta^2\xi^t(s)ds+f(u)=f(x), & (x,t)\in\Omega\times\mathbb{R}^+, \\ \xi_t^t=u_t-\xi_s^t, & (x,t,s)\in\Omega\times\mathbb{R}^+\times\mathbb{R}^+, \\ \xi=\Delta\xi=0(\ or\ \ \xi=\nabla\xi=0), \\ u=\Delta u=0(\ or\ \ u=\nabla u=0), & (x,s)\in\partial\Omega\times\mathbb{R}^+,\ t\geq 0, \\ \xi(x,0)=0,\ \xi^0(x,s)=\xi_0(x,s), \\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), \end{cases} \tag{2.4}$$

where

$$\begin{cases} u_0(x) = u_0(x,0),\ u_1(x) = \partial_t u_0(x,t)|_{t=0}, & x \in \Omega, \\ \xi_0(x,s) = u_0(x,0) - u(x,-s), & (x,s) \in \Omega \times \mathbb{R}^+. \end{cases}$$

For the sake of the existence of exponential attractor, we recall here some abstract results.

Definition 2.1[8] (Enhanced Flattening Property): Let X be a uniformly convex Banach space, for any bounded set B of X, there exist $k, l > 0, T > 0$ and a finite dimension subspace X_1 of X, such that

(1) $P_n\big(\cup_{s\geq t} S(t)B\big)$ is bounded, and

(2)$\|(I-P_n)\big(\cup_{s\geq t} S(t)x\big)\| \leq ke^{-lt} + k(n), \forall x \in B, \forall t \geq T.$

Here $P_n : X \to X_1$ is a bounded projector, n is the dimension of X_1 and $k(s)$ is a real-valued function satisfying

$$\lim_{s\to\infty} k(s) = 0.$$

Theorem 2.2[8]: Let B is a bounded absorbing set for $S(t)$ in X, then the following are equivalent:

(1) The measure of non-compactness is exponentially decaying for $\cup_{s\geq t} S(t)B$, i.e., there exist $k, l > 0$ such that

$$\rho\big(\cup_{s\geq t} S(s)B\big) \leq ke^{-lt}.$$

(2) For $S(t)$, there exist exponential attractors.

Theorem 2.3[8]: Assume the semigroup $\{S(t)\}_{t\geq 0}$ satisfies the enhanced flattening property, then the measure of non-compactness decays exponentially for the semigroup $\cup_{s\geq t} S(t)B$.

Theorem 2.4[8]: Let X be a uniformly convex Banach space, and $\{S(t)\}_{t\geq 0}$ be a continuous or norm-to-weak continuous semigroup in X. Then for $\{S(t)\}_{t\geq 0}$ there exist exponential attractors if the following condictions hold true:

(1) there is a bounded absorbing set $B \subset X$, and

(2) $S(t)$ satisfies the enhanced flattening property.

Theorem 2.5[2,10]: Let assumptions $(I_1) - (I_5)$ and g$\in L^2(\Omega)$ hold true.Then for any $(u_0, u_1, \xi_0) \in \mathcal{W}$ and any $T > 0$, there exists a unique solution u of (1.1) such that

$$u \in C([0,T]; V_2), u_t \in C([0,T]; H),$$

$$\xi^t \in C([0,T]; L^2_\mu(\mathbb{R}^+; V_2)).$$

Moreover, the solution continuously depends on the initial data on $\mathcal{W}$.

Lemma 2.6: In view of Theorem 2.5, problem (1.1) generates a C_0-semigroup $S(t) : \mathcal{W} \to \mathcal{W}$ in the space $\mathcal{W}$, where

$$S(t) : \{u_0, u_1, \xi_0\} \to \{u(t), u_t(t), \xi^t\}, t > 0.$$

III. Bounded Absorbing Set

Theorem 3.1: Let us assume $(I_1) - (I_5)$ and $\alpha > \frac{2\eta}{\sqrt{\lambda_1}} - \frac{\sqrt{\lambda_1}}{4}$. Then the semigroup $\{S(t)\}_{t\geq 0}$generated by (1.1) in the space $\mathcal{W}$ have a bounded absorbing set

$$\mathfrak{B}_0 = \{(u, u_t, \xi^t) \in \mathcal{W}; \|(u, u_t, \xi^t)\|_{\mathcal{W}} \leq R\},$$

i.e. there exists $R > 0$ possessing the property: for any bounded set $B \subset \mathcal{W}$ there exists $t_0 = t(B) > 0$ such that $\|S(t)y\|^2_{\mathcal{W}} = \|\big(u(t), u_t(t), \xi^t\big)\|^2_{\mathcal{W}} \leq R$ for all $t \geq t_0$ and $y_0 = (u_0, u_1, \xi_0) \in B$.

Proof: Multiplying (2.4) by u_t, and integrating over Ω, we obtain

$$\frac{d}{dt}\Big(\frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\triangle u\|^2 + \frac{\alpha}{2}\|\nabla u\|^2 + \frac{\beta}{4}\|\nabla u\|^4 + \frac{k^2}{2}\|u^+\|^2 + (F(u),1) - (g,u)\Big) + \gamma(|u_t|^r u_t, u_t) + (\xi^t, u_t)_{\mu,V_2} = 0. \quad (3.1)$$

Using (I_3), (2.4) and Hölder inequality, one has

$$(\xi^t, u_t)_{\mu,V_2} \geq \frac{1}{2}\frac{d}{dt}\|\xi^t\|^2_{\mu,V_2} + \frac{\delta}{2}\|\xi^t\|^2_{\mu,V_2}, \quad (3.2)$$

Thus, by combining (3.1) with (3.2), implies that

$$\frac{d}{dt}\mathcal{F}(t) + \frac{\delta}{2}\|\xi^t\|^2_{\mu,V_2} + \gamma\|u_t\|^{r+2}_{r+2} \leq 0, \quad (3.3)$$

where

$$\mathcal{F}(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\triangle u\|^2 + \frac{\alpha}{2}\|\nabla u\|^2 + \frac{\beta}{4}\|\nabla u\|^4 + \frac{k^2}{2}\|u^+\|^2 + \frac{1}{2}\|\xi^t\|^2_{\mu,V_2} + (F(u),1) - (g,u).$$

Integrating (3.3) over [t,t+1], we deduce that

$$\gamma\int_t^{t+1} \|u_t(s)\|^{r+2}_{r+2} ds \leq \mathcal{F}(t) - \mathcal{F}(t+1) = [H(t)]^2. \quad (3.4)$$

By Hölder inequality with $\frac{r}{r+2} + \frac{2}{r+2} = 1$, we have

$$\int_t^{t+1}\int_\Omega |u_t(s)|^2 dx ds \leq |\Omega|^{\frac{r}{r+2}} \int_t^{t+1} \|u_t(s)\|^2_{r+2} ds \leq \frac{1}{\gamma^{\frac{2}{r+2}}}|\Omega|^{\frac{r}{r+2}}[H(t)]^{\frac{4}{r+2}}, \quad (3.5)$$

which implies that there exist $t_1 \in [t, t+\frac{1}{4}]$ and $t_2 \in [t+\frac{3}{4}, t+1]$ such that

$$\|u_t(t_i)\|^2 \leq \frac{4|\Omega|^{\frac{r}{r+2}}}{\gamma^{\frac{2}{r+2}}}[H(t)]^{\frac{4}{r+2}}, i = 1, 2. \quad (3.6)$$

Then multiplying (2.4) by $u(t)$ and integrating over $\Omega \times [t_1, t_2]$, we get

$$\int_{t_1}^{t_2}\Big(\|\triangle u(s)\|^2 + \alpha\|\nabla u(s)\|^2 + \beta\|\nabla u(s)\|^4 + k^2\|u^+(s)\|^2 + (f(u), u) - (g, u)\Big)ds$$
$$= (u(t_1), u_t(t_1)) - (u(t_2), u_t(t_2)) + \int_{t_1}^{t_2}\|u_t(s)\|^2 ds - \gamma\int_{t_1}^{t_2}(|u_t|^r u_t, u)ds - \int_{t_1}^{t_2}(\xi^t, u)_{\mu,V_2} ds. \quad (3.7)$$

From Hölder, Young inequality and (1.2)-(2.1), we have

$$(g, u) \leq \frac{1}{\lambda_1}\|g\|^2 + \frac{1}{4}\|\triangle u\|^2, \quad (3.8)$$

$$(F(u), 1) \geq -\eta\|u\|^2 - K_1|\Omega| \geq -\frac{\eta}{\sqrt{\lambda_1}}\|\nabla u\|^2 - K_1|\Omega|. \quad (3.9)$$

Using (3.8) and (3.9), we assume $\mathcal{F}_g(t) = \mathcal{F}(t) + \frac{C_1}{8}$, where $C_1 = 8K_1|\Omega| + \frac{8}{\lambda_1}\|g\|^2$. Hence, when $\alpha > \frac{2\eta}{\sqrt{\lambda_1}} - \frac{\sqrt{\lambda_1}}{4}$ we conclude

$$\mathcal{F}_g(t) \geq \frac{1}{2}\|u_t\|^2 + \frac{1}{8}\|\triangle u\|^2 + \frac{1}{2}\|\xi^t\|^2_{\mu,V_2} \geq \frac{1}{8}\|(u, u_t, \xi^t)\|^2_{\mathcal{W}}. \quad (3.10)$$

Combining (I_2) and Young inequality, we infer

$$|(\xi^t, u)_{\mu,V_2}| \leq \frac{1}{2}\|\xi^t\|^2_{\mu,V_2} + \frac{\mu_0}{2}\|\triangle u\|^2. \quad (3.11)$$

Now by (3.11) and taking the sum of $\int_{t_1}^{t_2}\big[\frac{1}{2}\|u_t\|^2 + \frac{C_1}{8} + \frac{1}{2}\|\xi^t\|^2_{\mu,V_2} + (F(u),1)\big]ds$ in both sides of (3.7), we obtain

$$\int_{t_1}^{t_2}\mathcal{F}_g(s)ds \leq \|u(t_1)\|\|u_t(t_1)\| + \|u(t_2)\|\|u_t(t_2)\| + \frac{3}{2}\int_{t_1}^{t_2}\|u_t(s)\|^2 ds + \gamma\int_{t_1}^{t_2}\int_\Omega |u_t|^{r+1}|u| dx ds + \int_{t_1}^{t_2}\int_\Omega (F(u) - f(u)u) dx ds + \frac{\mu_0}{2}\int_{t_1}^{t_2}\|\triangle u(s)\|^2 ds + \int_{t_1}^{t_2}\|\xi^t\|^2_{\mu,V_2} ds + \frac{C_1}{8}.$$

From (2.1), (3.6) and (3.10), it follows that

$$\|u(t_i)\|\|u_t(t_i)\| \leq \frac{2|\Omega|^{\frac{r}{2(r+2)}}}{\gamma^{\frac{1}{r+2}}}[H(t)]^{\frac{2}{r+2}} \sup_{t\leq s\leq t+1}\|u(s)\|$$
$$\leq \frac{4\sqrt{2}|\Omega|^{\frac{r}{2(r+2)}}}{\gamma^{\frac{1}{r+2}}\sqrt{\lambda_1}}[H(t)]^{\frac{2}{r+2}} \sup_{t\leq s\leq t+1}\mathcal{F}_g^{\frac{1}{2}}(s)$$
$$\leq \frac{128|\Omega|^{\frac{r}{r+2}}}{\lambda_1\gamma^{\frac{2}{r+2}}}[H(t)]^{\frac{4}{r+2}} + \frac{1}{16}\sup_{t\leq s\leq t+1}\mathcal{F}_g(s), i = 1, 2. \quad (3.12)$$

By Hölder inequality, (3.4)-(3.10) and immersion $V_2 \hookrightarrow L^{p+2}(\Omega)$, we have

$$\int_{t_1}^{t_2}\int_\Omega |u_t|^{r+1}|u|dxds \le \Big(\int_{t_1}^{t_2}\int_\Omega |u_t|^{r+2}dxds\Big)^{\frac{r+1}{r+2}}\Big(\int_{t_1}^{t_2}\int_\Omega |u|^{r+2}dxds\Big)^{\frac{1}{r+2}} \le (\frac{H(t)^2}{\gamma})^{\frac{r+1}{r+2}}C_r \sup_{t\le s\le t+1}\|\Delta u(s)\| \le 2\sqrt{2}C_r(\frac{H(t)^2}{\gamma})^{\frac{r+1}{r+2}}\sup_{t\le s\le t+1}\mathcal{F}_g^{\frac12}(s) \le \frac{32C_r^2}{\gamma^{\frac{2(r+1)}{r+2}}}[H(t)]^{\frac{4(r+1)}{r+2}}+\frac{1}{16}\sup_{t\le s\le t+1}\mathcal{F}_g(s), \quad (3.13)$$

Combining (3.10) and Young inequality, we get

$$\frac{\mu_0}{2}\int_{t_1}^{t_2}\|\Delta u(s)\|^2 ds \le C_2+\frac{1}{32}\sup_{t\le s\le t+1}\mathcal{F}_g(s), \quad (3.14)$$

$$\int_{t_1}^{t_2}\|\xi^t\|^2_{\mu,V_2}ds \le C_3+\frac{1}{32}\sup_{t\le s\le t+1}\mathcal{F}_g(s). \quad (3.15)$$

In view of (1.3) and (3.5)-(3.15), it follows that

$$\int_{t_1}^{t_2}\mathcal{F}_g(s)ds \le \Big(C_4+C_5[H(t)]^{\frac{4r}{r+2}}\Big)[H(t)]^{\frac{4}{r+2}}+\frac14\sup_{t\le s\le t+1}\mathcal{F}_g(s) +(M_0+2M_1)|\Omega|+C_2+C_3+\frac{C_1}{8}, \quad (3.16)$$

where $C_4=\frac{256|\Omega|^{\frac{r}{r+2}}}{\lambda_1\gamma^{\frac{2}{r+2}}}+\frac{3|\Omega|^{\frac{r}{r+2}}}{2\gamma^{\frac{2}{r+2}}}, C_5=\frac{32C_r^2}{\gamma^{\frac{2(r+1)}{r+2}}}$. Using (3.4) and the mean value theorem, there exists $\tilde t\in(t_1,t_2)$ such that

$$\int_{t_1}^{t_2}\mathcal{F}_g(s)ds=\mathcal{F}_g(\tilde t)(t_2-t_1)\ge\frac12\mathcal{F}_g(t+1),$$

$$\mathcal{F}_g(t)\le[H(t)]^2+2\int_{t_1}^{t_2}\mathcal{F}_g(s)ds. \quad (3.17)$$

From (3.16), (3.17) and inequality $\frac{4}{r+2}\le 2$, we get

$$\mathcal{F}_g(t)\le(2C_4+2C_5[H(t)]^{\frac{4r}{r+2}}+[H(t)]^{2-\frac{4}{r+2}})[H(t)]^{\frac{4}{r+2}} \frac12\sup_{t\le s\le t+1}\mathcal{F}_g(s)+(2M_0+4M_1)|\Omega|+2C_2+2C_3+\frac{C_1}{4}. \quad (3.18)$$

Using the definition of $H(t)$, we obtain

$$2C_4+2C_5[H(t)]^{\frac{4r}{r+2}}+[H(t)]^{2-\frac{4}{r+2}}\le C_6=C_6(B).$$

Therefore, (3.18) can be rewritten as

$$\sup_{t\le s\le t+1}\mathcal{F}_g(s)\le 2C_6[H(t)]^{\frac{4}{r+2}}+(4M_0+8M_1)|\Omega| +4C_2+4C_3+\frac{C_1}{2}.$$

By inequality $(a+b)^n\le 2^n(a^n+b^n)$, we infer

$$\sup_{t\le s\le t+1}\mathcal{F}_g^{1+\frac{r}{2}}(s)\le C_7[\mathcal{F}(t)-\mathcal{F}(t+1)]+C_8^{\frac{r+2}{2}}, \quad (3.19)$$

where $C_7=2^{r+2}C_6^{\frac{r+2}{2}}$ and $C_8=C_1+(8M_0+16M_1)|\Omega|+8C_2+8C_3$ are constants depending on B. Thus, applying Nakao′s lemma(see [11,lemma2.1]), we obtain

$$\mathcal{F}_g(t)\le\begin{cases}\Big(\frac{r}{2C_7}(t-1)^++[\mathcal{F}_g(0)]^{-\frac{r}{2}}\Big)^{-\frac{2}{r}}+C_8, if\ r>0,\\ \mathcal{F}_g(0)e^{-\tau t}+C_8^{\frac{r+2}{2}}, \qquad if\ r=0,\end{cases} \quad (3.20)$$

where $\tau=\ln\big(\frac{1+C_7}{C_7}\big)>0$, $s^+=\frac{s+|s|}{2}$. Finally, in view of (3.10) and (3.20), it follows that

$$\|(u,u_t,\xi^t)\|_{\mathcal{W}}\le R. \quad (3.21)$$

IV. Exponential Attractor

From Theorem 2.4, now it remains to show that $S(t)$ satisfies the enhanced flattening property so as to prove the existence of exponential attractor.

Lemma 4.1: Let assumptions $(I_1)-(I_5)$ be valid. Then $f\in C^2(\mathbb{R},\mathbb{R}):H^2(\Omega)\to H^{1,q}(\Omega)(\forall\ q\ge1)$ is compact and continuous.

Theorem 4.2: Let assumptions $(I_1)-(I_5)$ and $r=0$ hold true. Then the corresponding semigroup $S(t)$ of problem (1.1) in the space $\mathcal{W}$ satisfies the enhanced flattening property.

Proof: Assume A in the space H exists eigenvalues $\{\lambda_i\}_{r=1}^\infty(0<\lambda_1\le\cdots\le\lambda_j\le\cdots,\ \lambda_j\to\infty$ as $j\to\infty)$and eigenfunctions$\{e_j\}_{j=1}^\infty$which are orthonormal in H and V_2 such that $Ae_j=\lambda_je_j(\forall j\in\mathbb{N})$. Let $H_n=span\{e_1,e_2,\cdots,e_n\}andP_n:H\to H_n$be an orthogonal projector. For any $(u,u_t,\xi^t)\in\mathcal{W}$, we write $(u,u_t,\xi^t)=(u_1,u_{1t},\xi_1^t)+(u_2,u_{2t},\xi_2^t)=y_1+y_2$ with $(u_1,u_{1t},\xi_1^t)=(P_nu,P_nu_t,P_n\xi^t)$. Set $r=0$, taking the inner product of (2.4) with $\omega_2=u_{2t}+\epsilon u_2(0<\epsilon<1)$ in H, we get

$$\frac12\frac{d}{dt}\Big(\|\omega_2\|^2+\|\Delta u_2\|^2+\alpha\|\nabla u_2\|^2+\frac{\beta}{2}\|\nabla u_2\|^4+k^2\|u_2^+\|^2\Big) +\epsilon(\epsilon-\gamma)(u_2,\omega_2)+(\gamma-\epsilon)\|\omega_2\|^2+\epsilon\|\Delta u_2\|^2+ +\alpha\epsilon\|\nabla u_2\|^2+\beta\epsilon\|\nabla u_2\|^4+k^2\epsilon\|u_2^+\|^2+(\xi^t,\omega_2)_{\mu,V_2} +(f(u),\omega_2)=(g(x),\omega_2). \quad (4.1)$$

Thanks to (2.1), (I_5) , Hölder and Young inequality, we find

$$\epsilon(\epsilon-\gamma)(u_2,\omega_2)+(\gamma-\epsilon)\|\omega_2\|^2+\epsilon\|\Delta u_2\|^2 \ge\epsilon(1-\frac{(\epsilon-\gamma)^2}{2\lambda_1})\|\Delta u_2\|^2+(\gamma-\frac{3\epsilon}{2})\|\omega_2\|^2, \quad (4.2)$$

$$(f(u),\omega_2)\le\frac{\gamma}{4}\|\omega_2\|^2+\frac{1}{\gamma}\|(I-P_n)f(u)\|^2, \quad (4.3)$$

$$(g(x),\omega_2)\le\frac{\gamma}{4}\|\omega_2\|^2+\frac{1}{\gamma}\|(I-P_n)g(x)\|^2. \quad (4.4)$$

Owing to (3.2), (3.11) and (4.1)-(4.4), we deduce

$$\frac12\frac{d}{dt}J(t)+Q(t)\le\frac1\gamma\|(I-P_n)f(u)\|^2+\frac1\gamma\|(I-P_n)g(x)\|^2, \quad (4.5)$$

where

$$J(t)=\|\omega_2\|^2+\|\Delta u_2\|^2+\alpha\|\nabla u_2\|^2+\frac\beta2\|\nabla u_2\|^4+k^2\|u_2^+\|^2+\|\xi^t\|^2_{\mu,V_2},$$

$$Q(t)=\Big(\epsilon(1-\frac{(\epsilon-\gamma)^2}{2\lambda_1})-\frac{\epsilon\mu_0}{2}\Big)\|\Delta u_2\|^2+(\frac\gamma2-\frac{3\epsilon}{2})\|\omega_2\|^2+\alpha\epsilon\|\nabla u_2\|^2 +\beta\epsilon\|\nabla u_2\|^4+k^2\epsilon\|u_2^+\|^2+(\frac\delta2-\frac\epsilon2)\|\xi^t\|^2_{\mu,V_2}.$$

Choosing $\epsilon>0$ small enough and setting $\alpha>-\frac{\sqrt{\lambda_1}}{2}$, we deduce that

$$J(t)\ge\|\omega_2\|^2+\frac12\|\Delta u_2\|^2+(\frac{\sqrt{\lambda_1}}{2}+\alpha)\|\nabla u_2\|^2+\frac\beta4\|\nabla u_2\|^4 +k^2\|u_2^+\|^2+\|\xi^t\|^2_{\mu,V_2}>0,$$

$$Q(t)\ge\frac\epsilon4J(t).$$

Hence,

$$\frac{d}{dt}J(t)+\frac\epsilon2J(t)\le\frac2\gamma\|(I-P_n)f(u)\|^2+\frac2\gamma\|(I-P_n)g(x)\|^2. \quad (4.6)$$

Using Lemma 4.1 and$g\in H$, for any $0<\epsilon<\sqrt{\frac{C}{\lambda_n}}$, there exists $N>0$ such that

$$\|(I-P_n)f(u)\|<\epsilon\le\sqrt{\frac{C}{\lambda_n}}, \quad (4.7)$$

$$\|(I-P_n)g(x)\|<\epsilon\le\sqrt{\frac{C}{\lambda_n}}, \quad (4.8)$$

for$n\ge N$.And when $t\ge t_0$, one can see that $\frac{d}{dt}J(t)+\frac\epsilon2J(t)\le\frac{C_9}{\lambda_n}$, and also applying Gronwall′s Lemma, we have

$$J(t)\le J(t_0)e^{-\frac\epsilon2(t-t_0)}+\frac{C_{10}}{\lambda_n},\ t\ge t_0. \quad (4.9)$$

Obviously, there exists $C_{11}>1$ such that

$$\|y_2(t)\|^2\le J(t)\le C_{11}\|y_2(t))\|^2.$$

So $\quad \|y_2(t)\|^2\le C_{11}\|y_2(t_0)\|^2e^{-\frac\epsilon2(t-t_0)}+\frac{C_{10}}{\lambda_n},\ t\ge t_0.$

Therefore, the enhanced flattening property holds true.

Due to Theorem 3.1 and Theorem 4.2, and combining with Theorem 2.4, we obtain our main results.

Theorem 4.3(Exponential attractor): Let assumptions $(I_1)-(I_5)$ and $r=0$ hold true. Then the corresponding semigroup $S(t)$ of problem (1.1) possess exponential attractors in $\mathcal{W}$.

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