Research on Transfer Alignment Method in Launch Inertial Coordinate System

Jie Li, Cong Wang, Shuai Chen

Abstract— **Due to its own characteristics, a certain type of cruise missile launched under an airborne platform chose to launch the inertial coordinate system as the navigation coordinate system. In order to eliminate the problem of redundant coordinate system conversion caused by the transfer alignment model under the traditional geographic coordinate system, In this paper, the inertial navigation error propagation equation is studied. According to the equation, the transfer alignment model in the launch inertial coordinate system is established. In the existing measurement matching algorithm, the "speed + measurement misalignment angle" matching algorithm is applied to the Model. This paper proposes a compensation method suitable for the model in terms of time delay and lever arm effect in transfer alignment, and uses digital simulation to verify the system model and compensation method.**

***Index Terms*—Launch inertial coordinate system, Transfer alignment, time delay, Lever arm effect**

I. Introduction

Air-based cruise missiles are one of the indispensable members of precision-guided weapons. They are equipped with high-precision navigation, guidance and control systems. Among them, the navigation system is generally composed of a combination of strapdown inertial navigation and satellite navigation. The normal work of strapdown inertial navigation needs to provide relatively accurate initial information. This process is often transmitted through the strapdown inertial navigation system carried by both the aircraft and the missile. Align to achieve.

Transfer alignment technology is relatively mature in both theoretical research and technical aspects. Using transfer alignment technology can achieve rapid initialization of the missile carrier strapdown inertial navigation system. However, so far, whether it is in the application of land-based, sea-based or space-based guided weapon equipment, it is customary to use the transfer alignment model in the geographic coordinate system, and for the launching inertial coordinate system or other inertial coordinate systems. There are relatively few studies on the transfer of domestic and foreign [1]. The traditional transfer alignment model is generally established in the geographic coordinate system. For the tactical missile system, the launching inertial system is used to establish the equation of motion of the missile in the inertial space, which is convenient to describe the force received by the projectile. Therefore, its navigation and guidance And control systems are often used to adopt inertial coordinate systems. The geographic coordinate system transfer alignment model is used for the transfer alignment of the main and sub strapdown inertial navigation systems, which brings about problems such as matrix calculation and rounding error of redundant coordinate system conversion, which is not conducive to the rapid launch of air-based missiles. The maneuverability of the missile launch. Domestic Han Pengxin [1] used the launch inertial coordinate system as the navigation coordinate system to study the transfer alignment model, and took a three-stage launch vehicle inertial navigation transfer alignment as an example, using the geographic coordinate system and the launch inertial coordinate system to transfer Alignment, and using the "velocity + attitude" and "angular velocity + acceleration" matching mode, through the analysis of simulation results, using the transmission inertial coordinate system as the navigation coordinate system transfer alignment model can also achieve transfer alignment The corresponding alignment accuracy of the geographic coordinate system can estimate the installation error angle more accurately [9].

This paper is based on the transmission alignment model studied by Han Pengxin, Jiang Xin [6] and others using the launching inertial coordinate system as the navigation coordinate system, which further simplifies the model without affecting the alignment accuracy, making the state equation change from 18 dimensions It is 15 dimensions, which greatly reduces the calculation time of Kalman filtering. In the existing transfer alignment matching mode, a new "speed + measurement misalignment angle" matching mode is added [5]. In addition, this paper takes the transfer alignment matching mode of "speed + measuring misalignment angle" as an example to study the time delay and lever arm effect that often exist in the process of transfer alignment [7-8]. An error compensation scheme suitable for the transmission alignment model in the launch inertial coordinate system is presented, and the compensation algorithm is verified by digital simulation.

II. Transfer Alignment Model in Launching Inertial System

Define the coordinate system: the main inertial missile body coordinate system is *m* system; the sub inertial missile body coordinate system is *s* system; the main inertial navigation coordinate system is *n* system; the sub inertial navigation coordinate system is n' system;

A. Velocity error equation

Differential equation of velocity in launching inertial coordinate system:

$$\dot{V}^{n'} = C_s^{n'} f^s \tag{1}$$

In the formula, $V^{n'}$ is the velocity vector in the navigation

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coordinate system, and $C_s^{n'}$ is the conversion matrix from the projectile coordinate system to the navigation coordinate system.

Differentiating the two sides of equation (1) gives:

$$\delta\dot{V}^{n'} = \left(\delta C_s^{n'}\right) f^s + C_s^{n'} \delta f^s \tag{2}$$

In the formula, $\delta V^{n'}$ is the speed error in the navigation coordinate system, and $\delta f^s = \begin{bmatrix}\nabla_x & \nabla_y & \nabla_z\end{bmatrix}^T$ is the part of the specific force measurement error, that is, the accelerometer drift error.

$$\delta C_s^{n'} = (\phi\times) C_s^{n'} \tag{3}$$

$(\phi\times)$ is the antisymmetric matrix corresponding to the attitude misalignment angle.

B. Attitude error equation

The error angle between the mathematical platform of the sub-Strapdown inertial navigation system and the navigation coordinate system is defined as the attitude misalignment angle, as follows:

$$\phi = \begin{bmatrix}\phi_x & \phi_y & \phi_z\end{bmatrix}^T \tag{4}$$

The attitude matrix differential equation is as follows:

$$\dot{C}_s^{n'} = C_s^{n'} \left(\omega_{n's}^s \times\right) \tag{5}$$

Among them, $\omega_{n's}^s \times$ is the antisymmetric matrix formed by the angular velocity $\omega_{n's}^s$ measured by the gyroscope.

Derivate formula (3) and bring formula (5) into it, we can get:

$$\delta\dot{C}_s^{n'} = \left(\dot{\phi}\times\right) C_s^{n'} + (\phi\times) C_s^{n'} \left(\omega_{n's}^s \times\right) \tag{6}$$

The partial differential of equation (5) can be obtained:

$$\delta\dot{C}_s^{n'} = (\phi\times) C_s^{n'} \left(\omega_{n's}^s \times\right) + C_s^{n'} \left(\delta\omega_{n's}^s \times\right) \tag{7}$$

Combining formula (6) and formula (7) can get:

$$\left(\dot{\phi}\times\right) = C_s^{n'} \left(\delta\omega_{n's}^s \times\right) C_{n'}^s \tag{8}$$

According to the nature of vector operations, we have:

$$\dot{\phi} = C_s^{n'} \delta\omega_{n's}^s \tag{9}$$

In the type, $\delta\omega_{n's}^s = \begin{bmatrix}\varepsilon_x & \varepsilon_y & \varepsilon_z\end{bmatrix}^T$ is the gyro drift error.

C. Equation of state

The attitude misalignment angle, the speed error of the sub inertial navigation relative to the main inertial navigation system, the gyro measurement error of the sub inertial navigation relative to the main inertial navigation system and the accelerometer measurement error, and the installation error of the sub inertial navigation system relative to the main inertial navigation system are selected as state vectors , which is

$$X = \begin{bmatrix}\phi & \delta V & \varepsilon & \nabla & \lambda\end{bmatrix}^T \tag{10}$$

Then the state equation of the system can be expressed as:

$$\begin{aligned}
&\dot{X} = FX + GW \\
&F = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & C_s^{n'} & 0_{3\times3} & 0_{3\times3} \\ F_1 & 0_{3\times3} & 0_{3\times3} & C_s^{n'} & 0_{3\times3} \\ & & 0_{9\times15} & & \end{bmatrix} \\
&F_1 = \begin{bmatrix} 0 & f_z^{n'} & -f_y^{n'} \\ -f_z^{n'} & 0 & f_x^{n'} \\ f_y^{n'} & -f_x^{n'} & 0 \end{bmatrix} \\
&G = \begin{bmatrix} C_s^{n'} & 0_{3\times3} \\ 0_{3\times3} & C_s^{n'} \\ 0_{9\times6} & \end{bmatrix}
\end{aligned} \tag{11}$$

D. Measurement equation

In domestic and foreign literature, the speed + measurement misalignment angle matching algorithm has been widely used in the transmission alignment of the geographic coordinate system as the navigation coordinate system. Under the premise that the installation angle is small, the speed + measurement misalignment angle matching algorithm has a simpler measurement equation construction and less calculation amount than other transfer alignment matching algorithms.

Define the installation angle as a small amount, then:

$$C_m^s = 1 - \lambda\times \tag{12}$$

In the formula, $\lambda\times$ is the antisymmetric matrix formed by the installation angle vector, and C_m^s is the conversion matrix of the main inertial missile body coordinate system to the sub-inertial missile body coordinate system.

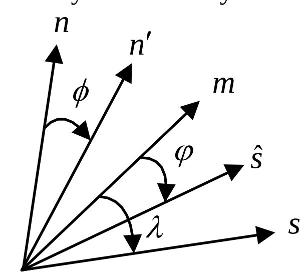

Fig.1 Relationship between coordinate systems

In the transfer alignment studied in this paper, it is assumed that the main inertial navigation system installed on the airborne platform is completely accurate, and the navigation coordinate system of the main inertial navigation system is considered to be an ideal coordinate system. Therefore, the calculation of the sub-inertial missile body coordinate system $\hat{s}$ is introduced. According to the definition of each coordinate system, the s coordinate system is the ideal coordinate system of the $\hat{s}$ coordinate system, the n coordinate system is the ideal coordinate system of the n' coordinate system, and the relationship between the coordinate systems As shown in Figure 1. The misalignment angle between the $\hat{s}$ coordinate system and the s coordinate system and the misalignment angle between the n' coordinate system and the n coordinate system are small and have the following relationship:

$$C_{n'}^n = I + \phi\times, C_n^{\hat{s}} = C_{n'}^s \tag{13}$$

The definition of the measurement misalignment angle is:

$$C_m^{\hat{s}} = C_n^{\hat{s}} C_m^n = I - \varphi \times \tag{14}$$

Take equation (13) into equation (14) and expand it to:

$$\begin{aligned} C_m^{\hat{s}} &= C_n^{\hat{s}} C_m^n = C_{n'}^s C_m^n = C_n^s C_{n'}^n C_m^n \\ &= C_m^s C_n^m C_{n'}^n C_m^n \\ &= (I - \lambda \times) C_n^m (I + \phi \times) C_m^n \end{aligned} \tag{15}$$

According to the relevant properties of the symmetric matrix, ignoring the second-order small quantity, the derivation is:

$$\begin{aligned} C_m^{\hat{s}} &= (I - \lambda \times) C_n^m (I + \phi \times) C_m^n \\ &\approx I + C_n^m \phi \times C_m^n - \lambda \times \\ &= I + (C_n^m \phi) \times - \lambda \times \end{aligned} \tag{16}$$

Taking equation (14) into equation (16) gives:

$$\begin{aligned} C_m^{\hat{s}} &= I + (C_n^m \phi) \times - \lambda \times \\ &= I - \varphi \times \end{aligned} \tag{17}$$

Then

$$\varphi = \lambda - C_n^m \phi \tag{18}$$

Therefore, the system observation vector is taken as

$$\begin{aligned} Z = [&V_{sx} - V_{mx} \quad V_{sy} - V_{my} \quad V_{sz} - V_{mz} \\ &\varphi_x \quad \varphi_y \quad \varphi_z]^T \end{aligned} \tag{19}$$

In the formula, V_{mx}, V_{my}, V_{mz} and V_{sx}, V_{sy}, V_{sz} are the velocity components of the main and sub-inertial navigation solutions under the launching inertial coordinate system; φ_x, φ_y, and φ_z can be calculated according to equations (13) and (14) Obtain:

$$\begin{aligned} \varphi_x &= C_{sm}(2,3) \\ \varphi_y &= -C_{sm}(1,3) \\ \varphi_z &= C_{sm}(1,2) \\ C_{sm} &= C_{n'}^s C_m^n \end{aligned} \tag{20}$$

The system measurement equation is:

$$\begin{aligned} Z &= HX + V \\ &= \begin{bmatrix} 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ -C_n^m & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} X + V \end{aligned} \tag{21}$$

III. Lever Arm Effect and Time Delay Compensation Scheme

A. Lever arm effect compensation scheme

In practical engineering applications, the main inertial navigation system is generally located at the center of the airborne platform, and the guided weapon is generally installed under the wing. There is a certain distance between the ballistic inertial navigation system and the main inertial navigation system. In the main and sub inertial navigation accelerometers, the specific forces to be sensed will be different, this phenomenon is the lever arm effect [9-11]. The lever arm effect will cause the speed error. If it is not compensated, the generated error will be introduced into the speed measurement. In order to ensure the accuracy of the transmission alignment, and consider the amount of calculation and the difficulty of implementation, this paper directly calculates the lever arm speed according to the length of the lever arm, and then compensates the main inertial navigation speed.

As shown in Figure 2, r is the position vector of the sub inertial navigation relative to the main inertial navigation system, O is the origin of the launching inertial coordinate system, R_s is the position vector of the sub inertial navigation to the origin, and R_m is the position of the main inertial navigation to the origin Vector.

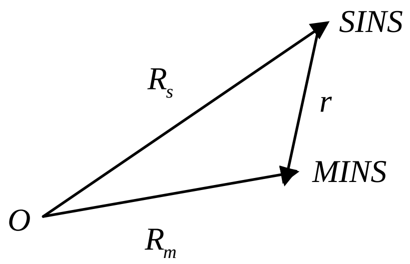

Fig.2 The geometric relationship of master and inertial navigation in coordinate system

The figure shows

$$R_s = R_m + r \tag{22}$$

Derivation on both sides

$$\left.\frac{dR_s}{dt}\right|_n = \left.\frac{dR_m}{dt}\right|_n + \left.\frac{dr}{dt}\right|_n \tag{23}$$

According to the Coriolis theorem

$$\left.\frac{dr}{dt}\right|_n = \left.\frac{dr}{dt}\right|_s + \omega_{ns}^s \times r \tag{24}$$

It can be obtained by bringing formula (24) into formula (23)

$$\left.\frac{dR_s}{dt}\right|_n = \left.\frac{dR_m}{dt}\right|_n + \left.\frac{dr}{dt}\right|_s + \omega_{nm}^m \times r \tag{25}$$

Without considering the deflection deformation, the relative position of the main sub-inertial navigation is fixed, so the position change rate of the sub-inertial navigation relative to the main inertial navigation is zero. Equation (25) is transformed into

$$V_s = V_m + C_m^n \omega_{nm}^m \times r \tag{26}$$

ω_{nm}^m is obtained from the output of the main inertial navigation gyro. If the lever arm r is known, the speed obtained by the main inertial navigation solution can be compensated by the above formula.

B. Time delay compensation scheme

In the process of transfer alignment, the main inertial navigation system needs to transmit the inertial navigation original data and other navigation parameters in real time. Due to the time-consuming transmission and other reasons, the system cannot receive the main inertial navigation information in time, and there is a certain delay, which affects the accuracy of the transfer alignment. Under high dynamic conditions, even in a short period of time, the position, speed and attitude of the carrier change extremely, especially the attitude information [2]. In order to accurately estimate the installation error angle after Kalman filtering, this paper uses delay time to compensate the main inertial navigation attitude matrix, that is, after the system receives the main inertial navigation attitude matrix at time $t - \Delta t$, it estimates the main inertial navigation attitude matrix at time t. When the delay time Δt is small enough, it can be known from the knowledge of calculus

$$\dot{C}_m^n(t-\Delta t)=\frac{C_m^n(t)-C_m^n(t-\Delta t)}{\Delta t} \tag{27}$$

Therefore

$$C_m^n(t)=C_m^n(t-\Delta t)+\dot{C}_m^n(t-\Delta t)*\Delta t \tag{28}$$

According to the differential equation of the direction cosine matrix

$$\dot{C}_m^n(t-\Delta t)=C_m^n(t-\Delta t)*\left(\omega_{nm}^m(t-\Delta t)\times\right) \tag{29}$$

We can get formula (29) into formula (28)

$$\begin{aligned}C_m^n(t)&=C_m^n(t-\Delta t)+\\&C_m^n(t-\Delta t)*\left(\omega_{nm}^m(t-\Delta t)\times\right)*\Delta t\end{aligned} \tag{30}$$

In the formula, $C_m^n(t)$ and $C_m^n(t-\Delta t)$ are the main inertial navigation attitude matrix at time t and time $t-\Delta t$.

In practical applications, an external clock can be used to measure the delay Δt. Since the transmission delay of the main and sub inertial navigation information is relatively small, the acquired Δt can be compensated for in the main inertial navigation attitude matrix by using equation (30). This method is simple in form and easier to implement.

IV. Simulation Verification

A. Model simulation verification

In order to verify the correctness of the improved 15-dimensional transfer alignment algorithm using the launch inertial coordinate system as the navigation coordinate system and the newly added "speed + measurement misalignment angle" matching algorithm, the simulation conditions are set as follows: the flight time of the aircraft is 60s; The initial position of the aircraft is latitude 32°, longitude 118°, altitude 500m; the initial speed of the aircraft is east speed 100m/s, north speed 100m/s, sky speed 0m/s; the initial attitude information of the aircraft (geographic coordinate system) pitch The angle is 0°, the roll angle is 0°, the heading angle is -45°; the main inertial gyroscope constant offset and random white noise are set to 0.01°/h, the accelerometer constant offset and random white noise are set to 0.01mg; sub-inertial navigation gyroscope constant bias and random white noise are set to 0.5°/h, accelerometer constant bias and random white noise are set to 1mg; the three-axis installation error angle of the projectile system is 1°, maneuvering The way is wing + level flight.

Establish the launch inertial coordinate system at the initial position, the launch azimuth is 45°, use the main and sub inertial navigation data generated by the above flight trajectory to perform the inertial navigation solution in the launch inertial coordinate system, construct the measurement of the solution value, and carry out Kalman After filtering, the following results are obtained:

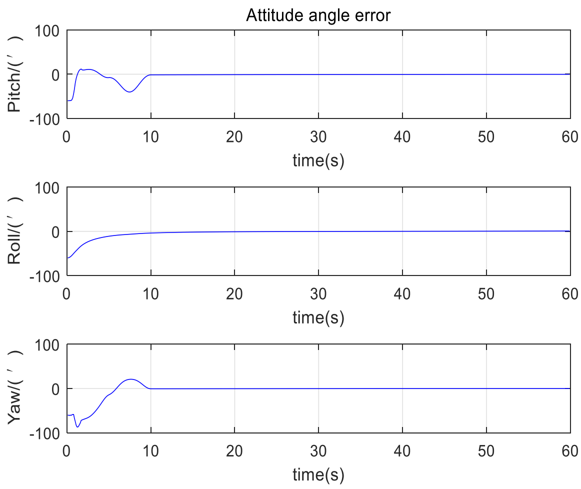

Fig.3 Error curve of attitude angle in launch inertial coordinate system

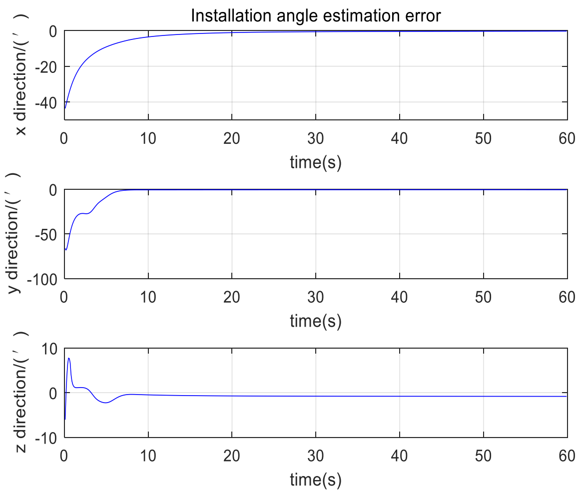

Fig.4 Three axial mounting angle estimation error curves

As can be seen from Figures 3 and 4, using the transfer alignment model under the launch inertial system and the "speed + measurement misalignment angle" matching algorithm, the Kalman filter convergence speed is fast, and the error curves can converge to $5'$ within 15s. At 30s, the errors of pitch angle, roll angle and heading angle are $1.340'$, $1.747'$ and $1.559'$, respectively; the estimated errors of the three axial installation angles are $1.819'$, $1.226'$ and $1.341'$ respectively. Based on the above data and images, it can meet the requirements of transfer alignment accuracy.

B. Compensation scheme verification

The delay time is set to 50ms, the length of the lever arm is 3m, 4m and 2m, the three-axis installation error angle of the projectile system is set to 0.8°, 1° and 0.6°, and the other simulation conditions are the same as described in Section *A*. The above lever arm effect and time delay factor consider the single factor's influence on the transmission alignment. The results of comparing the compensation and non-compensation are as follows:

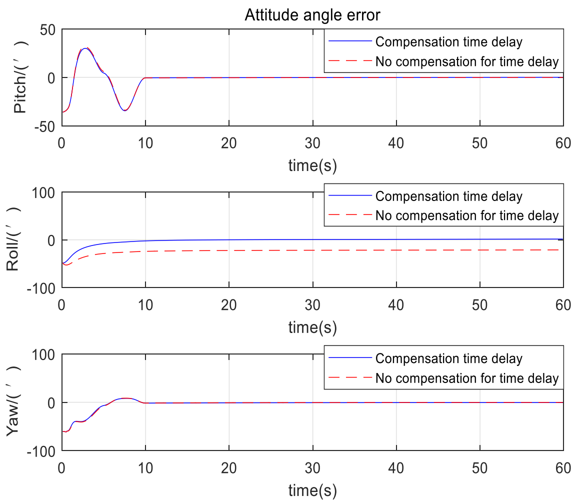

Fig.5 Comparison curve of attitude angle error with compensated and uncompensated time-delay in launch inertial coordinate system

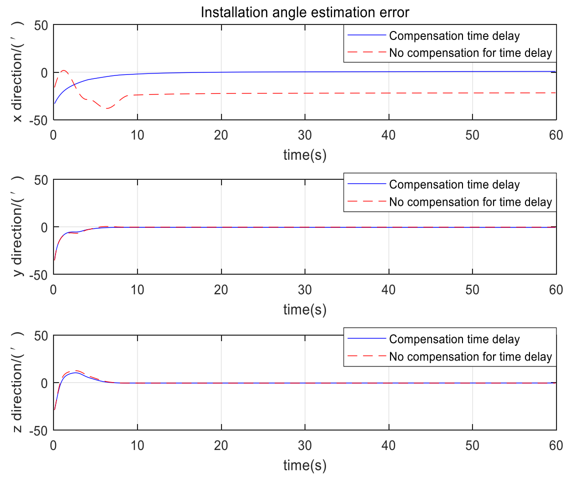

Fig.6 Comparison curve of three axial installation angle estimation errors with compensated and uncompensated time-delay

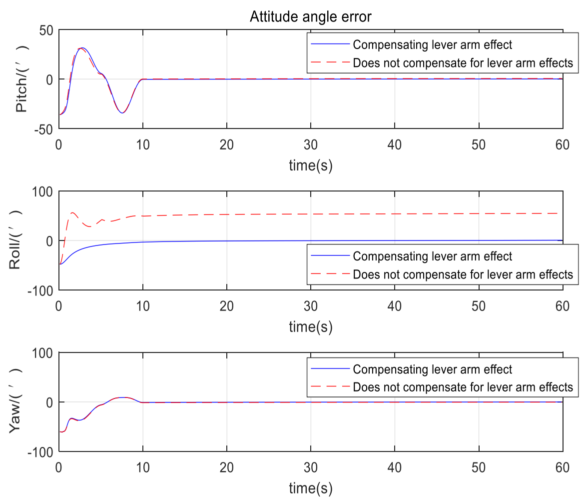

Fig.7 Comparison curve of attitude angle error with compensated and uncompensated lever-arm effect in launch inertial coordinate system

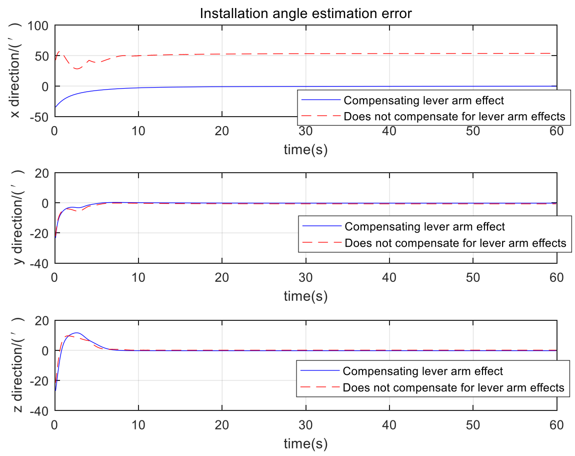

Fig.8 Comparison curve of three axial installation angle estimation errors with compensated and uncompensated lever-arm effect

From the above comparison curve, it can be seen that both time delay and lever arm effect have a greater impact on the roll angle, and less on the pitch angle and heading angle. The estimated value of the installation angle in the x direction produces a large error, which also causes a large error in the roll angle. The errors caused by both the time delay and the lever arm effect reach 22′ and 53′, respectively. Using the compensation method in this paper, the errors are all within 3′, which effectively reduces the errors and ensures the accuracy of transfer alignment.

V. Conclusion

This paper introduces the transfer alignment model in the launch inertial system in detail, and derives the “speed + measurement misalignment angle” matching algorithm. The simulation verifies that the above model can perform fast transfer alignment with high transfer alignment accuracy. By comparing the transfer alignment effects of compensated and uncompensated time delays and lever arm effects, the transfer alignment accuracy after compensation is significantly improved, and the compensation scheme proposed in this paper is verified. Due to the complexity of the aircraft's flight environment, the wing will be subjected to a certain external force to produce a deflection angle. Later, we will continue to study a deflection compensation scheme that can be used to transmit alignment under the launch inertial system, so that it can reduce Small or even eliminate the effect.

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