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Design a PID Regulator for Stabilizing Quadrotor

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Abstract— **Stabilizing Quadrotor is a main point needed to be solved before making this track along a desired trajectory. This paper present an explicit procedure to design a Propotional – Integral – Derivative (PID) regulator for stablizing Quadrotor. For more details, the mathematical model of Quadrotor is described under an engineering point of view, based on which a suitable PID controller(P, PI, PD, or PID) is designed and programmed on STM32 microcontroller. Besides, a good control feeback system needs a clean feedback signals, so effects of sensor noises are reduced by applying the complement filter. The performances of the control system and how sensor noises are well eliminated are going to be demonstrated by not only simulation but also experimental result.**

***Index Terms*— PID, Quadrotor, Complement filter, STM32 microcontroller.**

I. INTRODUCTION

Quadrotor, a typical unmanned aerial vehicle, has many applications in daily life such as in search and rescue, surveillance, and other applications. It attracts considerable attention from researchers, engineers. The quad-copter [1], [2], [3], [4] consists of 4 propellers arranged on "x" or "+"-shapes. The symmetry of the quad-copter body gives simplicity to the controller design as it can be controlled through varying the speed of the propellers [2]. The rotational speeds of four rotors are independent, so it's possible to control the pitch, roll, and yaw attitude of the vehicle.

Stabilizing quad-copter is the first mission before we can think about tracking along desired trajectories, and there are so many control strategies for balancing quad-copter in the air in which PID control algorithm [5] is the most popular due to its convenient. PID means Proportional – Integral – Derivative, and it functions to force the output of the plant to follow the expectation. Based on the engineer point of view, there are three Euler's angles, Roll, Pitch, Yaw, should be taken into account in order to stabilize the quad-copter hanging on the sky.

This paper presents how to understand the working principle of quadrotor physically, and then constructs the control scheme utilizing digital PID controllers for controlling each angle. In order to setup the experimental model, this paper mainly discusses about process to implement real model of quad-copter such as noise effect elimination of the sensor, and digital PD controller on microcontroller.

The paper is organized such that physically mathematical model of quad-copter is shown in section 2. Then control feedback system design is given in section 3. Section 4 demonstrates the experimental setup and results. Finally, some discussions and conclusions will be included in section 5.

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II. MATHEMATICAL MODEL OF QUADROTOR

A. Quadrotor dynamics

Figure 1 shows quadrotor frame system with a vehicle frame (x,y,z). The forces and moments on quadrotor are calculated by equation from (1) to (3).

$$F_i = k_f \omega_i^2 \, ; \; M_i = k_m \omega_i^2 \tag{1}$$

$$M_x = (F_1 - F_2) l \, ; \; M_y = (F_2 - F_4) l \tag{2}$$

$$\mathrm{w} = mg \tag{3}$$

In which F stand for forces and M index stand for moments. M_x and M_y are denoted for moments along the x-axis and y-axis respectively. l is the length from the rotor to the center of the quadrotor frame, w is a gravitational force caused by weight.

The motion of quadrotor can be analyzed by applying Newton's second law. For linear motion forces are calculated as a product of mass and linear acceleration, and torque are estimated as a product of inertial and angular acceleration in the rotational motion.

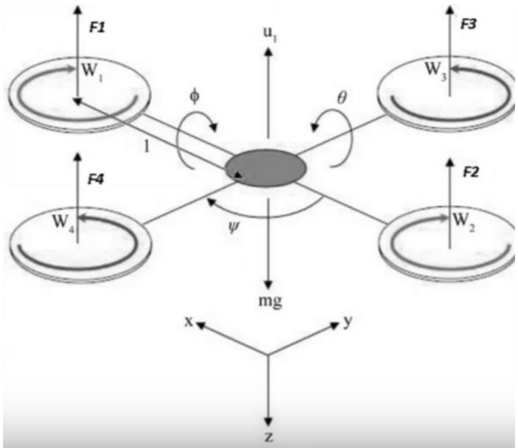

Figure 1: Diagram for analyzing dynamics of quadrotor.

There are some conditions which should be considered to control quadrotor: Hovering condition, rising condition, dropping condition. In rising condition known as take-off mode, the total force must be greater than the weight of the quadrotor, and all moments should be also zero.

$$mg < \sum F_i = F_1 + F_2 + F_3 + F_4 \tag{4}$$

So we get the equation of motion in case of the rising condition as (5).

$$m\ddot{r} = \sum F_i - mg > 0 \text{ or } F_1 + F_2 + F_3 + F_4 - mg > 0 \tag{5}$$

In dropping condition known as landing mode, the total force must be less than the weight of the quadrotor, and all moments should be also zero.

$$mg > \sum F_i = F_1 + F_2 + F_3 + F_4 \quad (6)$$

So we get the equation of motion in case of the rising condition as (7).

$$m\ddot{r} = \sum F_i - mg < 0 \text{ or } F_1 + F_2 + F_3 + F_4 - mg < 0 \quad (7)$$

Hovering condition means how the quadrotor hang on the air, in this condition total force should be balanced or total force produced by four propellers is equal to gravity force, and all moments produced are zero.

$$mg = \sum F_i = F_1 + F_2 + F_3 + F_4 \quad (8)$$

So we get the equation of motion in case of the hovering condition as (9).

$$m\ddot{r} = \sum F_i - mg = F_1 + F_2 + F_3 + F_4 - mg \quad (9)$$

In principle, two propellers (number 1, 3) rotate clockwise and two others rotate counter-clockwise (number 2, 4). When the quadrotor rotates in horizontal plane, it causes yaw motion. In other word, if the moments generated by one pair differ from the other pair, it will cause yaw motion. The yaw motion of quadrotor is described by the following equation,

$$I_{zz}.\ddot{\psi} = \sum M_i \; .$$

Table 1. Parameters due to dynamics of quadrotor

Symbols	Parameters	Values	Units
l	Length of the arm holding propellers	0.225	m
m	Total weight of the quad-copter	0.5	Kg
I_{xx}	Moment of inertial along x-axis	4.856×10^{-3}	Kg.m^2
I_{yy}	Moment of inertial along y-axis	4.856×10^{-3}	Kg.m^2
I_{zz}	Moment of inertial along z-axis	8.801×10^{-3}	Kg.m^2
k_f	Thrust (lift) factor	1.26×10^{-5}	
k_m	Drag factor	2.06×10^{-7}	

Similar to yaw motion, we can obtain roll and pitch motion when the quadrotor rotates around x, and y axis respectively.

$$I_{xx}.\ddot{\phi} = (F_3 - F_4)\,l\,;$$

$$I_{yy}.\ddot{\theta} = (F_1 - F_2)\,l$$

Hence we have equations of quadrotor motion as following,

$$I_{xx}.\ddot{\phi} = k_f l\left(\omega_3^2 - \omega_4^2\right) \quad (10)$$

$$I_{yy}.\ddot{\theta} = k_f l\left(\omega_1^2 - \omega_2^2\right)$$

$$I_{zz}.\ddot{\psi} = k_m.\left(\left(\omega_1^2 + \omega_2^2\right) - \left(\omega_3^2 + \omega_4^2\right)\right)$$

The parameters of the system are indicated in Table 1.

B. Brushless DC motor model

The mathematical model of BLDC is referenced from [6], it has the transfer function of the second order (11). The parameters of BLDC (shown in Table 2) are ownly identified by using lab equipments to measure such that the resistance, inductance of BLDC are measured by RLC measuring equipment.

$$G_{BLDC}(s) = \frac{1/k_e}{\tau_m.\tau_e.s^2 + \tau_m.s + 1} \quad (11)$$

Table 2. Parameters due to BLDC

Symbols	Definition	Value	Unit
k_e	EMF coefficient	8.409×10^{-3}	(v-s)/rad
k_t	Moment coefficient	0.14	N.m/A
J	Inertial of rotor	9.25×10^{-6}	Kg.m2
τ_m	Mechanical time constant	0.25	s
τ_e	Electrical time constant	0.05	s

Hence, the final transfer function of BLDC can be expressed as the following equation,

$$G_{BLDC}(s) = \frac{118.91}{(0.1649s + 1)(0.076s + 1)} \quad (12)$$

III. CONTROL SYSTEM DESIGN

A. Control scheme

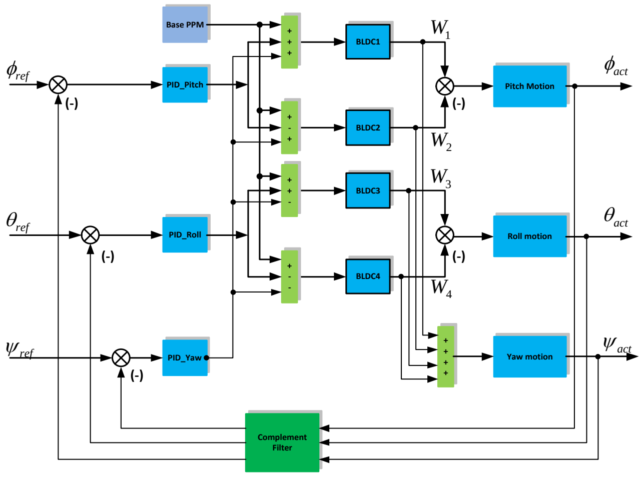

Figure 2: Control scheme for stabilizing three Euler's angles

In order to stabilize the quadrotor we need to care about controlling three Euler angles, in this paper PID controllers are applied to maintain three angles, roll, pitch and yaw, to follow the desired angle (often zero angle). The control scheme of the system is shown in Figure 2. The dynamic responses of roll and pitch angles are linearized and decoupled, so we can easily to design a controller for stabilizing each angle. The effects of yaw motion acted on roll and pitch motions are considered as a noise.

Controllers for Roll and Pitch angle: The roll angle can be controlled by adjusted the angular speed of rotor 1 and rotor 2. From control scheme (Figure 2) and equation (10), the mathematical model due to roll angle can be calculated by the following equation

$$\phi = \left[(\text{Base PPM} + \text{U}_1)G_{BLDC} - (\text{Base PPM - U}_1)G_{BLDC}\right].G_{dyn}$$

Therefore the mathematical model due to roll angle will be archived as following:

$$G_\phi(s) = 2G_{BLDC}.G_{dyn} = G_\phi(s)$$
$$= 2\times\frac{118.91}{(0.1649s+1)(0.076s+1)}\times\frac{k_f.l.k_v}{I_{xx}.s^2}$$

By approximating $\omega^2 = k_v.\omega$, the transfer function will be:

$$G_\phi(s) = 2\times\frac{118.91}{(0.1649s+1)(0.076s+1)}\times\frac{k_f.l.k_v}{I_{xx}.s^2} \quad (12)$$

Equation (12) we can be rewritten:

$$G_\phi(s) = \frac{k_\phi\times 237.82}{s^2(0.1649s+1)(0.076s+1)} \quad (13)$$

From (13), we see that this is fourth order system, and it is easily to see that the time constant 0.1649 is much bigger than 0.076. Since we can approximate (13) by (14)

$$G_\phi(s) = \frac{k_\phi\times 237.82}{s^2(0.24s+1)} \quad (14)$$

Now, we have the third order system, which does not guarantee that the system is always stable. Hence we need to add a controller to make the opened loop transfer function to get a form of the second order system in which integral part should be included. The second order system including the integral part means that the system is always stable, and no control deviation (steady state error) at the end. Thus, the PD controller should be chosen in this case, because the plant has itself an integral part.

$$G_{PD}(s) = k_p\left(1+sT_d\right)$$

Because the quadrotor has a symmetric construction, the pitch angle is the same as roll angle. The difference here is that the pitch angle is controlled by adjusting speed of motor 3 and 4. Hence the controller of pitch angle is also PD with the same parameters.

Controller for yaw angle: From control scheme (Figure 2) and equation (10), the mathematical model due to yaw angle can be calculated by the following equation

$$\psi = G_E.G_{dyn}$$
$$G_E = \left(\text{Base PPM + U}_1\text{+U}_3\right)+\left(\text{Base PPM - U}_1\text{+U}_3\right)$$
$$-\left(\text{Base PPM + U}_2\text{-U}_3\right)-\left(\text{Base PPM - U}_2\text{-U}_3\right)$$
$$\Rightarrow G_\psi(s) = 4G_{BLDC}.G_{dyn}$$

$$G_\psi(s) = 4\times\frac{118.91}{(0.1649s+1)(0.076s+1)}\times\frac{k_f.l.k_v}{I_{xx}.s^2} \quad (15)$$

Equation (15) we can be rewritten:

$$G_\psi(s) = \frac{k_\phi\times 475.64}{s^2(0.1649s+1)(0.076s+1)} \quad (16)$$

From (16), we see that this is fourth order system, and it is easily to see that the time constant 0.1649 is much bigger than 0.076. Since we can approximate (16) by (17)

$$G_\psi(s) = \frac{k_\phi\times 475.64}{s^2(0.24s+1)} \quad (17)$$

Similar to Roll and pitch angle's controllers, the PD controller should be chosen for yaw angle, because the plant has itself an integral part.

$$G_{PD}(s) = k_p\left(1+sT_d\right)$$

Discretization: In order to implement this controller on microcontroller (STM32) we need to discrete the control signal.

$$u(t) = k_p + T_d\frac{d}{dt}e(t)$$

$$\underline{discrete}\quad u_{k+1} \approx k_p + T_d\frac{e_{k+1}-e_k}{t_{k+1}-t_k} \approx k_p + T_d\frac{e_{k+1}-e_k}{h}$$

in which h is step size.

Parameter tuning: There are several method for tuning the controller parameters, and the most popular one is Zigler Nichol method. This method works well with almost plants, but it is depended on the experiences of the designer. In this paper we did a lot of experiments to figure out the parameters of kp and Td.

Simulation results: Before coming up with experimental setup, simulation is a good process to avoid violence. Matlab/ Simulink tool is our choice. Figure 3 indicates the roll angle response when applying PD controllers for three Euler's angles. The continuous line is stood for the reference value, and the dot line indicates the output response. The parameters of PD controller used in this simulation are defined experimentally (kp=7.78, and Td=0.613).

The roll angle is firstly set to be zero, then changed to negative ten degrees at 15s. The output response verifies that the PD controllers provide a good performance with no steady state error, and a fast response.

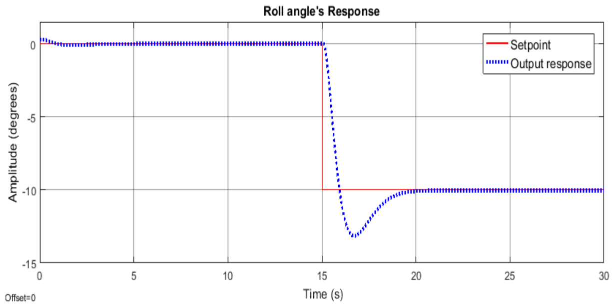

Figure 3: Roll angle's response.

IV. EXPERIMETAL SETUP

Our experiments were carried out in the Laboratory of Instrument and control department. Figure 5 shows the test bench for choosing parameters of controllers and the flight test on the campus of TNUT.

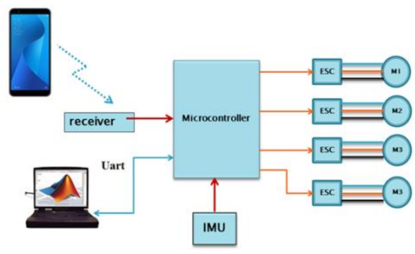

Figure 4: The root mean squared error of magnetic loss tangent the materials.

Figure 5: Test bench in Lab and flight test on TNUT

In this project, the main circuit is designed by our group members for implementing real-time control [7], [8]. Figure 4 indicates that the main circuit has functions of receiving the command from a smartphone or laptop, then sends control signals to drive BLDCs. Besides, it has a mission to send data back to the computer for visualizing the responses. The sensor selected to measure the angles is MPU6050 which includes gyroscope and accelerometer. While the main drawback of the gyroscope is drifted at low frequency, the accelerometer is affected by Gaussian noise at high frequency. Since the complement filter is introduced to solve this problem in this paper. The construction of complement filter contains low pass and high pass filter (Fig. 6).

The description of the complement filter for roll angle is given by (18):

$$\theta = \frac{1}{Ts+1}\theta_{accel} + \frac{Ts}{Ts+1}\frac{1}{s}\dot{\theta}_{gyro} \qquad (18)$$

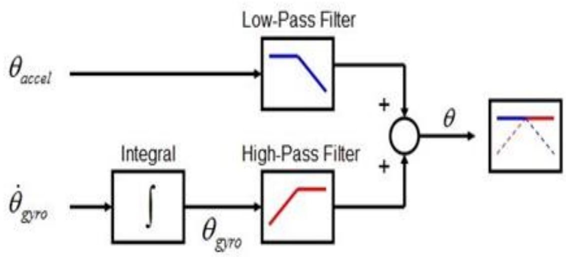

Figure 6. Complement filter structure

In discrete domain the complement filter is presented by equation (19):

$$\theta(t_{k+1}) = \alpha\left(\theta(t_k) + h.\dot{\theta}_{gyro}(t_{k+1})\right) + (1-\alpha)\theta_{accel}(t_{k+1}) \qquad (19)$$

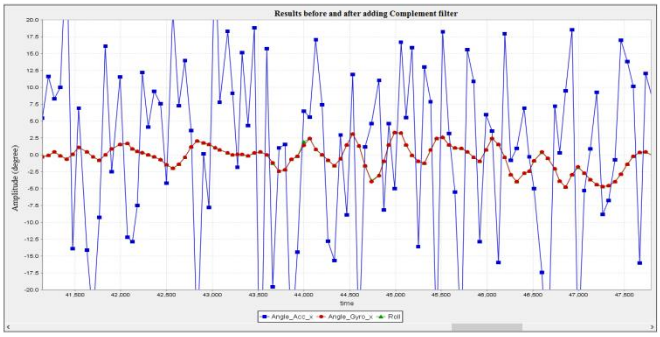

Figure 7: The roll angle before and after adding the complement filter.

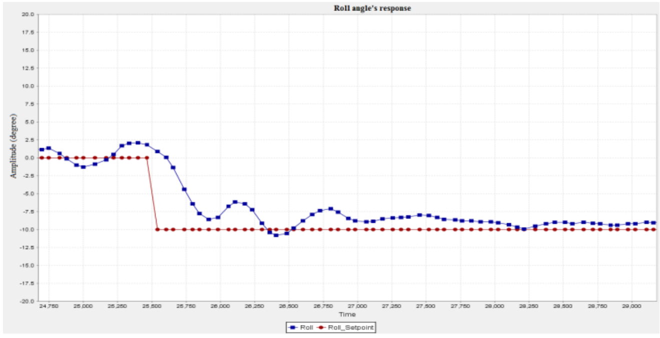

Figure 8: The roll angle's response on experimental setup

It is easy to see that, the signal is very clean and less oscillated after adding the complement filter. The control feedback system works well when the feedback signal is precise. The roll angle's response is indicated in Figure 8. The desired roll angle is set to be zero at the beginning, then changed to negative ten degrees at 25s. The roll angle's response follows the desired values after a short time..

V. DISCUSSION

In this paper, a technical point of view for understanding the dynamic characteristics of quadrotor was introduced. The mathematical model of BLDC was referenced from previous works, and the parameters of BLDC were defined by our own works. Based on the mathematical model of quadrotor system, the digital PD controllers were implemented on the STM32 platform fabricated by our group members. During the testing process, we realized that the noise affected by MPU6050 is a considerable problem that needed to be solved, and the complement filter was utilized to eliminate the noise. All results were proved not only simulation results but also experimental results. Finally, the flight test was done with a good performance. Although this paper has some good achievements, it is the beginning step. Therefore developing more control strategies [9] to improve the system performance is our future destination.

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