Asymptotic estimates for finite-time ruin probability of a bidimensional risk model based on entrance process

Zhankui Wang

***Abstract*— Consider a bidimensional risk model based on entrance process with constant force of interest in which the claim size from the same business are heavy-tailed and pairwise strong quasi-asymptotically independent, the two counting processes of different business satisfy a certain dependence structure. A precise asymptotic formula for the finite-time ruin probability is obtained.**

***Index Terms*— Bidimensional risk model; ruin probability; independence; asymptotic formula MSC(2010): — 60B12, 91B30**

I. Introduction

We know that the literature [1] put forward into a new model (LIG model) based on an entrance process and discussed asymptotic normality of the risk process. Furthermore, some scholars got some conclusions through the study of the LIG model. [2-4] investigated the one-dimensional risk model based on entrance processes. Recently, more attention has been paid to multi-dimensional risk models, especially bidimensional ones. [5] discussed the precise large deviations based on the entry process risk model in the independent case of multi-risk. [6] studied the ruin probability of a bidimensional risk model based on entrance processes with constant interest rate.

In this paper, we investigate finite-time ruin probability of a bidimensional risk model based on entrance processes, in which an insurance company operates two kinds of business. Suppose that the initial insurance fund for the $i-th$ class is x_i and S_j^i is entry time of the $j-th$ policy with $0<S_1^i<S_2^i<\cdots$ and $S_j^i=\sum_{k=1}^{j}\theta_k^i$, $i=1,2$. $\{(N_1(t),N_2(t))^{\mathrm{T}}$, $t\ge 0\}$ is a bidimensional renewal counting process. Here, $N_i(t)=\sup\{j\ge 0:S_j^i\le t\},t\ge 0,i=1,2$. For more detail of a bidimensional renewal counting process, we refer the reader to Examples 3.1 and 3.2 of [7]. Denote the mean function by $\lambda_i(t)=EN_k(t)$ with $\lambda_i(0)=0$ and $\lambda_i(t)<\infty$, and define the set $\Lambda_i=\{t>0:\lambda_i(t)>0\}=\{t>0:P(S_1^i\le t)\}$. If we set $t_-^i=\inf\{t>0:P(S_1^i\le t)>0\}$, then it is easy to see that $\Lambda_i=[t_-^i,\infty]$ if $P(S_1^i=t_-^i)>0$; or $\Lambda_i=(t_-^i,\infty]$ if $P(S_1^i=t_-^i)=0$, $i=1,2$. We denote the intersection set by $\Lambda=\Lambda_1\bigcap\Lambda_2$. Let the validity time of the $j-th$ policy be $\{C_j^i,j=1,2,\cdots\}$ with probability $P(C_j^i=\alpha_\ell^i)=p_\ell^i,\ell=1,2,\cdots,K^i$, where they are independent and identically distributed. The premium is $f_i(C_j^i)$ and $f_i(\cdot)$ is a strictly increasing function. D_j^i is claim time of the $j-th$ policy and independent and identically distributed function $H_i(\cdot)$. X_j^i is the $j-th$ claim size and identically distributed function $F_i(\cdot)$. Suppose that X_j^i have the same distributions X^i. For any time $t\ge 0$, the surplus process of the insurer can be described as

$$\begin{pmatrix}R_1(t)\\R_2(t)\end{pmatrix}=\begin{pmatrix}x_1e^{\delta t}\\x_2e^{\delta t}\end{pmatrix}+\begin{pmatrix}\sum_{i=1}^{N_1(t)}f_1(C_i^1)e^{\delta(t-S_i^1)}\\\sum_{j=1}^{N_2(t)}f_2(C_j^2)e^{\delta(t-S_j^2)}\end{pmatrix}-\begin{pmatrix}\sum_{i=1}^{N_1(t)}X_i^1e^{\delta(t-S_i^1-D_i^1)}I_{\{S_i^1+D_i^1\le t,D_i^1\le C_i^1\}}\\\sum_{j=1}^{N_2(t)}X_j^2e^{\delta(t-S_j^2-D_j^2)}I_{\{S_j^2+D_j^2\le t,D_j^2\le C_j^2\}}\end{pmatrix}. \quad (1)$$

where $\delta>0$ denotes the constant force of interest. We further assume that $\{X_j^1,j\ge 1\}$, $\{X_j^2,j\ge 1\}$ and $\{(N_1(t),N_2(t))^{\mathrm{T}},t\ge 0\}$ are mutually independent.

Define the finite-time ruin probabilities corresponding to risk model (1) as

$$\psi(x_1,x_2;t)=P(\tau_{\max}(x_1,x_2)\le t)$$

$$=P(\bigcap_{i=1}^{2}R_i(s)<0,0\le s\le t),$$

where

Zhankui Wang, School of Mathematics and Statistics, Northwest Normal University, Lanzhou, Gansu, China, Mobile No. 18894575336

$$\tau_{\max}(x_1, x_2)$$
$$= \inf\{t : \max\{R_1(t), R_2(t)\} < 0 \mid R_i(0) = x_i$$
$$, i = 1, 2\}.$$

In the rest of this paper, Section 2 presents our main results after introducing necessary preliminaries, Section 3 gives some lemmas, and Sections 4 gives the proofs of the main results.

II. PRELIMINARIES AND MAIN RESULTS

Definition 1. For a distribution F, denote its tail distribution by $\bar{F} = 1 - F$ and its upper Matuszewska index by

$$J_F^+ = -\lim_{x\to\infty} \frac{\log \bar{F}_*(x)}{\log x},$$

where

$$\bar{F}_*(x) = \liminf_{x\to\infty} \frac{\bar{F}(xy)}{\bar{F}(x)} \text{ for } y > 0,$$

which can be found in [8].A distribution F is said to be dominatedly-varying-tailed, denoted by $F \in D$, if for every fixed $y \in (0,1)$, $\bar{F}_*(x) < \infty$. Clearly, $F \in D$. A distribution F is said to be long-tailed, denoted by $F \in L$, if for every fixed $y \in R$, $\bar{F}(x+y) \sim \bar{F}(x)$ as $x \to \infty$, that is,

$$\lim_{x\to\infty} \frac{\bar{F}(x+y)}{\bar{F}(x)} = 1,$$

which can be found in [9].

Definition 2. If real valued random variables $X_i, i \geq 1$ with distribution functions $F_i, i \geq 1$ satisfy for any $i \neq j$

$$\lim_{\min\{x_i, x_j\}\to\infty} P(X_i > x_i \mid X_j > x_j) = 0,$$

Then we say $X_i, i \geq 1$ are pairwise strong quasi-asymptotically independent $(PSQAI)$, which can be found in [10].

Theorem 1. Consider the bidimensional risk model (1). Suppose that claim sizes, $\{X_j^i, j \geq 1\}$ be $PSQAI$ random variables with common distribution $F_i \in D \cap L$, $i = 1, 2$. If $N_1(t)$ and $N_2(t)$ are arbitrarily dependent, then for any fixed $\varepsilon > 0$ with $E[N_1(\varepsilon)N_2(\varepsilon)] > 0$. Then for $t \in \Lambda \cap [\varepsilon, T]$, we have

$$\psi(x_1, x_2; t)$$
$$\sim \int_0^t \int_0^t \prod_{i=1}^{2} \left(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})\right.$$
$$\left. dH_i(y)\right) dE[N_1(u_1)N_2(u_2)]. \quad (2)$$

III. SOME LEMMAS

The following lemma is an immediate corollary of Theorems 3.2 and 3.4 of [11].

Lemma 1. Let $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ be two renewal counting processes with their inter-arrival time $\theta_1^1, \theta_2^1, \cdots,$ and $\theta_1^2, \theta_2^2, \cdots,$ respectively. For any integer $i \geq 1$, denote by $S_i^1 = \sum_{j=1}^{i} \theta_j^1$ and $S_i^2 = \sum_{j=1}^{i} \theta_j^2$ the corresponding arrival times. If $\{(\theta_i^1, \theta_i^2); i \geq 1\}$ is a sequence of i.i.d. random vectors, then it holds for any $u_1 \geq 0$ and $u_2 \geq 0$ that

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} P(S_i^1 \leq u_1, S_j^2 \leq u_2) = E(N_1(u_1)N_2(u_2)).$$

The following lemma comes from Theorem 2.1 of [10]

Lemma 2. Assume that $\{X_j, 1 \leq j \leq n\}$ are n real-valued random variables with functions of distribution $F_j, 1 \leq j \leq n$. If $\{X_j, 1 \leq j \leq n\}$ are $PSQAI$ and $F_j \in D \cap L$ and $(c_1, \cdots, c_n) \in [a, b]^n$. Then

$$P\left(\sum_{j=1}^{n} c_j X_j > x\right) \sim \sum_{j=1}^{n} P(c_j X_j > x).$$

The following lemma comes from proposition 2.2.1 of [9]

Lemma 3. If a distribution $F \in D$, then for any $\beta > J_F^+$, there exist two positive constant C and D such that for all $x \geq y \geq D$,

$$\frac{\bar{F}(y)}{\bar{F}(x)} \leq C\left(\frac{x}{y}\right)^{\beta}.$$

Lemma 4. Under the conditions of Theorem 1, it holds that uniformly for $t \in \Lambda \cap [\varepsilon, T]$,

$$P\Bigg(\sum_{i=1}^{N_1(t)} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\leq t, D_i^1\leq C_i^1\}} > x_1,$$
$$\sum_{j=1}^{N_2(t)} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\leq t, D_j^2\leq C_j^2\}} > x_2\Bigg)$$
$$\sim \int_0^t \int_0^t \prod_{i=1}^{2} \left(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})\right.$$
$$\left. dH_i(y)\right) dE[N_1(u_1)N_2(u_2)]. \quad (3)$$

Proof. For arbitrarily fixed positive integer M, we split the left-hand side of (3) into three parts as

$$\left(\sum_{m=1}^{M}\sum_{n=1}^{M} + \sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty} + \sum_{m=M+1}^{\infty}\sum_{n=1}^{\infty} - \sum_{m=M+1}^{\infty}\sum_{n=M+1}^{\infty}\right)$$

$$\times P(\sum_{i=1}^{m} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$\sum_{j=1}^{n} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2,$$
$$N_1(t)=m,\ \ N_2(t)=n)$$

$$=\sum_{k=1}^{3} I_k(x_1,x_2;t) - I_4(x_1,x_2;t). \qquad (4)$$

We first deal with $I_1(x_1,x_2;t)$. Write $B_m^1=\{0\le z_1^1\le\cdots\le z_m^1\le t<z_{m+1}^1\}$, and $B_m^1=\{0\le \le\cdots\le z_m^1\le t<z_{m+1}^1\}$. Since the three sequences $\{X_j^i, j\ge 1\}$, $i=1,2$, and $\{(N_1(t),N_2(t)^{\mathrm{T}}, t\ge 0)\}$ are mutually independent, and using Lemma 2, we have that uniformly for $t\in\Lambda\bigcap[\varepsilon,T]$

$$I_1(x_1,x_2;t)$$
$$=\sum_{m=1}^{M}\sum_{n=1}^{M} P(\sum_{i=1}^{m} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$\sum_{j=1}^{n} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2,$$
$$N_1(t)=m,\ \ N_2(t)=n)$$
$$=\sum_{m=1}^{M}\sum_{n=1}^{M}\int\cdots\int_{B_m^1;B_n^2}$$
$$P(\sum_{i=1}^{m} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$\sum_{j=1}^{n} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2)$$
$$\times P(S_1^1\in dz_1^1,\cdots,S_{m+1}^1\in dz_{m+1}^1,$$
$$S_1^2\in dz_1^2,\cdots,S_{n+1}^2\in dz_{n+1}^2)$$
$$\square \sum_{m=1}^{M}\sum_{n=1}^{M}\sum_{i=1}^{m}\sum_{j=1}^{n}\int\cdots\int_{B_m^1;B_n^2}$$
$$P(X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1)$$
$$P(X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2)$$
$$\times P(S_1^1\in dz_1^1,\cdots,S_{m+1}^1\in dz_{m+1}^1,$$
$$S_1^2\in dz_1^2,\cdots,S_{n+1}^2\in dz_{n+1}^2)$$
$$=\sum_{m=1}^{M}\sum_{n=1}^{M}\sum_{i=1}^{m}\sum_{j=1}^{n} P(X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2, N_1(t)=m,$$
$$N_2(t)=n)$$
$$\square\ (\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} - (\sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty} + \sum_{m=M+1}^{\infty}\sum_{n=1}^{M}))$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n} P(X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2, N_1(t)=m,$$
$$N_2(t)=n)$$

$$=J_1(x_1,x_2;t)-J_2(x_1,x_2;t) \qquad (5)$$

For $J_1(x_1,x_2;t)$, uniformly for all $t\in\Lambda\bigcap[\varepsilon,T]$

$$J_1(x_1,x_2;t)$$
$$=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} P(X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1,$$
$$X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2, S_i^1\le t,$$
$$S_j^2\le t)$$
$$=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\int_0^t\int_0^t \prod_{i=1}^{2}(\sum\nolimits_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)})$$
$$dH_i(y))P(S_i^1\in du_1, S_j^2\in du_2)$$
$$=\int_0^t\int_0^t \prod_{i=1}^{2}(\sum\nolimits_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)})$$
$$dH_i(y))dE[N_1(u_1)N_2(u_2)]. \qquad (6)$$

As for $J_2(x_1,x_2;t)$, we have that for all $t\in\Lambda\bigcap[\varepsilon,T]$,

$$J_2(x_1,x_2;t)\le(\sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty}+\sum_{m=M+1}^{\infty}\sum_{n=1}^{\infty})$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n} P(X_i^1>x_1, X_j^2>x_2,$$
$$N_1(t)=m,\ \ N_2(t)=n)$$
$$=K_1(x_1,x_2;t)+K_2(x_1,x_2;t).$$

It holds that for all $t\in\Lambda\bigcap[\varepsilon,T]$,

$$K_1(x_1,x_2;t)$$
$$\le\sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} P(X_i^1>x_1, X_j^2>x_2, N_1(t)=m,$$
$$N_2(t)=n)$$
$$=P(X_i^1>x_1, X_j^2>x_2)\sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty} mnP(N_1(t)=m,$$
$$N_2(t)=n).$$
$$=\overline{F}(x_1)\overline{F}(x_2)E(N_1(t)N_2(t)I_{N_2(T)>M}).$$

We obtain from $H\ddot{o}lder's$ inequality that

$$\sup_{t\in\Lambda[\varepsilon,T],\ M\to\infty}\frac{E(N_1(t)N_2(t)I_{N_2(T)>M})}{E(N_1(t)N_2(t))}$$

$$\le \frac{(E[(N_1(T))]^2)^{\frac{1}{2}}(E[(N_2(T))^2 I_{\{N_2(T)>M\}}])^{\frac{1}{2}}}{E(N_1(\varepsilon)N_2(\varepsilon))} = 0 .$$

Then, by $F_i \in D, i=1,2$, we have

$$\lim_{M\to\infty}\lim_{\min\{x_1,x_2\}\to\infty}\sup_{t\in\Lambda\cap(0,T]} \frac{K_1(x_1,x_2;t)}{\int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})} \times \frac{1}{dH_i(y))dE[N_1(u_1)N_2(u_2)]}$$

$$\le \lim_{\min\{x_1,x_2\}\to\infty} \frac{\bar{F}(x_1)\bar{F}(x_2)}{\bar{F}_1(x_1e^{\delta T})\bar{F}_2(x_2e^{\delta T})} \times \frac{1}{\prod_{i=1}^{2}\sum_{\ell=1}^{K^i} p_\ell^i H_i(\alpha_\ell^i \wedge (t-u_i))}$$

$$\times \lim_{M\to\infty}\sup_{t\in\Lambda(0,T]} \frac{E(N_1(t)N_2(t)I_{N_2(T)>M})}{E(N_1(t)N_2(t))} = 0 . \qquad (7)$$

$K_2(x_1,x_2;t)$ can be dealt with in the same way. Thus, we obtain

$$\lim_{M\to\infty}\lim_{\min\{x_1,x_2\}\to\infty}\sup_{t\in\Lambda\cap(0,T]} \frac{J_2(x_1,x_2;t)}{\int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})} \times \frac{1}{dH_i(y))E(N_1(t)N_2(t))} = 0 . \qquad (8)$$

Substituting (6) and (8) into (5) leads to

$$I_1(x_1,x_2;t) \sim \int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y))dE[N_1(u_1)N_2(u_2)] . \qquad (9)$$

We next estimate $I_2(x_1,x_2;t)$. Choose some $\beta > \max\{J_{F_1}^+, J_{F_2}^+\}$. According to Lemma 3, uniformly for all $t\in\Lambda\cap[\varepsilon,T]$,

$$I_2(x_1,x_2;t) \le \sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty} mnP(X^1 > x_1/m)P(X^2 > x_2/n) \times P(N_1(t)=m,\ N_2(t)=n)$$

$$\le C_1C_2P(X^1 > x_1)P(X^2 > x_2)\sum_{m=1}^{\infty}\sum_{n=M+1}^{\infty} m^{\beta+1}n^{\beta+1} \times P(N_1(t)=m,\ N_2(t)=n)$$

$$\le CP(X^1 > x_1)P(X^2 > x_2) \times E(N_1(t)^{\beta+1}N_2(t)^{\beta+1}I_{N_2(T)>M}),$$

where C is some positive constant. Similarly to (8) we obtain

$$\lim_{M\to\infty}\lim_{\min\{x_1,x_2\}\to\infty}\sup_{t\in\Lambda\cap(0,T]} \frac{I_2(x_1,x_2;t)}{\int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})} \times \frac{1}{dH_i(y))dE[N_1(u_1)N_2(u_2)]} = 0 . \qquad (10)$$

In the same manner, we can prove

$$\lim_{M\to\infty}\lim_{\min\{x_1,x_2\}\to\infty}\sup_{t\in\Lambda\cap(0,T]} \frac{I_3(x_1,x_2;t)}{\int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})} \times \frac{1}{dH_i(y))dE[N_1(u_1)N_2(u_2)]}$$

$$\le \lim_{M\to\infty}\lim_{\min\{x_1,x_2\}\to\infty}\sup_{t\in\Lambda\cap(0,T]} \frac{I_4(x_1,x_2;t)}{\int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)})} \times \frac{1}{dH_i(y))dE[N_1(u_1)N_2(u_2)]} . \qquad (11)$$

Substituting (9), (10) and (11) into (4) the desired relation (3) holds uniformly for all $t\in\Lambda\cap[\varepsilon,T]$.

IV. PROOF OF MAIN RESULTS.

Proof of Theorem 1. We first deal with the asymptotic upper bound of $\psi(x_1,x_2;t)$. By Lemma 4, we have that uniformly for $t\in\Lambda\cap[\varepsilon,T]$,

$$\psi(x_1,x_2;t) \le P(\sum_{i=1}^{N_1(t)} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1\le t, D_i^1\le C_i^1\}} > x_1, \sum_{j=1}^{N_2(t)} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2\le t, D_j^2\le C_j^2\}} > x_2)$$

$$\sim \int_0^t\int_0^t\prod_{i=1}^{2}(\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i\wedge(t-u_i)} \bar{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y))dE[N_1(u_1)N_2(u_2)] . \qquad (12)$$

Then we discuss the asymptotic lower bound of $\psi(x_1,x_2;t)$. For simplicity, write

$$Z_k(t) = \sum_{i=1}^{N_k(t)} f_k(C_i^k)e^{-\delta S_i^k}, \quad k=1,2 .$$

For sufficiently large $N>0$, by Lemma 4,

$F_k \in L, k=1,2$, we obtain that uniformly for $t \in \Lambda \bigcap [\varepsilon, T]$,

$$\psi(x_1, x_2; t)$$

$$\square \int_0^\infty \int_0^\infty P(\sum_{i=1}^{N_1(t)} X_i^1 e^{-\delta(S_i^1+D_i^1)} I_{\{S_i^1+D_i^1 \le t, D_i^1 \le C_i^1\}} > x_1 + z_1, \sum_{j=1}^{N_2(t)} X_j^2 e^{-\delta(S_j^2+D_j^2)} I_{\{S_j^2+D_j^2 \le t, D_j^2 \le C_j^2\}} > x_2 + z_2) P(Z_1(t) \in dz_1, Z_2(t) \in dz_2)$$

$$\square \int_0^\infty \int_0^\infty \int_0^t \int_0^t \prod_{i=1}^{2} (\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y)) dE[N_1(u_1) N_2(u_2)] \times P(Z_1(t) \in dz_1, Z_2(t) \in dz_2)$$

$$\ge \int_0^N \int_0^N \int_0^t \int_0^t \prod_{i=1}^{2} (\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y)) dE[N_1(u_1) N_2(u_2)] \times P(Z_1(t) \in dz_1, Z_2(t) \in dz_2)$$

$$\square P(Z_1(t) \le N, Z_2(t) \le N) \int_0^t \int_0^t \prod_{i=1}^{2} (\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y)) dE[N_1(u_1) N_2(u_2)]$$

$$\square \int_0^t \int_0^t \prod_{i=1}^{2} (\sum_{\ell=1}^{K^i} p_\ell^i \int_0^{\alpha_\ell^i \Lambda(t-u_i)} \overline{F}_i(x_i e^{\delta(u_i+y)}) dH_i(y)) dE[N_1(u_1) N_2(u_2)]. \qquad (13)$$

A combination of (12) and (13) shows that (2) holds uniformly for all $t \le T$

REFERENCES

[1] Z. Li, J. Zhu, F. Chen, Study of a risk model based on the entrance process[J]. *Statistics and Probability Letters*. 2005, 72(1):1-10.

[2] Z. Li, X. Kong, A new risk model based on policy entrance process and its weak convergence properties [J]. *Applied Stochastic Models in Business and Industry*. 2007, 23(3): 235-246.

[3] H. Xiao, J. Tang, Asymptotic ruin probabilities of an entrance processes based risk model with interest force and regularly varying claims[J]. *Chinese Journal of Engineering Mathematics.* 2009, 26(6): 1126-1132.

[4] H. Xiao, Z. LI, W. Liu, The finite time ruin probability of a new risk model based on entrance process[J]. *Communications in Statistics-Theory and Methods.* 2013, 42(2): 336-345.

[5] F. Tang, J. Bai, Precise large deviations for aggregate loss process in a multi-risk model[J]. *J. Korean Math. Soc.* 2015, 52(3): 447-467.

[6] H. Xiao, L. Xie, Asymptotic ruin probability of a bidimensional risk model based on entrance processes with constant interest rate[J]. *Communications in Statistics-Theory and Methods.* 2018, 47(22): 5396-5417.

[7] H. Yang, J. Li, Asymptotic ruin probabilities for a bidimensional renewal risk model, *Stochastics*. 2017, 89(5): 687-708.

[8] Y. Yang, Y. Wang, Asymptotics for ruin probability of some negatively dependent risk models with a constant interest rate and dominatedly-varying-tailed claims[J]. *Statist. Probab. Lett.* 2010,80(3): 143-154.

[9] N. Bingham,C. Goldie, J. Teugels, Regular variation[M]. Cambridge: Cambridge University Press,1987.

[10] J. Li, On pairwise quasi-asymptotically independent random variables and their applications[J]. *Statist. Probab. Lett.* 2013,83(9):2081-2087.

[11] J. Hunter, Renewal theory in two dimensions: Basic results[J]. *Advances in Appl. Probability*. 1974, 6(2): 376–391.

Zhankui Wang, School of Mathematics and Statistics, Northwest Normal University, Lanzhou, China, Mobile 86- 18894575336.