Estimation the parameters of Odd Generalized Exponential-Gompertez distribution

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***Abstract*— In this paper, an odd generalized exponential-Gompertz (OGE-G) distribution which is capable of life tables to calculate death rates (failure) are considered. Based on simulated data from the OGE-G distribution, the problem of estimation of parameters under classical and Bayesian approaches are calculated. In this regard, the maximum likelihood estimates, and Bayes estimates under squared error loss function are obtained. Also 95% asymptotic confidence interval and highest posterior density interval estimates are calculated. The Monte Carlo simulation will be conduct to study and compare the performance of the various proposed estimators.**

***Index Terms*— Asymptotic confidence interval, Bayesian estimation, Odd generalized exponential-Gompertez distribution, Highest posterior density interval, Maximum likelihood estimation, Monte Carlo Markov Chain, Metropolis-Hasting algorithm..**

I. Introduction

El-Damcese et al. (2015) proposed a new model, called an odd generalized exponential-Gompertz (OGE-G) distribution and studied its properties. Some statistical properties of this distribution have been derived and discussed. The quantile, median, mode and moments of OGE-G are derived in closed forms. The distribution of the order statistics are discussed. Both point and asymptotic confidence interval estimates of the parameters are derived using the maximum likelihood method and obtained the observed Fisher information matrix. Also an application on a set of real data to compare the OGE-G with other known distributions such as Exponential, Generalized Exponential, Gompertz, Generalized Gompertz and Beta-Gompertz is given. Applications on a set of real data showed that the OGE-G is the best distribution for fitting these data sets compared with other distributions considered. The cumulative distribution function (cdf) of the OGE-G distribution is given by

$$F(x;\theta)=\left[1-e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right]^{\beta}\qquad x>0,\qquad \lambda,\alpha,\beta,\theta>0 \qquad (1)$$

where β is the shape parameter and (λ,α,θ) are the scale parameters. Figure (1) illustrated the behavior of the cdf of OGE-G distribution at $\alpha=2.5$ and $\alpha=0.5$ for some various values of λ,θ,β.

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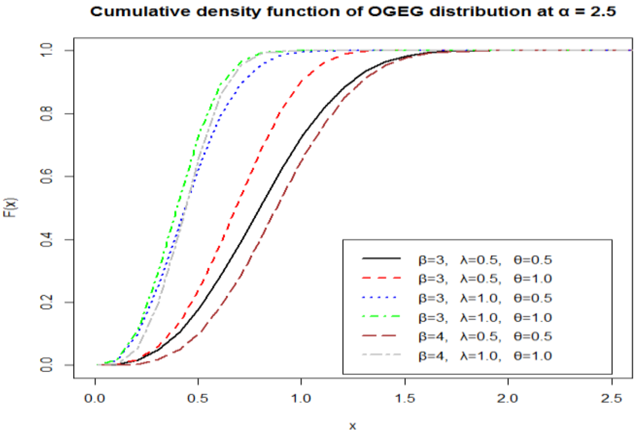
The probability density function (pdf) of the OGE-G distribution is given by

$$f(x;\theta)=\alpha\lambda\beta\left(e^{\left[\theta x+\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)\right]}\right)\left(e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right)\left[1-e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right]^{\beta-1} \qquad (2)$$

Figure (2) illustrated the behavior of the pdf of OGE-G distribution at $\alpha=2.5$ and $\alpha=0.5$ for some various values of λ,θ,β



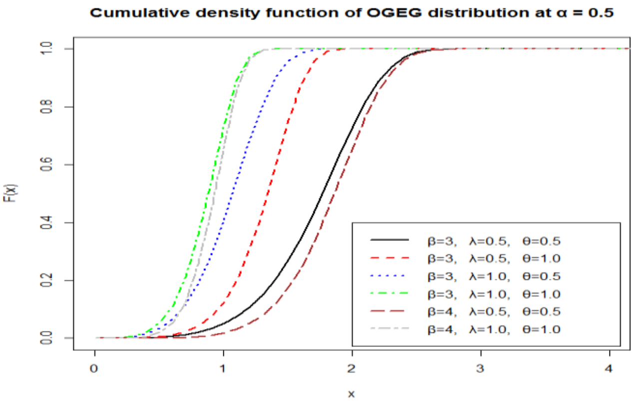
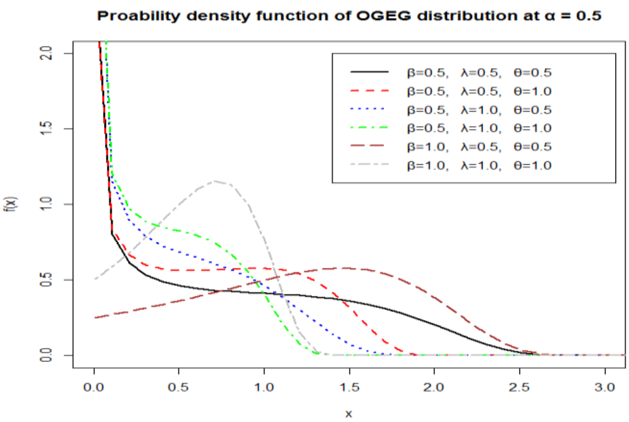


Figure 1: Cumulative function of OGE-G distribution at different parameters



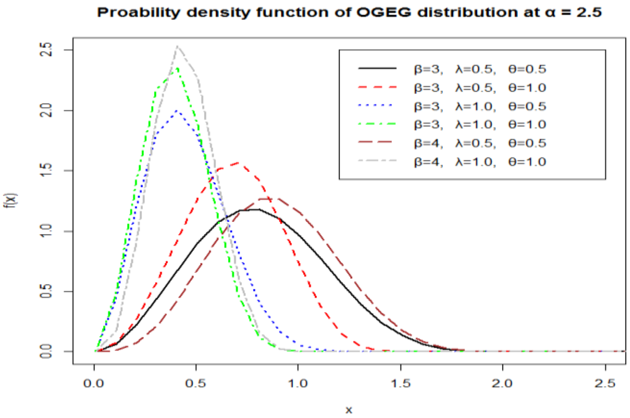


Figure 2: Density function of OGE-G distribution at different parameters

The hazard rate function of OGE-G distribution can be obtained from

$$h(x)=\frac{\alpha\lambda\beta\left(e^{\left[\theta x+\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)\right]}\right)\left(e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right)\left[1-e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right]^{\beta-1}}{1-\left[1-e^{-\alpha\left(e^{\frac{\lambda}{\theta}\left(e^{\theta x}-1\right)}-1\right)}\right]^{\beta}}$$

and its shape is illustrated in Figure (3) at $\beta=0.1$ and $\beta=0.5$ for some various values of α,λ,β.

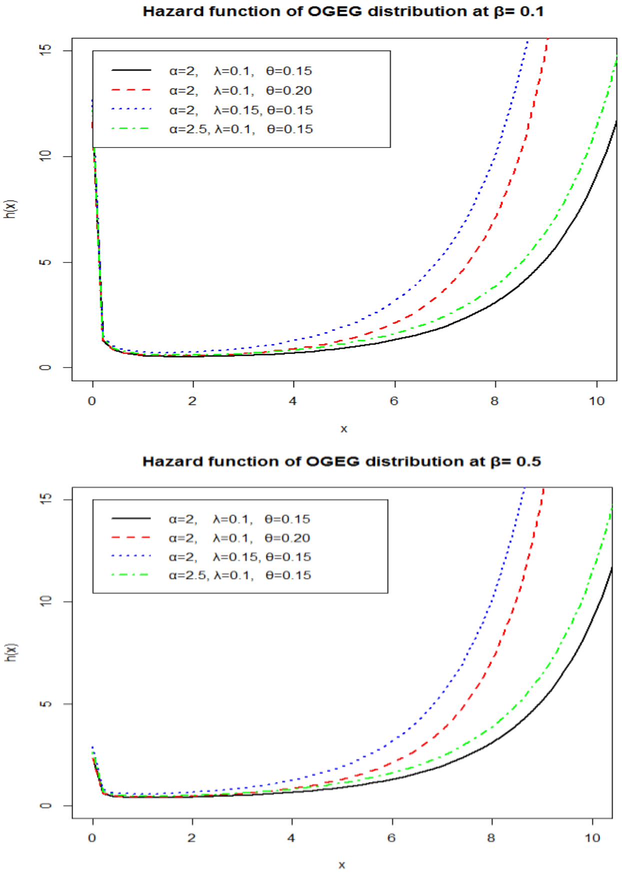


Figure 3: Hazard rate function of OGE-G distribution at different parameters

For parameter estimation of the unknown parameters of the OGE-G distribution $(\lambda,\alpha,\beta,\theta)$ there are two methods: Maximum likelihood estimation and Bayesian estimation.

II. Maximum Likelihood Estimation

Suppose that a random sample of n units whose lifetime follow an OGE-G distribution with cdf given in Eq. (1) and its pdf given in Eq. (2) and the likelihood function is defined as

$$L(\Theta)=\prod_{i=1}^{n} f\left(x_{(i)};\,\Theta\right)$$

Thus, the likelihood function for the OGE-G distribution can be written as

$$L(\lambda,\alpha,\beta,\theta)=(\alpha\beta\lambda)^n\exp\left(\sum_{i=1}^{n}\left[\theta x_i+\frac{\lambda}{\theta}\left(\exp(\theta x_i)-1\right)-\alpha\left(\exp\left(\frac{\lambda}{\theta}\left(\exp(\theta x_i)-1\right)\right)-1\right)\right]\right)\sum_{i=1}^{n}\left(1-\exp\left[-\alpha\left(\exp\left(\frac{\lambda}{\theta}\left(\exp(\theta x_i)-1\right)\right)-1\right)\right]\right)^{\beta-1}$$

Let

$$\psi_i=\exp\left(\frac{\lambda}{\theta}\left(\exp(\theta x_i)-1\right)\right)$$

and

$$\varphi_i=\exp(\theta x_i)-1$$

thus, the likelihood function can be rewritten as

$$L(\lambda,\alpha,\beta,\theta)=(\alpha\beta\lambda)^n\exp\left(\sum_{i=1}^{n}\theta x_i+\frac{\lambda}{\theta}\varphi_i-\alpha(\psi_i-1)\right)\sum_{i=1}^{n}\left(1-\exp(-\alpha(\psi_i-1))\right)^{\beta-1}$$

By taking logarithm of $L(\lambda,\alpha,\beta,\theta)$ to obtain log-likelihood $\mathcal{L}$ as

$$\mathcal{L}=n\log(\alpha)+n\log(\beta)+n\log(\lambda)+\sum_{i=1}^{n}\theta x_i+\frac{\lambda}{\theta}\varphi_i-\alpha(\psi_i-1)+(\beta-1)\sum_{i=1}^{n}\log\left(1-\exp(-\alpha(\psi_i-1))\right)$$

by differentiating the associated log-likelihood $\mathcal{L}$ with respect to λ,α,β and θ and equating them to zero, we get:

$$\frac{\partial\mathcal{L}}{\partial\lambda}=\frac{n}{\hat\lambda}-\sum_{i=1}^{n}\frac{\varphi_i\left(1-\hat\alpha\hat\psi_i\right)}{\hat\theta}+\left(\hat\beta-1\right)\left[\frac{\varphi_i\hat\psi_i\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)}{\hat\theta\left[1-\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)\right]}\right]=0$$

$$\frac{\partial\mathcal{L}}{\partial\alpha}=\frac{n}{\hat\alpha}+\sum_{i=1}^{n}\left(\hat\psi_i-1\right)+\left(\hat\beta-1\right)\left[\frac{\left(\hat\psi_i-1\right)\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)}{1-\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)}\right]=0$$

$$\frac{\partial\mathcal{L}}{\partial\beta}=\frac{n}{\hat\beta}+\sum_{i=1}^{n}\log\left(1-\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)\right)=0$$

$$\frac{\partial\mathcal{L}}{\partial\theta}=\sum_{i=1}^{n}\left(\frac{\hat\lambda x_i e^{\hat\theta x_i}}{\hat\theta}-\frac{\hat\lambda\varphi_i}{\hat\theta^2}\right)\left(1-\hat\alpha\hat\psi_i+\left(\hat\beta-1\right)\left[\frac{\hat\alpha\hat\psi_i\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)}{\left[1-\exp\left(-\hat\alpha\left(\hat\psi_i-1\right)\right)\right]}\right]\right)=0$$

where

$$\hat{\psi}_i = \exp\left(\frac{\hat{\lambda}}{\hat{\theta}}\left(\exp(\hat{\theta}x_i)-1\right)\right), \qquad \hat{\varphi}_i = \exp(\hat{\theta}x_i)-1$$

and $\hat{\lambda}, \hat{\alpha}, \hat{\beta}$ and $\hat{\theta}$ are the MLEs of λ, α, β and θ respectively. Now, the asymptotic variance-covariance matrix of the MLEs of λ, α, β and θ can be obtained by inverting the observed information matrix ($I(\lambda,\alpha,\beta,\theta)$), and is given by

$$I(\lambda,\alpha,\beta,\theta) = -\begin{bmatrix} E\left(\frac{\partial^2 \mathcal{L}}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\lambda\partial\alpha}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\lambda\partial\beta}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\lambda\partial\theta}\right) \\ E\left(\frac{\partial^2 \mathcal{L}}{\partial\alpha\partial\lambda}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\alpha\partial\beta}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\alpha\partial\theta}\right) \\ E\left(\frac{\partial^2 \mathcal{L}}{\partial\beta\partial\lambda}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\beta^2}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\beta\partial\alpha}\right) \\ E\left(\frac{\partial^2 \mathcal{L}}{\partial\theta\partial\lambda}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\theta\partial\alpha}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\theta\partial\beta}\right) & E\left(\frac{\partial^2 \mathcal{L}}{\partial\theta^2}\right) \end{bmatrix}_{|\lambda=\hat{\lambda},\alpha=\hat{\alpha},\beta=\hat{\beta},\theta=\hat{\theta}}$$

According to particular regularity conditions, the two-sided $100(1-\gamma)\%$, $0<\gamma<1$, asymptotic confidence intervals for the parameters λ, α, β and θ can be obtained.

III. Bayesian Estimation

For Bayesian parameter estimation consider squared error loss function. It is observed that if the scale parameter λ is known, the shape parameter α has a conjugate prior, which is a gamma prior. When both the parameters of the model are unknown, a joint conjugate prior for the parameters does not exist. We propose to use independent gamma priors for both λ and α having pdfs

$$\pi_1(\lambda) \propto \lambda^{a_1-1}\exp(-b_1\lambda) \qquad \lambda>0, \quad a_1,b_1>0$$
$$\pi_2(\alpha) \propto \alpha^{a_2-1}\exp(-b_2\alpha) \qquad \alpha>0, \quad a_2,b_2>0$$

and for parameter β we propose to use exponential prior with pdf

$$\pi_3(\beta) \propto \beta\exp(-c\beta) \qquad \beta>0, \quad c>0$$

where the hyper-parameters a_1, b_1, a_2, b_2, c are chosen to reflect the prior knowledge about the unknown parameters. The joint prior for λ, α and β is given by

$$\pi(\lambda,\alpha,\beta) = \pi_1(\lambda)\,\pi_2(\alpha)\,\pi_3(\beta)$$
$$\pi(\lambda,\alpha,\beta) \propto \beta\,\lambda^{a_1-1}\alpha^{a_2-1}\exp(-b_1\lambda-b_2\alpha-c\beta)$$

The corresponding posterior density given the observed data $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)$ can be written as

$$\pi(\lambda,\alpha,\beta|\boldsymbol{x}) = \frac{\pi(\lambda,\alpha,\beta)L(\lambda,\alpha,\beta)}{\int_0^\infty\int_0^\infty\int_0^\infty \pi(\lambda,\alpha,\beta)L(\lambda,\alpha,\beta)\,d\lambda\,d\alpha\,d\beta}$$

The posterior density function can be written as

$$\pi(\lambda,\alpha,\beta|\boldsymbol{x}) = K^{-1}(\beta^n\lambda^{n+a_1-1}\alpha^{n+a_2-1})\exp(-b_1\lambda-b_2\alpha-c\beta)$$
$$\exp\left(\sum_{i=1}^n\left[\theta x_i+\frac{\lambda}{\theta}(\exp(\theta x_i)-1)-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)$$
$$\sum_{i=1}^n\left(1-\exp\left[-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)^{\beta-1}$$

where

$$K=\int_0^\infty\int_0^\infty\int_0^\infty(\beta^n\lambda^{n+a_1-1}\alpha^{n+a_2-1})\exp(-b_1\lambda-b_2\alpha-c\beta)$$
$$\exp\left(\sum_{i=1}^n\left[\theta x_i+\frac{\lambda}{\theta}(\exp(\theta x_i)-1)-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)$$
$$\sum_{i=1}^n\left(1-\exp\left[-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)^{\beta-1}d\lambda\,d\alpha\,d\beta$$

Thus, the posterior density can be rewritten as

$$\pi(\lambda,\alpha,\beta|\boldsymbol{x}) \propto (\beta^n\lambda^{n+a_1-1}\alpha^{n+a_2-1})\exp(-b_1\lambda-b_2\alpha-c\beta)$$
$$\exp\left(\sum_{i=1}^n\left[\theta x_i+\frac{\lambda}{\theta}(\exp(\theta x_i)-1)-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)$$
$$\sum_{i=1}^n\left(1-\exp\left[-\alpha\left(\exp\left(\frac{\lambda}{\theta}(\exp(\theta x_i)-1)\right)-1\right)\right]\right)^{\beta-1}$$

The Bayes Estimator of any loss function, say $g(\lambda,\alpha,\beta)$ under the squared error, is given by

$$\tilde{g}(\lambda,\alpha,\beta) = \int_0^\infty\int_0^\infty\int_0^\infty g(\lambda,\alpha,\beta)\,\pi(\lambda,\alpha,\beta|\boldsymbol{x})\,d\lambda\,d\alpha\,d\beta \tag{4}$$

Unfortunately, Eq. (4) cannot be computed for general $g(\lambda,\alpha,\beta)$. Thus, we provide the approximate Bayes estimates of λ, α and β such as:

- Lindley's Approximation
- Importance Sampling
- Markov Chain Monte Carlo

3.1 Markov Chain Monte Carlo (MCMC)

Markov Chain Monte Carlo (MCMC) is a computer-driven sampling method. It allows one to characterize a distribution without knowing all of the distribution mathematical properties by random sampling values out of the distribution (Ravenzwaaij et al. (2016)).

MCMC is particularly useful in Bayesian inference because of the focus on posterior distributions which are often difficult to work with via analytic examination. In these cases, MCMC allows the user to approximate aspects of posterior distributions that cannot be directly calculated (e.g., random samples from the posterior, posterior means, etc.). To draw samples from a distribution using MCMC:

1. Starting with an initial guess: just one value that might be plausibly drawn from the distribution.
2. Producing a chain of new samples from this initial guess. Each new sample is produced by two simple steps:
   - Proposal: a proposal for the new sample is created by adding a small random perturbation to the most recent sample.
   - Acceptance: the new proposal is either accepted as the new sample, or rejected (in which case the old sample retained).

Proposal Distribution: A distribution for randomly generating new candidate samples, to be accepted or rejected. There are many ways of adding random noise to create proposals, and also different approaches to the process of accepting and rejecting, such as: Gibbs-sampling and Metropolis-Hastings algorithm.

3.2 Metropolis-Hastings Algorithm

Metropolis-Hastings (MH) algorithm is a useful method for generating random samples from the posterior distribution using a proposal density. To implement the MH algorithm we have to define a proposal distribution $q(\zeta'|\zeta)$ and an initial values $\zeta^{(0)}$ of the unknown parameters. For the proposal distribution, we consider a bivariate normal distribution, that is $q(\zeta'|\zeta) \equiv N_2(\zeta, S_\zeta)$, where $\zeta = (\lambda, \alpha, \beta)$ and S_ζ represent the variance-covariance matrix, we may get negative observations which are not acceptable. For the initial values, we guess an appropriate values to λ, α and β. Therefore, we propose the following steps of MH algorithm to draw sample from the posterior density $\pi(\lambda, \alpha, \beta|x)$ (Dey and Pradhan (2014)):

Step 1. Set initial value of ζ as $\zeta = \zeta^{(0)}$.

Step 2. For $i = 1,2,\dots,M$ repeat the following steps:

1. Set $\zeta = \zeta^{(i-1)}$.
2. Generate a new candidate parameter value δ from $N_2(\ln\zeta, S_\zeta)$.
3. Set $\zeta = \exp(\delta)$.
4. Calculate $\rho = \min\left(1, \frac{\pi(\zeta'|x)}{\pi(\zeta|x)}\right)$
5. Update $\zeta^{(i)} = \zeta'$ with probability ρ; otherwise set $\zeta^{(i)} = \zeta$.

The initial value for ζ is considered to be the MLE $\hat{\zeta} = (\hat{\lambda}, \hat{\alpha}, \hat{\beta})$ of $\zeta = (\lambda, \alpha, \beta)$. While, the selection of S_ζ is considered to be the asymptotic variance-covariance matrix $I^{-1}(\lambda, \alpha, \beta)$, where $I(.)$ is the Fisher information matrix. Notice that, the selection of S_ζ is an important issue in the MH algorithm where the acceptance rate is depends on upon this.

Finally, from the random samples of size M drawn from the posterior density, some of the initial samples (burn-in) can be discarded, and remaining samples can be further utilized to compute Bayes estimates. More precisely the Eq. (4) can be evaluated as

$$\tilde{g}_{MH}(\lambda, \alpha, \beta) = \frac{1}{M - l_B} \sum_{i=1}^{M} g(\lambda_i, \alpha_i, \beta_i) \quad (5)$$

where l_B represent the number of burn-in samples.

3.3 Highest Posterior Density

Suggesting utilizing the technique of Chen and Shao (1999) to calculate highest posterior density (HPD) interval estimates for the unknown parameters of the GIE distribution. The technique of Chen and Shao has been broadly utilized for constructing HPD intervals for the unknown parameters of the distribution of interest. In the present study, we will employ the samples drawn using the proposed MH algorithm to construct the interval estimates. More accurately, let us assume that $\Pi(\theta|x)$ denotes the posterior distribution function of θ. Let us further suppose that $\theta^{(p)}$ be the *p*th quantile of θ, that is, $\theta^{(p)} = \inf\{\theta : \Pi(\theta|x) \geq p\}$, where $0 < p < 1$. Notice that for a given θ^*, a simulation consistent estimator of $\Pi(\theta^*|x)$ can be estimated as

$$\Pi(\theta^*|x) = \frac{1}{M - l_B} \sum_{i=l_B}^{M} I_{\theta \leq \theta^*}$$

Here $I_{\theta \leq \theta^*}$ is the indicator function. Then the corresponding estimate is obtained as

$$\hat{\Pi}(\theta^*|x) = \begin{cases} 0 & \text{if } \theta^* \leq \theta_{(l_B)} \\ \sum_{j=l_B}^{i} w_j & \text{if } \theta_{(i)} \leq \theta^* \leq \theta_{(i+1)} \\ 1 & \text{if } \theta^* > \theta_{(M)} \end{cases}$$

where $w_j = \frac{1}{M - l_B}$ and $\theta_{(j)}$ are the ordered values of θ_j.

Now, for $i = l_B, \dots, M$, $\theta^{(p)}$ can be approximated by

$$\hat{\theta}^{(p)} = \begin{cases} \theta_{(l_B)} & \text{if } p = 0 \\ \theta_{(i)} & \text{if } \sum_{j=l_B}^{i-1} w_i < p < \sum_{j=l_B}^{i} w_i \end{cases}$$

Now to obtain a $100(1-p)\%$ HPD credible interval for θ, let

$$R_j = \left(\hat{\theta}^{\left(\frac{j}{M}\right)}, \hat{\theta}^{\left(\frac{j+(1-p)M}{M}\right)}\right), \qquad j = l_B, \dots, [p_M],$$

here $[a]$ denotes the largest integer less than or equal to a. Then choose R_{j^*} among all the R'_js such that it has the smallest width.

IV. Simulation Study

The aim of this section is to set a comparison the performance of the different methods of estimation discussed in the previous sections. A simulation study is employed to check the behavior of the proposed methods as well as to assess the statistical performances of the estimators, by using *R*-statistical programming language for calculation. Further, utilizing *bbmle* and *HDInterval* packages to compute MLEs and HPD interval in *R*-language.

A Monte Carlo simulation study is employed to compare the performance of proposed methods of estimation. This method simulates 1000 generating data from OGE-G distribution with initial values:

- Case I: $\lambda = 0.5,\ \alpha = 2.5,\ \beta = 3,\ \theta = 0.5$

Case II: $\lambda = 0.5,\ \alpha = 2,\ \beta = 4,\ \theta = 0.5$

Based on the generated data, firstly, we calculate maximum likelihood estimates and associated 95 % asymptotic confidence interval estimates. Note that the initial guess values are considered to be same as the true parameter values while obtaining maximum likelihood estimates.

For Bayesian estimation, we calculate Bayes estimates using the MH algorithm under the informative prior. These priors are then plugged-in to calculate the desired estimates. While utilizing MH algorithm, we take into account the maximum likelihood estimates as initial guess values, and the associated variance–covariance matrix. At the end, we discarded 1000 burn-in samples among the overall 5000 samples generated from the posterior density, and subsequently obtained Bayes estimates, and HPD interval estimates utilizing the technique of Chen and Shao (1999).

All the average estimates for both methods are reported in Table 1 and Table 2. Further, the first row represents the average estimates and interval estimates, and in the second row, associated means square errors (MSEs) and average interval lengths (AILs) with coverage percentages (CPs) are reported. The convergence of MCMC estimation for λ, α and β can be showed in figure (4) and figure (5).

From tabulated values it can be noticed that depending on MSEs, higher values of *n* lead to better estimates. It is also noticed that the performance of the Bayes estimates obtained are better than the MLE estimates. It can also be noticed that the AILs and associated CPs of HPD intervals of Bayes estimates are better than those of MLE estimates.

Table 1: Estimated values, interval estimates, MSEs, AILs and CPs for MLE and Bayesian (MCMC) for number of simulation 5000

Initial: $\lambda = 0.5 \quad \alpha = 2.5 \quad \beta = 3 \quad \theta = 0.5$					
n	**Parameters**	**MLE**		**Bayesian (MCMC)**	
		Estimate MSE	**Asy CI AIL/CP**	**Estimate MSE**	**HPD interval AIL/CP**
25	λ	0.7057 (1.2143)	(0.0562, 4.2319) 4.1757 / 95.50	0.4264 (0.0131)	(0.3352, 0.5369) 0.2016 / 98.70
	α	3.5399 (9.5847)	(0.1046, 10.809) 10.7046 / 97.10	2.5989 (3.9400)	(0, 6.5460) 6.5460 / 97.80
	β	3.7582 (13.4985)	(1.089, 14.479) 13.390 / 97.50	3.3288 (0.9026)	(1.7431, 5.1585) 3.4154 / 96.60
	θ	0.6762 (0.6772)	(0, 2.5202) 2.5202 / 96	----	----
50	λ	0.8056 (1.6330)	(0.1140, 4.8760) 4.7619 / 96.50	0.50137 (0.0016)	(0.4207, 0.5844) 0.16366 / 97.60
	α	3.3264 (6.6586)	(0.0751, 9.0352) 8.9601 / 97.50	2.4846 (3.4647)	(0, 5.9960) 5.9960 / 97.70
	β	3.2991 (3.7233)	(1.4116, 8.2259) 6.8142 / 97	3.5121 (0.7700)	(1.9673, 4.8140) 2.8466 / 95.90
	θ	0.53720 (0.3831)	(0, 1.7028) 1.7028 / 97.20	----	----
75	λ	0.8102 (1.3124)	(0.0986, 4.3928) 4.2942 / 98	0.4809 (0.0015)	(0.4131, 0.5426) 0.1295 / 97.20
	α	3.2606 (6.1628)	(0.1004, 8.9703) 8.8698 / 97.50	2.4359 (2.7433)	(0, 5.7272) 5.7272 / 97.50
	β	3.0900 (2.5756)	(1.5337, 6.5564) 5.0227 / 98.50	3.5305 (0.6046)	(2.5516, 4.6725) 2.1209 / 97.20
	θ	0.50902 (0.3165)	(0, 1.5884) 2.4073 / 97.50	----	----
100	λ	0.90965 (1.5107)	(0.1595, 4.3399) 4.1804 / 95.50	0.5166 (0.0013)	(0.4552, 0.5778) 0.1226 / 97.40
	α	2.9454 (4.3171)	(0.0934, 7.2565) 7.1631 / 97.80	2.5776 (2.9922)	(0, 5.7052) 5.7052 / 96.80
	β	2.9488 (1.0926)	(1.5884, 5.2357) 3.6473 / 97.50	3.0011 (0.1742)	(2.1712, 3.7708) 1.5996 / 96.00
	θ	0.4567 (0.3128)	(0, 1.3232) 1.3232 / 97.50	----	----
Asy CI: Asymptotic confidence interval, AIL: Average interval length, CP: Coverage probability					

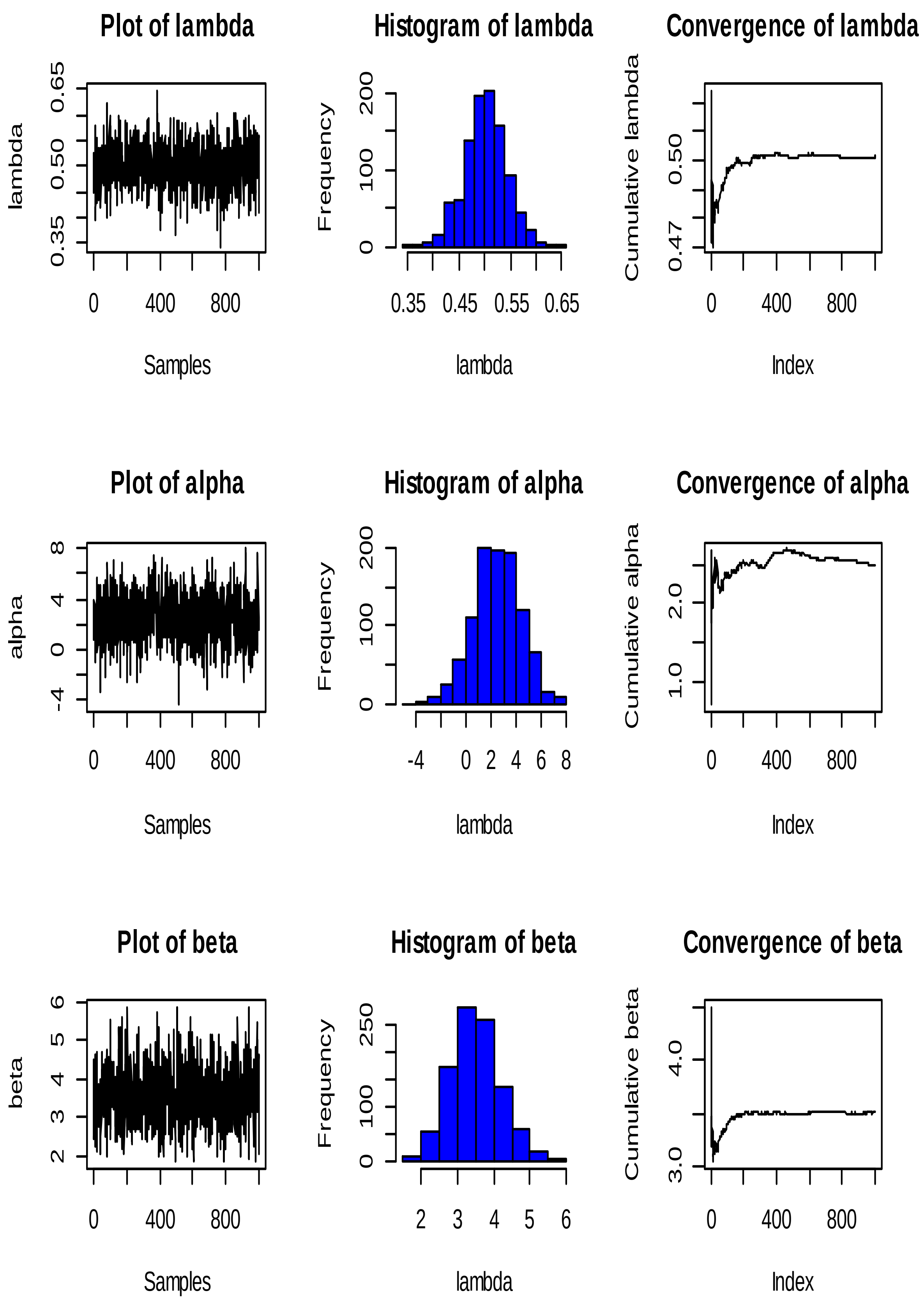


Figure 4: Convergence of MCMC estimation for λ, α and β when $n = 50$

Table 2: Estimated values, interval estimates, MSEs, AILs and CPs for MLE and Bayesian (MCMC) for number of simulation 5000

Initial: $\lambda = 0.5 \quad \alpha = 2 \quad \beta = 4 \quad \theta = 0.5$					
n	**Parameters**	**MLE**		**Bayesian (MCMC)**	
		Estimate MSE	**Asy CI AIL/CP**	**Estimate MSE**	**HPD interval AIL/CP**
25	λ	0.67919 (0.9188)	(0.10442, 3.5922) 3.4877 / 97.50	0.4749 (0.00336)	(0.3695, 0.5675) 0.19798 / 97.00
	α	2.9322 (6.95297)	(0.07706, 9.2593) 9.1822 / 97.50	1.9853 (4.3739)	(0, 5.9908) 5.9908 / 97.60
	β	5.9520 (42.8007)	(1.2524, 26.2899) 25.0375 / 96	3.4495 (1.0806)	(1.8197, 5.1523) 3.3326 / 96.50
	θ	0.6567 (0.6587)	(0, 2.2211) 2.2211 / 95	----	----
50	λ	0.70250 (0.8883)	(0.1565, 3.8144) 3.6579 / 97.40	0.51179 (0.0018)	(0.43175, 0.5972) 0.1655 / 98.30
	α	2.8277 (4.9761)	(0.0778, 7.7629) 7.6850 / 98	1.9718 (3.4255)	(0, 5.3129) 5.3129 / 96.80
	β	4.6691 (12.0931)	(1.7066, 13.6274) 11.9208 / 97.50	3.6497 (0.66417)	(2.2955, 5.1080) 2.8124 / 97
	θ	0.56298 (0.2958)	(0, 1.5913) 1.5913 / 97.50	----	----
75	λ	0.8119 (1.1105)	(0.2099, 3.8430) 3.6331 / 97.50	0.47658 (0.00147)	(0.41790, 0.5375) 0.11969 / 97.60
	α	2.5221 (3.4426)	(0.0715, 6.3853) 6.3137 / 98	1.9292 (2.5634)	(0, 5.1501) 5.1501 / 97.80
	β	4.2047 (5.6127)	(1.8958, 10.4970) 8.6011 / 97.50	3.7200 (0.41486)	(2.5351, 4.8078) 2.2726 / 96.20
	θ	0.4996 (0.2617)	(0, 1.3773) 1.3773 / 97.90	----	----
100	λ	0.8458 (1.1221)	(0.22346, 3.6774) 3.4539 / 98.30	0.47266 (0.00153)	(0.41568, 0.52452) 0.1088 / 97.40
	α	2.4117 (2.8903)	(0.0768, 5.9538) 5.8770 / 98.50	2.0456 (2.486)	(0, 4.9778) 4.9778 / 96.80
	β	4.0663 (3.3185)	(1.9830, 8.6559) 6.6729 / 98	3.8436 (0.63456)	(2.4418, 4.1761) 1.7342 / 96.60
	θ	0.4684 (0.2302)	(0, 1.1928) 1.1928 / 98.10	----	----
Asy CI: Asymptotic confidence interval, AIL: Average interval length, CP: Coverage probability					

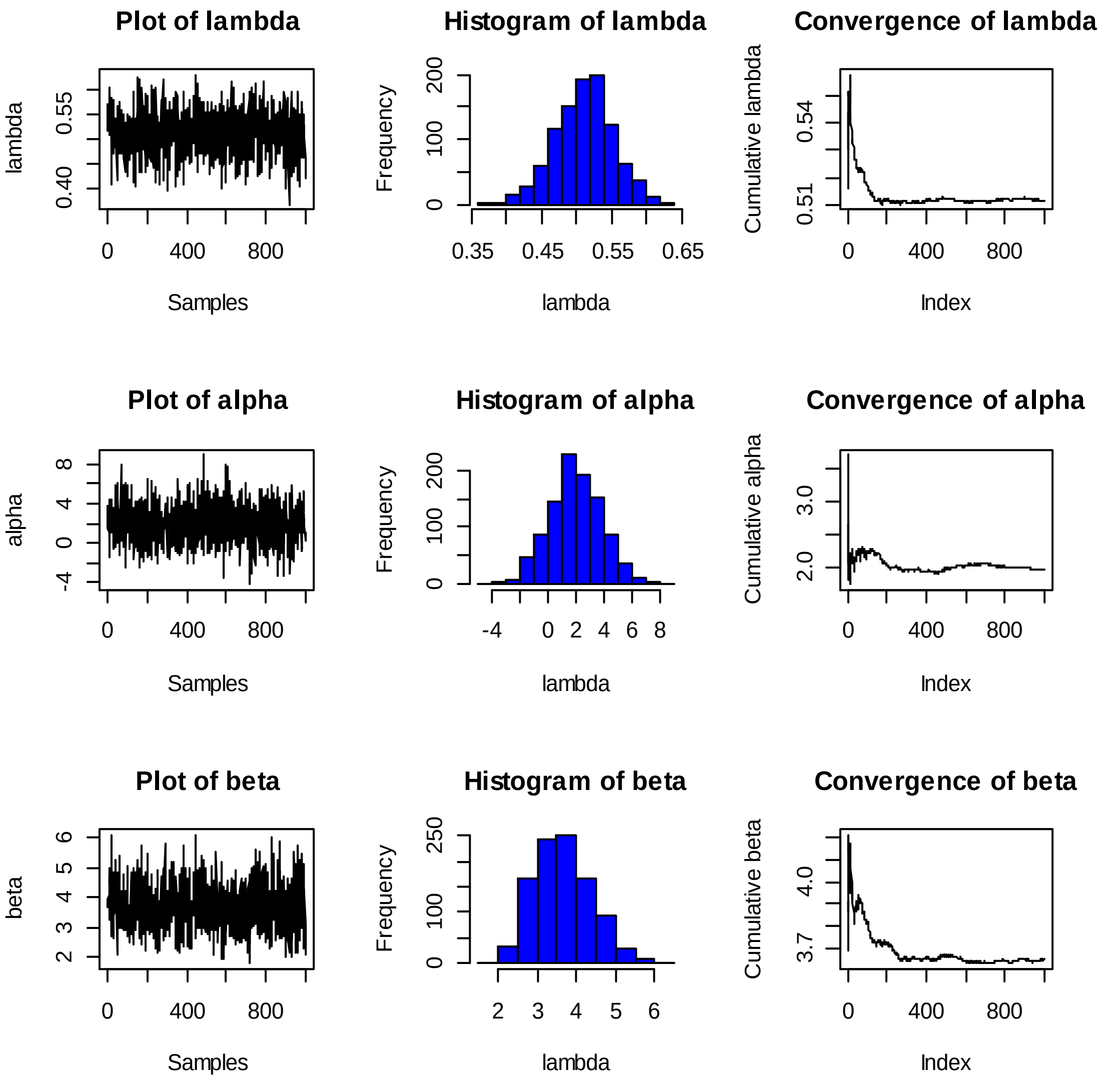


Figure 5: Convergence of MCMC estimation for λ, α and β when $n = 50$

V. Conclusion

In this paper, the problem of estimation of the odd generalized exponential-Gompertz (OGE-G) distribution from classical and Bayesian viewpoint, were studied maximum likelihood estimates and associated asymptotic confidence interval estimates for the unknown parameters of a OGE-G distribution. Then, we calculated Bayes estimates and the corresponding HPD interval estimates were derived under informative priors. Using simulation study indicates that the performance of Bayes estimates is better MLE estimates. Though squared error loss function under Bayesian set up, yet other loss functions can also be considered. This research can also be extended to design of progressive censoring sampling plan and other censoring schemes can also be considered.

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