

Relation between peak value and acoustic damping of sound pressure level of one dimensional sound field partitioned with perforated plates

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Abstract— In an industry field, the acoustic characteristics such as a resonance frequency and an acoustic damping ratio are often required. The response test due to a random noise wave and the sweep test due to a sinusoidal wave can be performed in those cases. In vibration theory, it is well known that the peak value at resonance is inversely proportional to the damping ratio. Is it true even in the acoustic theory? If it can be true the acoustic damping ratio can be obtained simply by the peak sound pressure level after the relation between the peak value and the damping ratio is examined only once. In this study, the sound pressure level of the one dimensional duct with the perforated plate will be calculated by the transfer matrix method in various parameters such as the aperture ratio, the hole's length and the set position of the plate. These calculations are conducted for the first mode, the second mode and third mode. And the relation between the peak value and the damping ratio are examined.

Index Terms— Acoustic characteristics, Damping ratio, Resonance frequency, Transfer matrix method, Perforated plate

I. INTRODUCTION

A perforated plate is widely used as an acoustic absorption material for compressors [1] and acoustic barriers for roads and railways[2]. It has been confirmed that the perforated plate suppressed the self- sustained tone generated from heat exchangers like a boiler by using it on a duct wall[3]~[6]. The effects of the perforated plate on the acoustic natural frequency of an one dimensional sound field partitioned with the perforated plate has been studied in our previous investigations [7],[8]. It was clarified that the acoustic natural frequency of the one dimensional sound field partitioned with the perforated plate becomes lower as the aperture ratio becomes smaller. The acoustic characteristics such as a resonant frequency and an acoustic damping ratio of an one dimensional sound field like a duct has often been required. In that case, the random excitation or the sine wave sweep test has been performed to obtain the acoustic characteristics.

By the way, we have a question that whether the response value at resonance is inversely proportional to the acoustic damping ratio similar to the vibration theory or not. On the other hand, we have the thought it is natural. But it is not clear in the present stage. If it is possible, the peak value of the sound pressure level at resonance can be obtained easily without obtaining the acoustic damping ratio which is hard to obtain.

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Then, in this study, the sound pressure level is calculated for each parameter of the perforated plate such as the aperture ratio, hole's length and the setting position in the duct and the peak value and acoustic damping ratio for 1st, 2nd and 3rd modes are obtained by the analysis. The relation between the peak value and the acoustic damping will be examined. The analysis is performed by using the Transfer Matrix Method and Melling's results[9],[10].

II. ANALYTICAL METHOD

Figure1 shows an analytical model of the duct with a perforated plate. The numbers show the varying points of the cross section of the duct and the perforated plate position. Indicating the state vector at each numbering position as $[P_i \ U_i]^T$ ($i=1\sim 4$), the equation (1) can be obtained as the relation between both state vectors of first and terminal positions. Where P_i , U_i are the sound pressure and the volume velocity respectively.

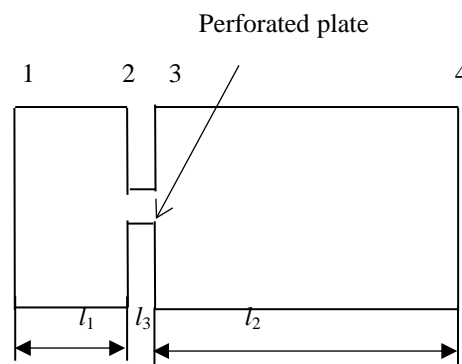


Fig.1 Analytical model of 1D duct

$$\begin{bmatrix} P_4 \\ U_4 \end{bmatrix} = \begin{bmatrix} \cos kl_2 & jZ_2 \sin kl_2 \\ j\frac{1}{Z_2} \sin kl_2 & \cos kl_2 \end{bmatrix} \begin{bmatrix} \cos kl_3 & jZ_3 \sin kl_3 \\ j\frac{1}{Z_3} \sin kl_3 & \cos kl_3 \end{bmatrix} \times \begin{bmatrix} \cos kl_1 & jZ_1 \sin kl_1 \\ j\frac{1}{Z_1} \sin kl_1 & \cos kl_1 \end{bmatrix} \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \quad (1)$$

Given the unit forced displacement at the left end of the duct and the boundary condition of the right end is given as follows because the right end of the duct is closed.

$$U_1=1, \ U_4=0 \quad (2)$$

And the center matrix of the equation (1) which relates the state vectors of before and behind the perforated plate can be given as follows.

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$$\begin{bmatrix} 1 & Z_R + jZ_i \\ 0 & 1 \end{bmatrix} \quad \text{Where } Z = z/S \quad (3)$$

Here Melling's equation (4) is used as the acoustic impedance before and behind the perforated plate.

$$\begin{aligned} z &= (2\mu l_3 / r_0) \sqrt{\omega\rho/2\mu}(1+j) + j\omega\rho l_3 \\ &= (2\mu l_3 / r_0) \sqrt{\omega\rho/2\mu} + j\omega\rho l_3 \left[1 + \frac{1}{r_0} \sqrt{\frac{2\mu}{\omega\rho}} \right] \end{aligned} \quad (4)$$

As a result, A11 ~ A22 of equation (1) become as follows.

$$\begin{aligned} A11 &= \cos kl_2 \cos kl_1 - \frac{Z_i}{Z_1} \cos kl_2 \sin kl_1 - \frac{Z_2}{Z_1} \sin kl_2 \sin kl_1 \\ &\quad + j \frac{Z_R}{Z_1} \cos kl_2 \sin kl_1 \\ A12 &= Z_R \cos kl_2 \cos kl_1 \\ &\quad + j [Z_1 \cos kl_2 \sin kl_1 + Z_i \cos kl_2 \cos kl_1 + Z_2 \sin kl_2 \cos kl_1] \\ A21 &= -\frac{Z_R}{Z_1 Z_2} \sin kl_2 \sin kl_1 + j \left[\frac{1}{Z_2} \sin kl_2 \cos kl_1 \right. \\ &\quad \left. - \frac{Z_i}{Z_2 Z_1} \sin kl_2 \sin kl_1 + \frac{1}{Z_1} \sin kl_1 \cos kl_2 \right] \\ A22 &= \left[\cos kl_2 \cos kl_1 - \frac{Z_1}{Z_2} \sin kl_2 \sin kl_1 - \frac{Z_i}{Z_2} \sin kl_2 \cos kl_1 \right] \\ &\quad + j \frac{Z_R}{Z_2} \sin kl_2 \cos kl_1 \end{aligned}$$

The equation (5) can be obtained as the sound pressure P4 of the right end of the duct by using these equations.

$$P4 = \frac{-A_{11}A_{22} + A_{12}A_{21}}{A_{21}} \quad (5)$$

The sound pressures were calculated for the aperture ratios 1%, 4% and 16%. And calculated for the hole's length l_3 of 0.0023m, 0.0046m, 0.1m and 0.2m for each aperture ratio. Moreover we calculated the sound pressures for three perforated plate positions (1) $l_1=0.334m$, $l_2=0.5m$, (2) $l_1=0.434m$, $l_2=0.4m$, (3) $l_1=0.634m$, $l_2=0.2m$ for three aperture ratios and four the perforated plate positions. r_0 is used the radius that makes a circle of the same area as the entire holes area. Namely $r_0 = \sqrt{(0.2m \times 0.2m \times \phi) / \pi}$. This treatment should be noticed not to be able to obtain the correct acoustic damping ratio and this treatment is only used to clarify the relationship between the peak sound pressure and the acoustic damping ratio. The frequency resolution is 10Hz.

III. ANALYTICAL RESULTS

i FREQUENCY RESPONSES

Figure2 ~ Figure4 show the frequency responses of P4 (Right end pressure) for three perforated plate positions (1), (2) and (3) described above. The left figures show the frequency responses for the aperture ratio $\phi=1\%$ and right figures for $\phi=16\%$. Notification 334*500 shows the perforated plate position and this indicates the perforated plate is set at the position of 334mm from the left end of the duct. Sum of two numbers is 834mm constant and it is the total length of the duct.

ii ACOUSTIC DAMPING RATIO

It is well known in the vibration theory that the peak value at resonance is inversely proportional to the damping ratio. Is it true in the acoustic field like the one dimensional duct examined here? We think that it is difficult to answer the question immediately. Then the acoustic damping ratio is obtained by using the half power method. The equation of half power method is given as follows.

$$\zeta = \Delta f / 2 / f_n \quad (6)$$

Where Δf is frequency difference of f_1 and f_2 ($\Delta f = f_2 - f_1$) and these are the frequencies at which the 3dB lower than the peak value. f_n is the natural acoustic frequency. Suffix n indicates mode order.

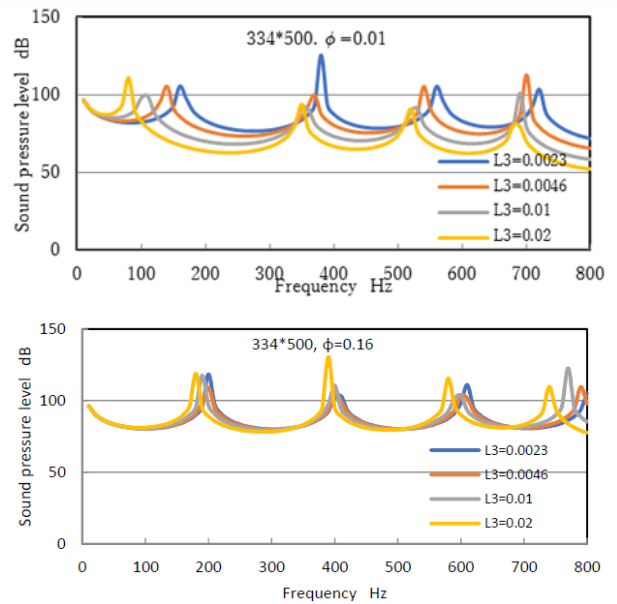


Fig.2 Frequency responses for duct of 334*500

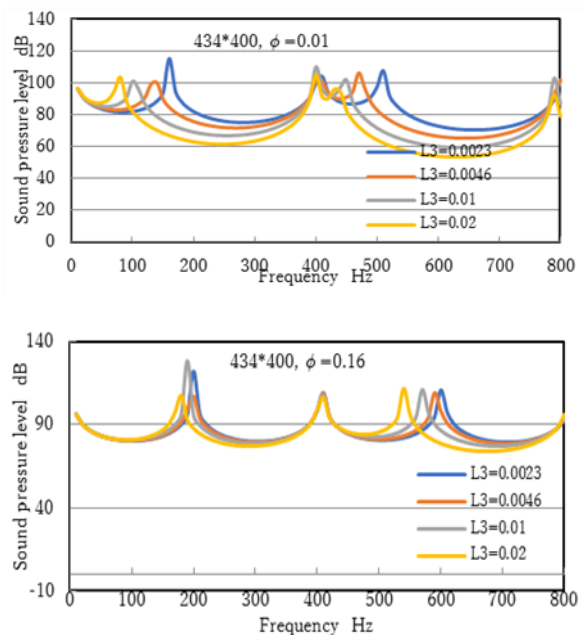


Fig.3 Frequency responses for duct of 434*400

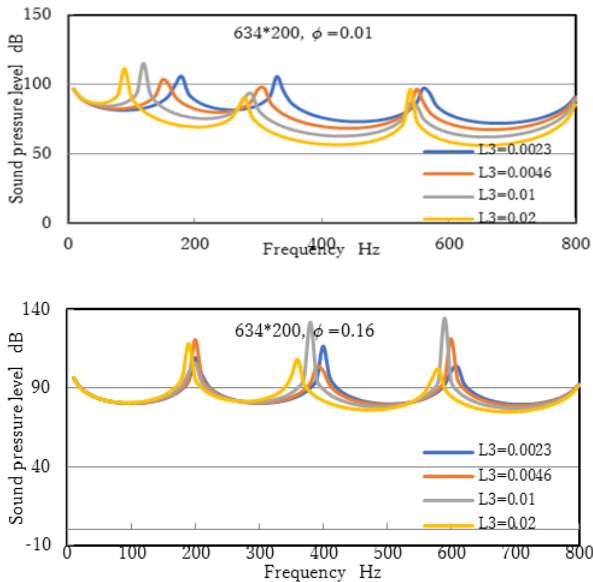


Fig.4 Frequency responses for duct of 643*200

Then the calculations were performed by varying Δf as best suited to the acoustic damping to be able to catch the each mode peak value. The calculation results of acoustic damping are shown in table1 to table6. These are the results in the case of the perforated plate position of 434*400. Table1 and 2 show the results from the peaks of 1st mode, Table 3 and 4 from the peaks of 2nd mode and Table 5 and 6 from the peaks of 3rd mode. The odd number of the table show the case of $\phi = 0.01$ and the even number of the table show the case of $\phi = 0.16$. Where ϕ is the aperture ratio. The values of right column (hatching) show the product of the sound pressure p and the acoustic damping ζ . From these tables, the product value is about 3277for 1st mode regardless of the perforated plate positions and hole's length. And 1570 for 2nd mode, 1094 for 3rd mode respectively. These values are mean values of four values for L3. It can be said that the peak sound pressure p is inversely proportional to the acoustic damping ζ and the product value of p and ζ for each mode is inversely proportional to the mode order.

Table 1 434*400, $\phi = 0.01$, 1st mode

L3	P(Pa)	ζ	$P \times \zeta$
0.0023	7103951.7	0.000446	3165
0.0046	3767038	0.000892	3360
0.01	2016044	0.00158	3185
0.02	1339676.7	0.002493	3340

Table 2 434*400, $\phi = 0.16$, 1st mode

L3	P(Pa)	ζ	$P \times \zeta$
0.0023	54890856	6.48E-05	3559
0.0046	27669417	0.000114	3158
0.01	13091819	0.000237	3104
0.02	6990367.4	0.000478	3344

Table 3 434*400, $\phi = 0.01$, 2nd mode

L3	p(Pa)	ζ	$P \times \zeta$
0.0023	295120923	5.40E-06	1594
0.0046	147570653	1.11E-05	1632
0.01	69502432	2.22E-05	1541
0.02	36475395	3.96E-05	1444

Table 4 434*400, $\phi = 0.16$, 2nd mode

L3	p(Pa)	ζ	$P \times \zeta$
0.0023	2.35E+09	6.74E-07	1585
0.0046	1.178E+09	1.35E-06	1589
0.01	543876132	2.95E-06	1602
0.02	273526873	5.78E-06	1580

Table 5 434*400, $\phi = 0.01$, 3rd mode

L3	p(Pa)	ζ	$P \times \zeta$
0.0023	4560369.2	0.000235	1072
0.0046	2834653.6	0.0003757	1065
0.01	2333458.1	0.0004767	1112
0.02	2760577.9	0.0004048	1117

Table 6 434*400, $\phi = 0.16$, 3rd mode

L3	p(Pa)	ζ	$P \times \zeta$
0.0023	32998941	3.29E-05	1084
0.0046	16904409	6.59E-05	1113
0.01	8521185.1	0.0001268	1081
0.02	5432503.3	0.0002032	1104

IV. PHYSICAL UNDERSTANDING DUE TO EQUATION

Consider the lateral vibration of a bar which is equivalent to the acoustic phenomenon of an one dimensional sound field like the duct. The equation of motion without damping can be described by the equation (7).

$$-\rho A \frac{\partial^2 u}{\partial t^2} + AE \frac{\partial^2 u}{\partial x^2} = 0 \quad (7)$$

Where u is the displacement.

Consider the viscous coefficient c ($N \cdot s/m^3$) per unit area on this equation, we got the next equation.

$$-\rho A \frac{\partial^2 u}{\partial t^2} - cA \frac{\partial u}{\partial t} + AE \frac{\partial^2 u}{\partial x^2} = 0 \quad (8)$$

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Dividing both sides by ρA

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} - \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{Where } \gamma = \frac{c}{\rho}$$

Here putting $a^2 = \frac{E}{\rho}$

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} = 0 \quad (9)$$

Putting $u_i(x,t) = \varphi_i(x) \cdot q_i(t)$ (i : mode order) and substituting this to the equation (9)

$$\varphi_i \frac{d^2 q_i}{dt^2} + \gamma \varphi_i \frac{dq_i}{dt} - a^2 \frac{d^2 \varphi_i}{dx^2} \cdot q_i = 0 \quad (10)$$

Multiplying φ_j (Mode function) to both sides and integrating 0 ~ 1, We got the next equation.

$$\int_0^l \varphi_i \varphi_j \cdot dx \frac{d^2 q_i}{dt^2} + \gamma \int_0^l \varphi_i \varphi_j \cdot dx \frac{dq_i}{dt} - a^2 \int_0^l \frac{d^2 \varphi_i}{dx^2} \cdot \varphi_j \cdot dx \cdot q_i = 0$$

From the orthogonality of the mode, $\int_0^l \varphi_i \varphi_j \cdot dx = \begin{cases} i \neq j & 0 \\ i = j & A \end{cases}$ (11)

And performing the partial integration and considering the boundary conditions $\varphi_i(0) = \varphi_i(l) = 0$

$$\int_0^l \frac{d^2 \varphi_i}{dx^2} \cdot \varphi_j \cdot dx = \frac{d\varphi_i}{dx} \cdot \varphi_j \Big|_0^l - \int_0^l \frac{d\varphi_i}{dx} \cdot \frac{d\varphi_j}{dx} dx = - \int_0^l \left(\frac{d\varphi_i}{dx} \right)^2 dx = -B$$

Then

$$\begin{aligned} A_i \frac{d^2 q_i}{dt^2} + \gamma A_i \frac{dq_i}{dt} + a^2 B_i \cdot q_i &= 0 \\ \frac{d^2 q_i}{dt^2} + \gamma \frac{dq_i}{dt} + a^2 \frac{B_i}{A_i} \cdot q_i &= 0 \end{aligned} \quad (12)$$

Where putting $a^2 \frac{B_i}{A_i} = \omega_i^2$, $\gamma = 2\zeta_i \omega_i$

$$\frac{d^2 q_i}{dt^2} + 2\zeta_i \omega_i \frac{dq_i}{dt} + \omega_i^2 \cdot q_i = 0$$

We can obtain the equation of motion for one degree of freedom vibration system. The solution of the forced vibration in the case of given the steady external force to the right hand side becomes as follows.

$$q_i = \frac{f_i / m_i \omega_i^2}{\sqrt{(1 - u_i^2)^2 + (2\zeta_i u_i)^2}} \quad (13)$$

Where $f_i = \int_0^l F \cdot \varphi_i dx$

The steady state solution can be obtained as follows.

$$u_i(x,t) = \varphi_i(x) \cdot q_i(t) \quad (14)$$

From the above theoretical development, the damping ratio ζ_i of mode i is inversely proportional to the natural circular frequency ω_i due to $\gamma = 2\zeta_i \omega_i$ when γ is constant. And the damping ratio becomes inversely proportional to the mode order as the natural frequency is proportional to the mode order. This is coincident with the result derived from the calculation result. As a result, it was reasonable that the

damping to the one dimensional sound field partitioned with the perforated plate can be given like the equation (8).

The mode orthogonality as shown in the equation (11) is even true for the case of one dimensional sound field partitioned with the perforated plate. Figure 5 (a) and (b) are 1st mode and 2nd mode shapes of the pressure obtained by the reference [6] respectively. The horizontal axis shows the position from the origin and the vertical axis shows the normalized sound pressure. The length of this duct is 500mm. It can be seen from this figure that the sound pressure suddenly drops at the position of the perforated plate.

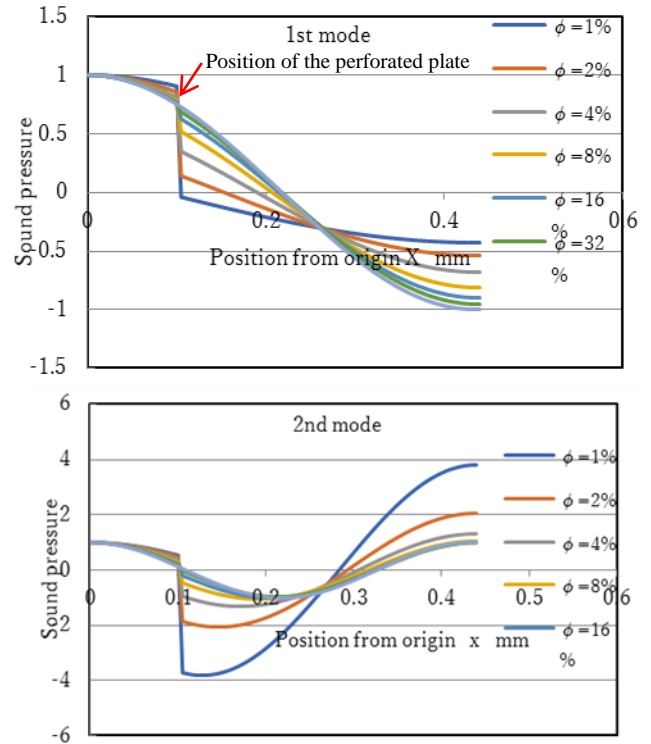


Fig.5 1st and 2nd modes of sound pressure

The result calculated by the equation (5) shows that the value in the case of $i=j$ is 1/100 order smaller than that in the case of $i \neq j$. Namely the orthogonal condition is true for this case.

Table 7 Verification of orthogonality condition

φ	0.01	0.02	0.04	0.08	0.16	0.32	1
$i \neq j$	-0.00212	-0.00161	-0.00124	-0.00095	-0.00073	-0.00616	-0.00301
$i = j$	0.127607	0.133895	0.151384	0.174766	0.194437	0.207706	0.222594

V. CONCLUSIONS

In this paper, theoretical analysis was conducted to clarify the relation between the peak sound pressure level at resonance and the acoustic damping ratio. In general, the peak value at resonance is inversely proportional to the damping ratio in the vibration theory. This is the purpose to confirm that the peak value at resonance is inversely proportional to the damping ratio in also acoustic fields. We got the following findings from the theoretical consideration,

- (1) The pressure at resonance is inversely proportional to the acoustic damping ratio in even acoustic fields.

- (2) It is true that the mode order bear an inverse relation to the acoustic damping ratio.

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