# The design of a low - cost and missile - borne distributed SINS/GNSS integrated navigation

# system

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Abstract— At present, the missile navigation task machine is developing towards the direction of low cost, but at the same time of reducing cost, it is inevitably faced with the problem of decreasing computing speed and storage capacity of the task machine, resulting in insufficient computing time. In order to solve this contradiction, this paper proposes a distributed algorithm based on the equivalent rotation vector method and it is used to appropriately increase inertial navigation calculating cycle, at the same time non-communicative errors can be reduced. The conventional centralized Kalman filter is separated into IMU several sampling cycles with the advantage of high inertial navigation refresh rate. In this way, not only the navigation precision but also the computation time margin of the combined navigation is guaranteed. Digital simulation shows that the speed and position accuracy of the GNSS/SINS distributed filtering algorithm proposed in this paper is similar to the conventional Kalman filtering algorithm, and the attitude error decreases. This is a good solution to the problem of insufficient computing time and the result is good.

*Index Terms*— Distributed filtering, GNSS/SINS, Kalman filter, Rotation vector

#### I. INTRODUCTION

SINS (Strapdown inertial navigation system) and GNSS (Global Navigation Satellite System) are two common navigation and positioning methods, which have their own advantages and disadvantages. As a mature and classic navigation method, SINS has good autonomy and high short-time accuracy. It can realize all-weather autonomous three-position positioning speed measurement, but the error will accumulate over time. GNSS system has the characteristics of high precision, all-weather and the error does not diverge with time. But the work of GNSS receiver is greatly affected by the external environment, prone to interference and artificial control. In cities, canyons, forests and other areas where blocked, the signal will attenuate or lose the lock, resulting in the decline of positioning accuracy, or even unable to work properly [1].

The combined GNSS/SINS navigation system overcomes the disadvantages of the two systems, and makes the combined system more accurate than the separate systems.

In the integrated navigation system, it is required that the time of data transmission, strapdown calculating, combined filtering and guidance control must be completed within one inertial navigation solution cycle, and some redundant idle time must be set aside. Otherwise, the high dynamic motion of ballistic missiles will inevitably have large errors. However, with the low cost of the missile-borne aircraft, the calculation time is extended. Therefore, this paper adopts the equivalent rotation vector method to correct the strapdown algorithm of quaternion, and on this basis, proposes a distributed filtering algorithm that can effectively solve the problem of long period of combined navigation. Finally, the digital simulation is carried out and good results are obtained.

#### II. OVERALL SYSTEM DESIGN

The hardware platform of the integrated navigation system adopts DSP(TMS6000 series)+FPGA. In this system, the refresh rate of IMU is 400Hz, the data update cycle of SINS is 2.5ms, the data transmission adopts the form of pulse number, and the data update cycle of GNSS is ls. The function of FPGA is to convert the number of SINS pulses into angle increment and generate 2.5ms interrupt. So it can transmit the pulse data to DSP every 2.5ms. DSP solves SINS with input angular velocity and acceleration information, and integrates them with GNSS information. Then the navigation and positioning information is sent to the guidance control module to control the missile to accurately hit the target. At the same time, all the above information is transmitted to the monitoring computer through Ethernet

The overall system structure is shown follows.



Figure.1 Overall system

#### III. EQUIVALENT ROTATION VECTOR METHOD

In the finite rotation of a rigid body, the instantaneous angular position of the rigid body is related to the rotation order [2]. Its rotation synthesis does not conform to the

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exchange rule of vector addition, that is, the finite rotation of a rigid body is non-commutative.

In the traditional quaternion method, usually we use the angular velocity vector integral as  $\Delta \theta = \int_{t}^{t+\Delta t} \omega dt$ , that is, the attitude Angle change is assumed to be small and linear. When the motion of the vector is not fixed axis rotation, that is, when the direction of the vector w changes in space, the former formula does not hold. Therefore, rotation noncommutative error is produced.

In 1971.Boetz proposes the concept of equivalent rotation vector. The vector  $\Phi(t) = \begin{bmatrix} \Phi_x & \Phi_y & \Phi_z \end{bmatrix}^T$  can uniquely determine the motion posture of the rigid body at time interval. The equivalent rotation vector method can effectively compensate the non-commutative error, which is especially suitable for high dynamic environment:

The differential equation described by the rotation vector is as follows:

$$\dot{\Phi} \approx \omega + \frac{1}{2} \Phi \times \omega + \frac{1}{12} \Phi \times (\Phi \times \omega)$$
 (1)

In practical engineering applications, we ignore the higher order terms as [3]:

$$\dot{\Phi} = \omega + \frac{1}{2} \Phi \times \omega \tag{2}$$

According to the different angular rate sampling times in a single attitude update period, different sub-sample algorithms can be obtained, such as the monad sample and twin sample method, etc. The relation between angular rate sampling times N and subsample number M is N = M + 1 [4]. According to the different iteration times of rotation vector in a single attitude update cycle, the algorithm can be divided into single-loop and multi-loop algorithms.

From the engineering application and the navigation accuracy, this paper will mainly use the four-subsample algorithm and introduce it.

In this paper, the attitude quaternion is modified by using the equivalent rotation vector algorithm of four subsamples so as to update the strapdown matrix. The flow chart of equivalent rotation vector method calculation process is as follows:



Figure.2 Calculation procedure of equivalent rotation vector method

The specific calculation steps are as follows:

Step One: Angle incremental extraction.

The system uses 400Hz optical fiber IMU, as angular

incremental output. Five gyro samples can be taken within a single attitude resolution period (including the angular rate sampling value at the time of the previous attitude update).

Step Two: calculate the equivalent rotation vector. The rotation vector formula [5] is as follows:

$$\Phi(h) = \theta + k_1(\theta_1 \times \theta_2 + \theta_3 \times \theta_4) + k_2(\theta_1 \times \theta_3 + \theta_2 \times \theta_4) + k_3\theta_1 \times \theta_4 + k_4\theta_2 \times \theta_3$$
(3)

$$k_1 = \frac{736}{945}, k_2 = \frac{334}{945}, k_3 = \frac{526}{945}, k_4 = \frac{654}{945}$$
 (4)

Among them,  $\theta_1 \sim \theta_2 \sim \theta_3$  and  $\theta_4$  are angular increments within the time period of  $\begin{bmatrix} t_k & t_k + \frac{1}{4}h \end{bmatrix}$ ,  $\begin{bmatrix} t_k + \frac{1}{4}h & t_k + \frac{2}{4}h \end{bmatrix}$ ,  $\begin{bmatrix} t_k + \frac{2}{4}h & t_k + \frac{3}{4}h \end{bmatrix}$  and  $\begin{bmatrix} t_k + \frac{3}{4}h & t_{k+1} \end{bmatrix}$  respectively.

Step Three: Calculate the recursive estimator of quaternion from the rotation vector.

The recursive algorithm of quaternion sub-expression is:

$$q(h) = \cos\left(\frac{|\Phi|}{2}\right) + \frac{\Phi}{|\Phi|}\sin\left(\frac{|\Phi|}{2}\right)$$
(5)

Where,  $\phi$  is the amplitude of the rotation vector, i.e.

$$f = \sqrt{\boldsymbol{\Phi}_x^2 + \boldsymbol{\Phi}_y^2 + \boldsymbol{\Phi}_z^2} \tag{6}$$

Step Four: rotation vector corrects quaternion [6].

The rotation vector is used to correct the recursive calculation of quaternions, as shown in equation (7), where " $\otimes$  " is the multiplication of quaternions:

$$Q(t_{k+1}) = Q(t_k) \otimes q(h) \tag{7}$$

Finally, the attitude angle information of carrier can be obtained according to the updated quaternion.

#### IV. CENTRALIZED KALMAN FILTERING

Aiming at the characteristics of vertical strike and high firing height of ballistic missile, a combined navigation algorithm for velocity and position combination in launch inertial system is proposed. The launching inertial coordinate system coincides with the launching coordinate system at the moment of missile launch and is then fixed in the inertial space. In the launch inertial system under inertial navigation, pitching Angle range of [- 180  $^{\circ}$ , 180  $^{\circ}$ ], meet the characteristics of vertical against ballistic missiles.

In this paper, the SINS/GNSS loose combination mode was adopted, and the navigation system adopted the transmitting inertial coordinate system as the navigation coordinate system. The selected state variables are:

$$X = \begin{bmatrix} \varphi_x & \varphi_y & \varphi_z & \delta V_x & \delta V_y & \delta V_z & \delta x & \delta y \\ \delta z & \varepsilon_x & \varepsilon_y & \varepsilon_z & \nabla_x & \nabla_y & \nabla_z \end{bmatrix}^T$$
(8)

In the above equation,  $\varphi_x \cdot \varphi_y \cdot \varphi_z$  is the attitude

misalignment Angle;  $\delta V_x \cdot \delta V_y \cdot \delta V_z$  is the velocity error of the three axes in the launching inertial coordinate system;  $\delta x \cdot \delta y \cdot \delta z$  Is the position error of the three axes in the launching inertial coordinate system;  $\nabla_x \cdot \nabla_y$  $\cdot \nabla_z \pi \mathcal{E}_x \cdot \mathcal{E}_y \cdot \mathcal{E}_z$  are accelerometer constant bias and gyro constant drift in the projectile coordinate system.

According to the error propagation model of the SINS subsystem [7], the system dynamic model can be obtained as follows:

$$\begin{cases} X_{k} = \Phi_{kk-1} X_{k-1} + \Gamma_{k-1} W_{k-1} \\ Z_{k} = H_{k} X_{k} + V_{k} \end{cases}$$
(9)

Where,  $\Phi_{k,k-1}$  is the one-step state transition matrix;  $\Gamma_{k-1}$  is the system noise driving matrix;  $H_k$  is the system measurement matrix;  $V_k$  is the measurement noise matrix of the system;  $W_k$  is the system noise matrix.

Meanwhile, the system meets the following conditions:

$$\begin{cases} E[W_{k}] = 0, Cov[W_{k}, W_{j}] = E[W_{k}W_{j}^{T}] = Q_{k}\delta_{kj} \\ E[V_{k}] = 0, Cov[V_{k}, V_{j}] = E[V_{k}V_{j}^{T}] = R_{k}\delta_{kj} \\ Cov[W_{k}, V_{j}] = E[W_{k}V_{j}^{T}] = 0 \end{cases}$$
(10)

Where,  $Q_k$  is the variance matrix of system noise,  $R_k$  is the variance matrix of system measurement noise,  $Q_k$  and  $R_k$  are positive definite matrices [8].

The discrete kalman filter equation [9] is as follows:

(1) One-step state prediction:

$$\hat{X}_{k|k-1} = \Phi_{k,k-1} \hat{X}_{k-1}$$
(11)

(2) State estimation:

$$\hat{X}_{k} = \hat{X}_{k|k-1} + K_{k}(Z_{k} - H_{k}\hat{X}_{k|k-1})$$
(12)

(3) The filter gain:

$$K_{k} = P_{k/k-1}H_{k}^{T}(H_{k}P_{k/k-1}H_{k}^{T} + R_{k})^{-1}$$
(13)

(4) One-step prediction of mean square error:

$$P_{k,k-1} = \boldsymbol{\Phi}_{k,k-1} P_{k-1} \boldsymbol{\Phi}_{k,k-1}^{T} + \boldsymbol{\Gamma}_{k-1} Q_{k-1} \boldsymbol{\Gamma}_{k-1}^{T}$$
(14)

(5) Estimated mean square error:

$$P_{k} = (I - K_{k}H_{k})P_{k/k-I}(I - K_{k}H_{k})^{T} + K_{k}R_{k}K_{k}^{T}$$
(15)

The above is the basic equation of discrete kalman filter. According to the kalman filter equations, as long as the initial value of the system is given in advance and  $Z_k$  is measured according to the system quantity, the estimated value of the state variable at the time can be obtained recursively [10].

### V. DISTRIBUTED FILTERING

At the time of combined filtering, the projectile DSP computer needs to complete the tasks of measurement data transmission analysis, strapdown solution, combined filtering solution, guidance control solution and so on at the same time.

For the inertial navigation system of medium and high frequency (for example, 400Hz and above), the sampling period of inertial group is less than the solution time of combined filtering, so the condition cannot be met, which will affect the real-time performance and navigation accuracy of the combined system.

In view of the real-time problem of filtering generated by one-step filtering in the high frequency inertial navigation system, A stepwise filtering method is proposed, whose basic filtering equation is the same as described above.

This paper mainly describes the four-subsample rotation vector algorithm under the stepwise filtering. The inertial navigation system frequency is 400Hz and strapdown solution period is 10ms.In the optimization mode of the hardware platform mentioned above, the combined filtering process takes about 4.17ms, and other calculation processes cannot obtain sufficient time allowance, so it can be decomposed into four sub-processes, and the time of each sub-process is shown in the table:

Table.1 Time consumption of each sub-process of combined filtering

The process sequence number	The subprocess of Combined filter	Time consuming
First	Constructing the equation of state of continuous system	0.15ms
Second	Discretization of the equation of state	1.10ms
Third	Kalman filtering solution	2.75ms
Fourth	Filtering correction	0.17ms

Thus, several steps of kalman filtering can be separated and distributed in each angular increment extraction cycle of inertial navigation. The sequence diagram of stepwise filtering operation is shown below:



Figure.3 Filtering sequence diagram

For inertial navigation system with higher output frequency and multi-sub-sample rotation vector algorithm such as Gemini sample and tri-sub sample, the stepwise filtering method is also applicable.

#### VI. THE SIMULATION TEST

In order to verify the performance of the distributed kalman filter described in this paper, the system model is established and the algorithm is verified under the Matlab software simulation environment.

The track generator is used to generate the running track of a high dynamic missile and output real-time IMU and GNSS information at the same time.

In the simulation, the GPS refresh rate was set as 1Hz, the position RMSE (Root mean square error) was 10m, and the speed RMSE was 0.1m/s. The refresh rate of IMU is 400Hz, the zero deviation and zero deviation stability of gyroscope are both 1°/h, the zero deviation and zero deviation stability of accelerometer are 0.5mg and 0.15mg, and the total simulation time is 600s.



Figure.4 The launch trajectory in inertial coordinate system

A comparison is made between the conventional one-step kalman filter using Picard successive approximation method to solve quaternions and the distributed kalman filter using equivalent rotation vector method described in this paper.



Figure.5 Velocity error comparison diagram



Figure.6 Position error comparison diagram



Figure.7 Yaw angle error comparison diagram

As can be seen from the above figure, the attitude angle is updated by using the equivalent rotation vector method described in this paper, and the state vector of integrated navigation is modified in real time. The effect is much better than that of the conventional kalman filter, with smaller error and smoother curve. The accuracy of attitude angle is within  $0.3^{\circ}$ . At the same time, the position and velocity errors of the two methods are roughly the same, the position accuracy is within 3m and the speed accuracy is within 0.15m/s after normal convergence of filtering. This effect is good.

#### VII. CONCLUSION

Low-cost development of bomb-borne computers is a hot spot of military development. In this paper, from the point of view of solving the problem of insufficient filtering time, the equivalent rotation vector method is used to appropriately increase the computational period of inertial navigation, and several steps of kalman filtering are divided into several IMU output cycles. In this way, the margin of combined navigation calculation time is guaranteed while the navigation precision is guaranteed. This method can be applied to the navigation system of actual missile. Digital simulation shows that the distributed filtering algorithm of GNSS/SINS in inertial system of launch presented in this paper has the same position speed accuracy as the conventional kalman filtering algorithm, and the attitude error is reduced to some extent, which solves the problem of insufficient computing time and has better effect.

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