Effects of Chip Atomic Clock on Positioning Accuracy of BDS/GPS Dual Mode Receiver

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Abstract—Traditional BDS/GPS dual-mode receiver uses quartz crystal vibration as the clock source for the system to provide positioning and navigation services, but quartz crystal vibration has many defects and deficiencies in the long-term stability, the frequency drift is large, will bring certain impact on the positioning accuracy. Based on Chip level Atomic clocks (Chip Scale Atomic Clock, CSAC) instead of the traditional semiconductor, the Atomic Clock is small in size, low power consumption, can provide the frequency stability of a long period of time, its frequency deviation is less than 0.2 Hz caused even smaller, the Chip Atomic clocks, instead of the traditional quartz crystal can be greatly reduced by the local Clock frequency drift caused by doppler frequency shift, can be widely used in miniature system. Dual mode receiver position calculating of clock difference to solve out at the same time, the atomic clocks replacement semiconductor chip after vibration can make the receiver clock and satellite clock consistent as far as possible, to locate at the same time when calculating the space position dilution of precision PDOP value are also associated with satellite clock error, chip atomic clocks, can obviously improve the localization accuracy of system.

Index Terms—chip atomic clock, dual-mode receiver, PDOP, positioning accuracy, quartz crystal oscillator.

I. INTRODUCTION

The global satellite navigation system has global, all-weather, continuous 3d positioning and navigation capabilities. The space constellation part of GPS consists of BDS space constellation is slightly different from that of GPS, mainly including mid-orbit satellites, geostationary orbit satellites and inclined geosynchronous orbit satellites [2-3]. The fusion of BDS and GPS can increase the number of visible satellites and improve the distribution of satellites, thus improving the positioning accuracy and continuity of the system [4]. BDS/GPS receiver uses quartz crystal oscillator to drive RF front-end sampling and complete positioning calculation function. Traditional quartz crystal oscillator has been widely used due to its advantages such as small size, low price and integrability, but it has great shortcomings in long-term stability.

Atomic clock can provide long-term stable frequency, but it has the problems of high power consumption, large size and high cost [5]. It’s very difficult for the users to use while chip atomic clock can provide long-term stable frequency, and has the advantages of small size and low power consumption.

In BDS/GPS dual-mode receiver, pseudo-range observation is adopted for positioning function. The local clock difference of the receiver is an unknown quantity to be solved for the pseudo-range positioning equation [6]. Therefore, in addition to the positioning function, the receiver can also be timed for the user. This usually makes the crystal oscillator with general frequency stability as the frequency and time source of the receiver, thus reducing the production cost of the receiver. From this point of view, the receiver has to solve the clock difference of the receiver simultaneously for the purpose of positioning. If the clock of the receiver can be synchronized with the time of the satellite as much as possible, the receiver needs at least three visible satellites to achieve three-dimensional positioning. PDOP value of satellite positioning accuracy is a function of clock error, and PDOP value also reflects the accuracy of spatial position. It is necessary to use the chip atomic clock with high frequency accuracy and high frequency stability to replace the common crystal oscillator (TXCO and OXCO) as the time-frequency of the satellite navigation system.

II. BDS/GPS DUAL MODE RECEIVER FUSION LOCALIZATION METHOD

A. Analysis of pseudo - distance fusion positioning principle

Pseudo-range observation is defined as the difference between the local time of receiving satellite signal and the time of transmitting satellite signal multiplied by the speed of light [7].

\[ \rho = c(\tau - \tau^s) \]  \hspace{1cm} (1)

Where, \( c \) represents the speed of light, \( \tau \) represents the local time, \( \tau^s \) and represents the time of satellite signal transmission.

Can be further written as:

\[ \rho = r + c(\delta \tau - \delta \tau^s) + I + T + \varepsilon_\rho \] \hspace{1cm} (2)

Where, \( r \) represents the geometric distance between the satellite navigation receiver and the satellite; \( \delta \tau \) represents the clock error information of the satellite navigation receiver; \( \delta \tau^s \) represents the satellite clock error information; \( I \) and \( T \) represents the ionospheric delay and tropospheric delay in atmospheric propagation delay; \( \varepsilon_\rho \) represents the pseudo-distance measurement error.

In addition, besides the pseudo-range, the carrier phase is also the basic measurement value obtained from the satellite signal, and its observation equation is expressed as:

\[ \phi = r + c(\delta \tau_c - \delta \tau^s_c) + I + T + \lambda N + \varepsilon_\phi \] \hspace{1cm} (3)
Where, $\phi$ denotes the carrier phase value converted to distance, $\lambda$ denotes the carrier wavelength, $N$ denotes the circularity ambiguity and $E_\theta$ denotes the carrier phase measurement noise converted to distance.

As mentioned above, system time should be processed uniformly in fusion positioning. The difference between GPS time and Beidou time can be expressed as $\delta t_A$.

$$\delta t_A = t_{GPS} - t_{BDS}$$  \hspace{1cm} (4)

$t_u$ is the expressions of the clock difference between the local time of the satellite navigation receiver and the Beidou time and the GPS time.

$$\delta t_{u,BDS} = t_u - t_{BDS}$$  \hspace{1cm} (5)

$$\delta t_{u,GPS} = t_u - t_{GPS}$$  \hspace{1cm} (6)

Combined with equations (4) ~ (6), we can get:

$$\delta t_{u,BDS} = \delta t_{u,GPS} + \delta t_A$$  \hspace{1cm} (7)

Due to the time difference between GPS and BDS, $\rho^{(\text{BDS})}_c$ and $\rho^{(\text{GPS})}_c$ are the pseudo-distance measurement information of BDS and GPS. They have different physical definitions after the pseudo-distance measurement information is corrected [8].

The pseudo-distance measurement information of the Beidou navigation system cannot be regarded as equivalent GPS pseudo-distance measurement information, which can be solved by substituting the single GPS positioning equation.

**B. Fusion localization method for measuring system time difference at receiver end**

The difference value of system time is measured in real time by the receiver itself. Assuming that the time difference is an unknown quantity, the pseudo-distance observation formula of GPS is the same as formula (3), and the pseudo-distance observation equation of BDS is $\delta t_A$:

$$r^{(\text{BDS})}_c + \delta t_{u,GPS} + \delta t_A = \rho^{(\text{BDS})}_c - r^{(\text{BDS})}_c$$  \hspace{1cm} (8)

Among them:

$$\rho^{(\text{BDS})}_c = \rho^{(\text{BDS})}_c + \delta t^{(\text{BDS})}_u - T^{(\text{BDS})}_u$$  \hspace{1cm} (9)

The simultaneous equation is used to establish a linearized fusion positioning equation as follows:

$$\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta \delta t_{u,GPS} \\
\delta t_A
\end{bmatrix} = \begin{bmatrix}
-I_{x}^{(\text{GPS})} & -I_{y}^{(\text{GPS})} & -I_{z}^{(\text{GPS})} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\rho^{(\text{BDS})}_c \\
\rho^{(\text{GPS})}_c \\
\rho^{(\text{GPS})}_c \\
\rho^{(\text{BDS})}_c \\
\end{bmatrix}$$  \hspace{1cm} (10)

The participation of BDS measurement values increases the time difference as a state variable, and the formula (8) is substituted into the formula (9).

$$\rho^{(\text{BDS})}_c + \delta t_{u,BDS} = \rho^{(\text{GPS})}_c - E_\theta^{(\text{GPS})}$$  \hspace{1cm} (11)

Formula (10) can be equivalent to the following fusion positioning equation:

$$\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta \delta t_{u,GPS} \\
\delta t_A
\end{bmatrix} = \begin{bmatrix}
-I_{x}^{(\text{GPS})} & -I_{y}^{(\text{GPS})} & -I_{z}^{(\text{GPS})} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\rho^{(\text{BDS})}_c \\
\rho^{(\text{GPS})}_c \\
\rho^{(\text{GPS})}_c \\
\rho^{(\text{BDS})}_c \\
\end{bmatrix}$$  \hspace{1cm} (12)

This method does not take advantage of the system time difference broadcast value, and introduces the fifth unknown quantity into the BDS/GPS dual-mode fusion positioning equation, which increases the reliability of dual-mode positioning [9].

**III. CSAC ASSISTED POSITIONING ALGORITHM OF BDS/GPS RECEIVER**

**A. Working principle of chip atomic clock**

The circuit structure of chip atomic clock mainly consists of physical system, MCU processor, temperature control circuit, control drive circuit and microwave synthesis circuit. The integrated design of MUC processor, DAC and peripheral analog circuit forms a special chip of chip-level atomic clock SIP (System in a Package) which contains digital devices and simulator components [10]. The miniaturization design of the physical system adopts the micromachining technology and processes the physical system as a whole, including the micro atomic gas chamber, micro-optical system, micro magnetic field coil, magnetic shielding structure and thermal insulation structure.

Based on the phenomenon of CPT atomic clock, and its principle block diagram as shown in figure 1, a bundle of over 3.4 GHz microwave modulated laser beam from a semiconductor laser light, laser beam through a quarter wavelength wave plate adjustment for coherent circularly polarized light, the right hand (or left hand) carry microwave signal into a closed bubbles, polarized light interacts with alkali metal rubidium again [11]. The CPT coherent state is completed when the wavelength of the modulated laser is the corresponding wavelength of the alkali metal atom, and the frequency of microwave is equal to that of the rubidium atom. The "bright line" and "dark line" appeared in the transmission spectrum and the fluorescence spectrum respectively, and the laser wavelength and microwave frequency were adjusted back.
Two servo circuits were needed to realize the two modulation processes: one circuit was locked on the crystal oscillator, which was used to generate the accurate microwave modulation frequency and the reference frequency of output 10MHz, and the other loop locks the laser wavelength.

**B. Chip atomic clock algorithm design**

The frequency accuracy of the chip atomic clock is generally 4 orders of magnitude higher than the oscillation rate of the temperature-supplemented crystal and 2 orders of magnitude higher than that of the thermostatic crystal oscillator. Therefore, when the chip atomic clock is used as the clock source of the satellite navigation receiver, the clock frequency of the satellite navigation receiver can be guaranteed to be stable and reliable for a period of time. Because the clock error calculated by the satellite navigation receiver is independent of each other in each positioning solution process, the clock error information within a period of time is retained as historical data. Based on the clock error information, a clock error model is established to predict the clock error information within a period of time. The expression is:

\[
T - T_s = \alpha_0 + \alpha_1(T_s - t_0) + \alpha_2(T_s - t_0)^2
\]

Where, \(\alpha_0, \alpha_1, \alpha_2\) respectively represent the relative deviation, clock error, and clock drift information, and \(t_0\) represent the reference epoch information.

Set:

\[
T = [T_1(t), T_2(t), ..., T_n(t)]^T
\]

\[
T_s = [T_1(s), T_2(s), ..., T_n(s)]^T
\]

\[
\alpha = [\alpha_0, \alpha_1, \alpha_2]^T
\]

\[
R = \begin{bmatrix}
1 & T_1 - t_0 & (T_1 - t_0)^2 \\
1 & T_2 - t_0 & (T_2 - t_0)^2 \\
... & ... & ... \\
1 & T_n - t_0 & (T_n - t_0)^2
\end{bmatrix}
\]

Comprehensive formula (23) - (35), can obtain:

\[
T - T_s = RX
\]

The least square method is adopted to carry out the optimal estimation of the above formula. Let \(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2\) respectively represent the optimal estimation of parameters \(\alpha_0, \alpha_1, \alpha_2\) and can estimate the clock difference of satellite navigation receiver in a period of time in the future. The expression is:

\[
T - T_s = \hat{\alpha}_0 + \hat{\alpha}_1(T_s - t_0) + \hat{\alpha}_2(T_s - t_0)^2
\]

**C. Influence of chip atomic clock on positioning accuracy**

The local clock difference of the receiver is an unknown quantity to be solved by the pseudo-distance positioning equation. The positioning equation of the satellite receiver is as follows:

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta \delta t_n
\end{bmatrix} = G^{-1} \begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\Delta \rho_4
\end{bmatrix}
\]

Suppose the following definition:

\[
X = [\Delta x \ \Delta y \ \Delta z \ \Delta \delta t_n]^T \quad \rho = [\Delta \rho_1 \ \Delta \rho_2 \ \Delta \rho_3 \ \Delta \rho_4]^T
\]

According to equation (14), we can get:

\[
\varepsilon_s = G^{-1} \varepsilon_\rho
\]

Where, \(\varepsilon_s\) represents the error of position and clock error, and \(\varepsilon_\rho\) represents the pseudo-distance residual. For the covariance of \(X\)
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\[
\text{Cov}(X) = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\
\sigma_{yx}^2 & \sigma_y^2 & \sigma_{yz}^2 \\
\sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_z^2 \\
\end{bmatrix}
\]

(20)

Where, the diagonal variables \(\sigma_x^2, \sigma_y^2, \sigma_z^2\) respectively represent the error variance of position and clock error. The covariance matrix can also be expressed as:

\[
\text{Cov}(X) = E[\mathbf{e}_x \mathbf{e}_x^T] = G^{-1} \text{Cov}(\mathbf{p}) G^T
\]

(21)

Assuming that the pseudo-distance residual error between each satellite is not correlated, then:

\[
\text{Cov}(\mathbf{p}) = \sigma_p^2 \mathbf{I}
\]

(22)

Therefore:

\[
\text{Cov}(X) = \sigma_p^2 [G^T G]^{-1} = \sigma_p^2 \mathbf{H}
\]

(23)

Where, \(\mathbf{H}\) denotes the weight system matrix, from which the positioning precision factor (DOP) can be obtained from \(\mathbf{H}\).

GDOP (geometric dilution of precision) can be described as:

\[
\text{GDOP} = \sqrt{\text{Tr}(\mathbf{H})}
\]

(24)

Can also be expressed as:

\[
\text{GDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_{\delta_u}^2} / \sigma_p
\]

(25)

In addition, the expression and direct relationship of PDOP (spatial position dilution of precision), TDOP (clock error dilution of precision), HDOP (horizontal dilution of precision) and VDOP (elevation dilution of precision) are shown as follows:

\[
\text{PDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} / \sigma_p
\]

(26)

\[
\text{TDOP} = \sigma_{\delta_u} / \sigma_p
\]

\[
\text{GDOP}^2 = \text{PDOP}^2 + \text{TDOP}^2
\]

\[
\text{HDOP} = \sqrt{\sigma_x^2 + \sigma_y^2} / \sigma_p
\]

\[
\text{VDOP} = \sigma_z / \sigma_p
\]

\[
\text{PDOP}^2 = \text{HDOP}^2 + \text{VDOP}^2
\]

(27)

For the BDS/GPS dual-mode system, which has two clock difference information \(\Delta \delta t_{u,\text{GPS}}\) and \(\Delta \delta t_{u,\text{BDS}}\), 10MHz output by CSAC is used to replace the original 16.369MHz crystal as the local clock source, and the positioning equation can be expressed as:

\[
\begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\end{bmatrix}
= \begin{bmatrix}
-\mathbf{f}^{(1)}(u_{k-1}) \\
-\mathbf{f}^{(2)}(u_{k-1}) \\
-\mathbf{f}^{(3)}(u_{k-1}) \\
\end{bmatrix}
\Delta \mathbf{t}
\]

\[
+ \Delta \delta t_{u,\text{GPS}} \mathbf{I} + \Delta \delta t_{u,\text{BDS}} \mathbf{I}
\]

(28)

For the covariance of \(X_3\):

\[
\text{Cov}(X_3) = \text{Cov}(G_{x_3}^{-1} [\mathbf{p} - \Delta \delta t_{u} \mathbf{I}])
\]

\[
= G_{x_3}^{-1} [\sigma_p^2 \mathbf{I} + \sigma_p^2 \mathbf{I}^T G_{x_3}^{-T}]
\]

\[
= \sigma_p^2 [(G_{x_3} G_{x_3}^T)^{-1} + \sigma_p^2 G_{x_3} G_{x_3}^T \mathbf{I}^T G_{x_3}^{-T}]
\]

(29)

Where, \(\sigma_p^2\) represents the variance of clock error.

Referring to the definition of PDOP in equation (26), the same is true:

\[
\text{PDOP} = \sqrt{\text{Tr}((G_{x_3} G_{x_3}^T)^{-1} + \frac{\sigma_p^2}{\sigma_p^2} G_{x_3} G_{x_3}^T \mathbf{I}^T G_{x_3}^{-T})}
\]

(30)

Define \(k^2 = \frac{\sigma_p^2}{\sigma_{\delta_u}^2}\), then PDOP value can be expressed as:

\[
\text{PDOP} = \sqrt{\text{Tr}((G_{x_3} G_{x_3}^T)^{-1} + k^2 G_{x_3} G_{x_3}^T \mathbf{I}^T G_{x_3}^{-T})}
\]

(31)

PDOP value can be obtained as a function of clock error, and PDOP value also reflects the accuracy of spatial position. It is necessary to use CSAC with high frequency accuracy and high frequency stability to replace common crystal oscillator (TXCO and OXCO) as the time-frequency system of satellite navigation system [12].

IV. TEST RESULTS

In order to better verify the influence of CSAC on receiver positioning, the receiver experiment was conducted with if data collector and MATLAB software. The intermediate frequency data collector uses the 16.369MHz output of quartz crystal and 10MHz output of CSAC as the sampling rate through US3.0, and the collected intermediate frequency data as the input of the software receiver for positioning solution and data analysis.

Figure 2 PDOP value comparison
The comparison diagram of position error is shown in figure 3-5.

Compared with BDS/GPS receiver, CSAC assisted BDS/GPS receiver reduces PDOP value and improves positioning accuracy: latitude accuracy is improved by 0.84m (1σ), longitude accuracy by 0.96m (1σ), and height accuracy by 1.38m (1σ).

The above experiments show that the addition of chip atomic clock can improve the performance of dual-mode receiver, especially the positioning accuracy. With the development of microminiaturization of chip atomic clock, it will be more widely used in receivers in the future.

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