

# Analytical calculation of tube counts and geometric characteristics of tube layouts of heat exchangers

Andrew Ch. Yiannopoulos

**Abstract**— The optimum design of shell-and-tube heat exchangers is an interesting subject in engineering, because it is interrelated with constructional, operational and economic issues. In the present work a detailed geometric consideration of tube layouts is conducted and a method for precise calculation of tube counts is given. Furthermore, some issues concerning the design of tube-sheets and baffles are examined. The calculations were performed through a mathematical approach by introducing a characteristic parameter, the value of which depends on the number of tubes and therefore can be given in tables. A simple working equation was also derived for the calculation of shell diameter of a heat exchanger for a given number of tubes, tube diameter and tube pitch. The results, compared with those obtained by approximate relationships or tables available in the literature, showed that the method of the present study is complete, rapid and with the best accuracy.

**Index Terms** — Heat exchanger, Square pattern, Triangular pattern, Tube counts, Tube layout.

## I. INTRODUCTION

The study of heat exchangers is of great interest, because the conversion of heat is widely used in industries or many other facilities. Among them chemical industries, refineries, power plants and factories for production of many kinds of products are the main places where the heat exchangers are used. In the early years the design of heat exchangers was based on empirical methods, so they had either low efficiency or many clogging and fouling problems. A systematic methodology was presented in 1950 by Kern [1], which was established as the common practice for designing heat exchangers for the next years and named “Kern’s method”. He gave the calculation of heat transfer coefficients, pressure drop and many other constructional features of heat exchangers, such as shell dimensions, tube patterns, tube counts etc. In 1963 the professor Bell [2], in the University of Delaware in USA, by accomplishing a project, presented a more precise method than Kern’s method, which is commonly described as the “Bell-Delaware Method”. Bell took into account some more conditions and parameters that had not been included in the Kern’s method. Among them are the leakage effects of fluids due to by-pass or due to clearances between baffles and shell or between baffle holes and tubes etc. For tube counts Bell applied the available tables. Serna and Jiménez [3] adopted Bell’s method and presented a compact formulation for the design and optimization of heat exchangers. For the calculation of tube counts they used an empirical expression suggested earlier by Taborek [4].

For large number of tubes the simple method of plotting the layout and counting the tubes is very cumbersome and time-consuming. In Wolverine Data Book III, by Thome [5], as well as in the Books by Kakac and Liu [6], Shah and Sekulic [7], Serth and Lestina [8], one can find some approximate analytical expressions or tables available for the estimation of tube counts, which often cover only certain standard combinations of pitch, tube diameter, and layout parameters. There are also many Handbooks, in which several working methods for designing heat exchangers are given. The most common of them is the Perry’s Chemical Engineers Handbook [9] and the Coulson & Richardson’s Chemical Engineering, Vol. 6, by Sinnott [10]. In these Handbooks, a detailed analysis for heat exchangers is included, as well as tube patterns and tables with tube counts, and other technical characteristics. It is of interest the work of Phadke [11], who suggested a mathematical method to predict the number of tubes and presented tables with tube counts for various combinations of tube pattern parameters. His method can accommodate any configuration of tube layout. For the design of shell-and-tube heat exchangers, Muralikrishna and Shenoy [12], Serna-Gonzalez *et al.* [13], Sahin *et al.* [14], Fettaka *et al.* [15] calculated the number of tubes by using approximate empirical equations. The Tubular Exchanger Manufacturer’s Association (TEMA) [16] has standardised the heat exchangers and published rules and technical specifications available for the scientific community. Tan and Fok [17] and Than *et al.* [18], in the design of heat exchangers adopted the empirical expressions of TEMA standards for tube counts, whereas Guo *et al.* [19] used a different empirical equation which is not accurate enough.

In the present work a detailed geometric consideration is conducted to predict either the total number of tubes (tube counts) or the number of tubes on particular rows and/or in the baffle window. To do this a precise method is developed for the calculation of geometric characteristics for several tube layouts, namely the square and triangular tube pattern. The method is based on the mathematical computation of distances of all holes from the center of a circular tube-sheet. For each tube pattern there are discrete circles which contain a certain number of holes for a given pitch size. The number of holes varies slightly if there is or not a hole at the center of the cross-section. To the best of our knowledge, the distinction and detailed consideration of two cases, with or without a hole at the center of a tube-sheet, is first examined in the present study. For the solution, a characteristic parameter  $f$  was introduced, which depends only on the number of holes and tube pattern. This proved to be a great advantage because parameter  $f$  can be tabulated and used directly to determine the bundle or shell diameter, if the number of tubes has been formerly defined through thermal consideration of the heat exchanger.

A. Ch. Yiannopoulos, Department of Mechanical Engineering, Technological Education Institution of Western Greece, M. Alexandrou 1, Patras 26334, Greece, Phone: +30-2610-369084, Fax: +30-2610-369198,

## II. METHOD OF ANALYSIS

## A. Tube layouts

The following analysis deals with the shell-and-tube heat exchangers, which have a number of tubes fitted onto circular tube-sheets at the two ends. The tube-sheets have an equal number of holes to accommodate the tubes. The most common tube layouts are the square and triangular patterns, which are divided further into two forms. The one has a hole at the center of the cross-section, whereas the other not. These configurations are examined separately, because the method of analysis and the results differ slightly. To distinguish them we introduce the abbreviations:

SQ-1 = 1<sup>st</sup> kind of square pattern with a hole at the center.

SQ-2 = 2<sup>nd</sup> kind of square pattern without a hole at the center. The intersection of axes is placed at the center of the square.

TR-1 = 1<sup>st</sup> kind of triangular pattern with a hole at the center.

TR-2 = 2<sup>nd</sup> kind of triangular pattern without a hole at the center. The intersection of axes is placed at the center of the equilateral triangle.

In the square patterns SQ-1 and SQ-2 the tube rows are parallel to the orthogonal axes ( $x$ ,  $y$ ) and they have the same tube pitch  $t$  along  $x$  and  $y$  directions, as in Figs. 1(a) and 1(b). The triangular patterns TR-1 and TR-2 are formed by equilateral triangles with rows parallel to ( $x'$ ,  $y'$ ) axes, which are inclined by  $60^\circ$  with respect to  $x$  axis, and they have the same tube pitch  $t$  along  $x$ ,  $x'$  and  $y'$  directions, as in Figs. 2(a) and 2(b).

## B. Number of holes included in a circle

For a given shell diameter there is a limit to the number of tubes that can fit, which depends on heat exchanger design parameters, as the tube diameter, pitch, layout and clearances. The objective of this section is to find how many holes of diameter  $d_0$  can be included into a circle of diameter  $D$ . Obviously the number of holes depends on pitch  $t$  and therefore the diameter  $D$  is related with the size of  $t$ . This relation can be expressed through a characteristic parameter  $f$  which depends on the number of holes that can be accommodated on a tube-sheet. To calculate the parameter  $f$  two positive integers  $A$  and  $B$  are defined, which characterise the coordinates on ( $x$ ,  $y$ ) axes for square patterns and on ( $x'$ ,  $y'$ ) axes for triangular patterns, as shown in the cross sections of Fig. 3. It is easier to work on a basic section of the circle (shaded area), where the coordinates are positive, and then to extend the calculation and obtain the total number of holes included in the circle. Taking into account the geometry of each separate pattern, the equations for the coordinates are:

$$\text{– For SQ-1: } \quad x = A t \quad y = B t \quad (1)$$

$$\text{– For SQ-2: } \quad x = \frac{2A-1}{2} t \quad y = \frac{2B-1}{2} t \quad (2)$$

$$\text{– For TR-1: } \quad x' = A t \quad y' = B t \quad (3)$$

$$\text{– For TR-2: } \quad x' = \frac{3A-1}{3} t \quad y' = \frac{3B-2}{3} t \quad (4)$$

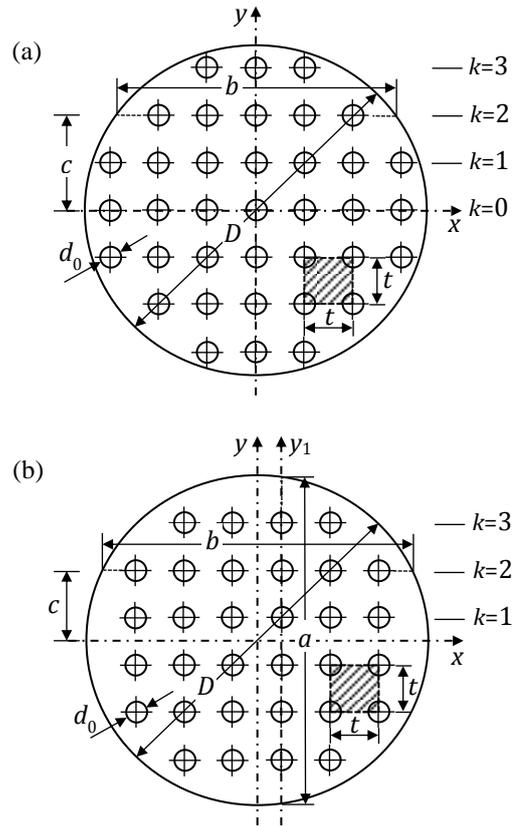


Fig. 1: Square pattern, (a) with a hole at the center (SQ-1), (b) without a hole at the center (SQ-2)

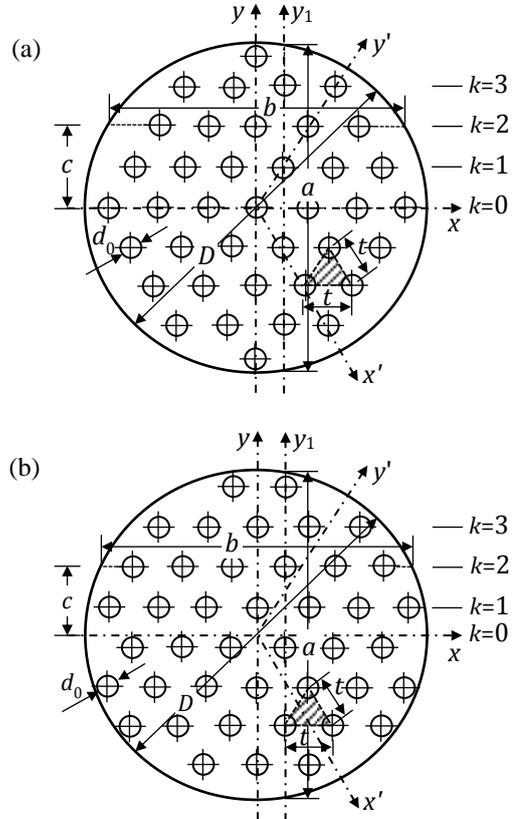


Fig. 2: Triangular pattern, (a) with a hole at the center (TR-1), (b) without a hole at the center (TR-2)

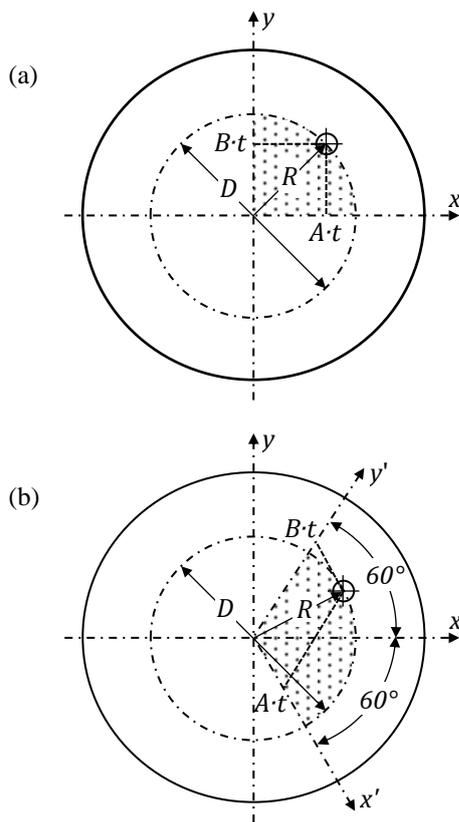


Fig. 3: Axes and coordinates, (a) for square pattern SQ-1, (b) for triangular pattern TR-1

For the examined patterns the characteristic number  $f$  is expressed with the following relationships:

– For SQ-1:  $f = 2\sqrt{A^2 + B^2}$  (5)

$A = 0, 1, 2, 3, \dots$ , and  $B = 0, 1, 2, 3, \dots$

– For SQ-2:  $f = \sqrt{(2A - 1)^2 + (2B - 1)^2}$  (6)

$A = 1, 2, 3, \dots$ , and  $B = 1, 2, 3, \dots$

– For TR-1:  $f = 2\sqrt{A^2 + B^2 - AB}$  (7)

$A = 0, 1, 2, 3, \dots$ , and  $B = 0, 1, 2, 3, \dots$

– For TR-2:

$f = \frac{2}{3}\sqrt{(3A - 1)^2 + (3B - 2)^2 - (3A - 1)(3B - 2)}$  (8)

$A = 1, 2, 3, \dots$ , and  $B = 1, 2, 3, \dots$

After the above definitions and derivations we can create a simple equation, which is valid for all patterns, and can be written as:

$D = f t$  (9)

Using equations (5) to (8) we can calculate the value  $f$  for each shaded area of Fig. 3, which corresponds to a certain number of holes into this area. Then taking into account that the circle includes 4 times more holes for the square patterns SQ-1 and SQ-2, and 3 times more holes for the triangular patterns TR-1 and TR-2, we can find the total number of

holes included in the circle  $D$ . To find the relation between  $f$  and number  $n$  of holes in  $D$  we proceed as follows:

*Step 1:* We write a matrix containing the values of integers  $A$  and  $B$ , and the values  $f$  are calculated using (5) to (8). As an example the calculation for pattern SQ-1 is presented in Table I. The values of  $f$  are computed for all combinations of integers  $A$  and  $B$ , but in the table are shown only the values for  $A \geq B$  due to symmetry. The table is extended to large values of  $A$  and  $B$ , whereas a small part of it is shown here.

Table I. Values  $f$  for pattern SQ-1

A	0	1	2	3	4	5	...
B							
0	0	$2\sqrt{1}$	$2\sqrt{4}$	$2\sqrt{9}$	$2\sqrt{16}$	$2\sqrt{25}$	
1		$2\sqrt{2}$	$2\sqrt{5}$	$2\sqrt{10}$	$2\sqrt{17}$	$2\sqrt{26}$	
2			$2\sqrt{8}$	$2\sqrt{13}$	$2\sqrt{20}$	$2\sqrt{29}$	
3				$2\sqrt{18}$	$2\sqrt{25}$	$2\sqrt{34}$	
4					$2\sqrt{32}$	$2\sqrt{41}$	
5						$2\sqrt{50}$	
⋮							

*Step 2:* We find the number  $i$  of holes at the circumference of the complete circle  $D$ , which corresponds to any combination of integers  $A$  and  $B$  in the matrix. We take into account that for the center corresponds 1 hole, for  $x$ ,  $y$  axes and diagonal axes correspond 4 holes, whereas for the rest combinations correspond 8 holes.

*Step 3:* We sort the values  $f$  by increasing order together with the corresponding values  $i$  and then we find the values of cumulative  $i$ , say  $n$ , which are the sum of holes that included in the particular circle  $D$ . The results are shown in Table II.

Table II. Values  $f$  sorted by increasing order together with  $i$  and number of holes  $n$  for pattern SQ-1

$f$	$i$	$n$ as cumulative $i$
0	1	1
$2\sqrt{1}$	4	5
$2\sqrt{2}$	4	9
$2\sqrt{4}$	4	13
$2\sqrt{5}$	8	21
$2\sqrt{8}$	4	25
$2\sqrt{9}$	4	29
$2\sqrt{10}$	8	37
$2\sqrt{13}$	8	45
$2\sqrt{16}$	4	49
$2\sqrt{17}$	8	57
$2\sqrt{18}$	4	61
⋮	⋮	⋮

Proceeding with the same manner we can find the number  $n$  of the holes for the patterns SQ-2, TR-1 and TR-2. It is clear that each  $n$  corresponds to a certain  $f$ , which is different for each particular pattern. The computations were carried out by a computer program for an increasing large number  $n$  and the results have been tabulated in the Appendix.

### C. Shell inside diameter

The design of heat exchangers must include the formation of tube-sheets. Therefore, if the number  $n$  of tubes is given, using (9) we can obtain directly the diameter  $D$  which refers to the center line of the outer tubes of the bundle. However, the formation of tube-sheets needs some more details to account for, as for instance, the diametric clearance  $C$  between tube bundle and shell, the clearance along a diameter to fit a separation plate of flow etc. Such considerations are important because they influence the magnitude of the inside diameter of the shell.

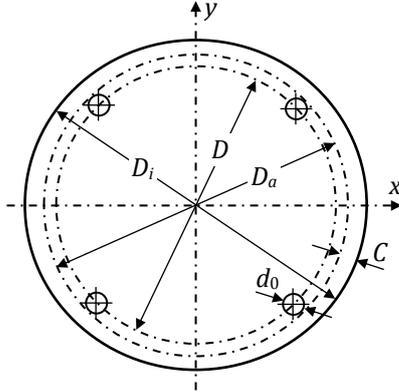


Fig. 4: Geometry of a tube-sheet

Fig. 4 shows the geometry of a tube-sheet fitted into the shell of a heat exchanger. If we add in (9) the outer diameter  $d_0$  of the tubes and the proper clearance  $C$ , we obtain the complete working equation for the heat exchanger, as:

$$D_i = f t + d_0 + 2 C \quad (10)$$

where  $D_i$  is the shell inside diameter. The tube bundle diameter  $D_b$  is given by (10) for  $C=0$ . It can be noticed that using the appended tables with the characteristic parameter  $f$ , the calculations become easier and more precise than other approximate solutions.

### D. Number of rows of a tube-sheet

For the study of heat exchangers it is important to know how many rows of tubes should be placed on a tube-sheet and how many holes should be drilled on each row. In this section we calculate the number of rows parallel to  $x$  axis for each particular pattern. In the square patterns SQ-1 and SQ-2 the horizontal rows are identical to the vertical rows, due to symmetry with respect to  $x$  and  $y$  axis. In the triangular patterns TR-1 and TR-2 the horizontal rows are not identical to the inclined rows parallel to  $x'$  and  $y'$  axis, so there is some difference in the procedure.

**Square pattern SQ-1.** Fig. 1(a) shows the geometry of SQ-1 pattern. To find the number of tube rows we work on  $y$  axis and above  $x$  axis. We divide the radius  $R$  by the pitch  $t$  and we obtain a number from which we take the integer part. Then we replace  $R$  by  $D/2$  and using (9) we can write the equation for the number of rows above  $x$  axis, say  $m_1$ , as:

$$m_1 = \text{Int} \left[ \frac{f}{2} \right] \quad (11)$$

The total number of rows that can be fitted on a tube-sheet is then calculated by duplicating  $m_1$  and adding one row for

that on  $x$  axis. So we obtain the total number  $m$  of tube rows parallel to  $x$  axis on a tube-sheet as:

$$m = 1 + 2 \text{Int} \left[ \frac{f}{2} \right] \quad (12)$$

**Square pattern SQ-2.** Fig. 1(b) shows the geometry of SQ-2 pattern. To find the number of tube rows for this pattern we work similarly above  $x$  axis but on  $y_1$  axis. Since there is no row on  $(x, y)$  axes we divide  $a/2+t/2$  by pitch  $t$  and we take the integer part from the result. The distance  $a$  is the length of the vertical chord on  $y_1$  axis and is given by the equation:

$$a = 2 \sqrt{R^2 - \left(\frac{t}{2}\right)^2} \quad (13)$$

We replace  $R$  by  $D/2$  and using (9) we have the number of rows above  $x$  axis, say  $m_1$ , as:

$$m_1 = \text{Int} \left[ \frac{1+\sqrt{f^2-1}}{2} \right] \quad (14)$$

The total number  $m$  of rows of the tube-sheet is then obtained by duplicating the number  $m_1$ , as:

$$m = 2 \text{Int} \left[ \frac{1+\sqrt{f^2-1}}{2} \right] \quad (15)$$

**Triangular pattern TR-1.** The number of tube rows for this pattern is determined with the same manner. We work separately on  $y$  and  $y_1$  axis and above  $x$  axis. As Fig. 2(a) shows, the distance between rows parallel to  $x$  axis is equal to the height  $h$  of the equilateral triangle, which is:  $h = t\sqrt{3}/2$ .

(i) On  $y$  axis. We divide  $R$  by  $2h$  and we find the number of rows, say  $m_{1(y)}$ , as:

$$m_{1(y)} = \text{Int} \left[ \frac{\sqrt{3}f}{6} \right] \quad (16)$$

(ii) On  $y_1$  axis. We use (13) and divide  $a/2+h$  by  $2h$  and the number of rows, say  $m_{1(y_1)}$ , is:

$$m_{1(y_1)} = \text{Int} \left[ \frac{3+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (17)$$

Therefore, the number of rows above  $x$  axis is the sum of the two numbers  $m_{1(y)}$  and  $m_{1(y_1)}$ , so we have the equation:

$$m_1 = \text{Int} \left[ \frac{\sqrt{3}f}{6} \right] + \text{Int} \left[ \frac{3+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (18)$$

The total number  $m$  of rows parallel to  $x$  axis of the tube-sheet is then obtained by duplicating the number  $m_1$  and adding one unit for the row on  $x$  axis, as:

$$m = 1 + 2 \left\{ \text{Int} \left[ \frac{\sqrt{3}f}{6} \right] + \text{Int} \left[ \frac{3+\sqrt{3}\sqrt{f^2-1}}{6} \right] \right\} \quad (19)$$

**Triangular pattern TR-2.** In this case the  $x$  axis passes through the center of the equilateral triangle, as Fig. 2(b) shows, and since there is no symmetry about  $x$  axis, we work on  $y$  and  $y_1$  axis separately for calculations above or below  $x$  axis. The distance between rows parallel to  $x$  axis is also equal to the height  $h$  of the equilateral triangle. Then we proceed according to the previous manner:

(i) Above  $x$  axis

We examine the horizontal rows of the holes on  $y$  axis and divide  $R+h+h/3$  by  $2h$ , so we find the number of them, as:

$$m_{1(y)} = \text{Int} \left[ \frac{4+\sqrt{3}f}{6} \right] \quad (20)$$

After that we examine the horizontal rows on  $y_1$  axis. We use (13) and divide  $a/2+h/3$  by  $2h$ , so we find the number of them, as:

$$m_{1(y_1)} = \text{Int} \left[ \frac{1+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (21)$$

We add (20) and (21) and obtain the number  $m_1$  of all rows above  $x$  axis, which is:

$$m_1 = \text{Int} \left[ \frac{4+\sqrt{3}f}{6} \right] + \text{Int} \left[ \frac{1+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (22)$$

(ii) Below  $x$  axis

We examine the horizontal rows on  $y$  axis and divide  $R+2h/3$  by  $2h$ , so we find the number of rows, as:

$$m_{2(y)} = \text{Int} \left[ \frac{2+\sqrt{3}f}{6} \right] \quad (23)$$

We continue with the examination of the horizontal rows on  $y_1$  axis and using (13) we divide  $a/2+h+2h/3$  by  $2h$ . The number of rows is:

$$m_{2(y_1)} = \text{Int} \left[ \frac{5+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (24)$$

We add (23) and (24) and obtain the number  $m_2$  of all rows below  $x$  axis, as:

$$m_2 = \text{Int} \left[ \frac{2+\sqrt{3}f}{6} \right] + \text{Int} \left[ \frac{5+\sqrt{3}\sqrt{f^2-1}}{6} \right] \quad (25)$$

The total number  $m$  of rows parallel to  $x$  axis of the tube-sheet is then obtained as the sum of the two numbers  $m_1$  and  $m_2$ :

$$m = m_1 + m_2 \quad (26)$$

It is worth noting that number  $m$  of all patterns depends only on parameter  $f$ .

#### E. Number of holes on any row

The number of holes that can be fitted on any row of the tube-sheet is also calculated separately for each pattern. For this case we examine the rows parallel to  $x$  axis, and define  $c$  as the distance of a particular row from  $x$  axis and  $b$  as the length of the chord at  $c$ , as shown in Figs. 1 and 2.

**Square pattern SQ-1.** We work on rows above  $x$  axis. The rows are numbered by  $k$ , so the central row on  $x$  axis takes the number  $k=0$  and the last takes the number  $k=m_1$ . Considering the geometry of Fig. 1(a) and replacing  $R=D/2$  in (9), we write the following relationships:

$$b = 2\sqrt{R^2 - c^2} \quad R = \frac{1}{2}ft \quad c = kt \quad (27)$$

We take into account (27) and divide the half width  $b/2$  of the chord by  $t$  to obtain a number from which we take the

integer part. Then we duplicate this integer and add one unit for the hole on  $y$  axis, so we have the number  $z_k$  of the holes on a particular row  $k$ , as:

$$z_k = 1 + 2 \text{Int} \left[ \sqrt{\left(\frac{f}{2}\right)^2 - k^2} \right] \quad (28)$$

where  $k = 0, 1, 2, 3, \dots, m_1$ , and  $m_1$  is given by (11).

The number of holes on the central row, namely on  $x$  axis, is obtained from (28) for  $k=0$ , as:

$$z_0 = 1 + 2 \text{Int} \left[ \frac{f}{2} \right] \quad (29)$$

The arrangement SQ-1 is symmetrical about  $x$  and  $y$  axis, so the number  $m$  of rows is equal to the number  $z_0$  of holes at the central row, i.e.  $z_0 = m$ , and thus (29) is identical to (12).

**Square pattern SQ-2.** We work as before on rows above  $x$  axis. The rows are numbered by  $k$ , the first row takes  $k=1$  and the last  $k=m_1$ . From the geometry of Fig. 1(b) we have the relationships:

$$b = 2\sqrt{R^2 - c^2} \quad R = \frac{1}{2}ft \quad c = \frac{2k-1}{2}t \quad (30)$$

We use (30) and divide  $b/2+t/2$  by  $t$ , and then we take the integer part from the result. We duplicate the integer and the number  $z_k$  of the holes on any row  $k$  is:

$$z_k = 2 \text{Int} \left[ \frac{1+\sqrt{f^2-(2k-1)^2}}{2} \right] \quad (31)$$

where  $k = 1, 2, 3, \dots, m_1$ , and  $m_1$  is given by (14).

The number of holes on the first row, near  $x$  axis, is obtained from (31) for  $k=1$ , as:

$$z_1 = 2 \text{Int} \left[ \frac{1+\sqrt{f^2-1}}{2} \right] \quad (32)$$

We note that the number  $m$  of rows is equal to the number  $z_1$  of holes of the first row, since the layout SQ-2 is symmetrical about  $x$  and  $y$  axes, which means  $z_1 = m$ , and therefore (32) is identical to (15).

**Triangular pattern TR-1.** We work as above on rows above  $x$  axis. The rows are numbered by  $k$ , the central row takes  $k=0$  and the last  $k=m_1$ . From the geometry of Fig. 2(a) we have the relationships:

$$b = 2\sqrt{R^2 - c^2} \quad R = \frac{1}{2}ft \quad c = \frac{\sqrt{3}}{2}kt \quad (33)$$

We observe that for  $k = 0, 2, 4, \dots$ , we have holes on  $y$  axis and for  $k = 1, 3, 5, \dots$ , we have holes on  $y_1$  axis. So we work separately if  $k$  is an odd or even number.

– For  $k = 1, 3, 5, \dots$ , taking into account (33) and dividing  $b/2+t/2$  by  $t$ , we obtain a number from which we take the integer part. We duplicate this integer and write the number  $z_k$  of the holes on a particular row  $k$ , as:

$$z_k = 2 \text{Int} \left[ \frac{1+\sqrt{f^2-3k^2}}{2} \right], \quad k = 1, 3, 5, \dots \quad (34)$$

– For  $k = 0, 2, 4, \dots$ , and following the same way we divide  $b/2$  by  $t$  to obtain a number and the integer part of it. We duplicate it and add one unit for the hole on  $y$  axis. The

number  $z_k$  of the holes on a particular row  $k$  is:

$$z_k = 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - 3k^2}}{2} \right], \quad k = 0, 2, 4, \dots \quad (35)$$

**Triangular pattern TR-2.** The analysis of this pattern differs if the rows are above or below  $x$  axis. So we work separately for these two cases. The rows are numbered by  $k$ , the first row takes  $k=1$  and the last  $k=m_1$ . From the geometry of Fig. 2(b) we can write the same relationships as in TR-1 for  $b$  and  $R$ , but there is a difference in  $c$ .

$$\text{– Above } x \text{ axis:} \quad c = \frac{(3k-1)\sqrt{3}}{6} t \quad (36)$$

$$\text{– Below } x \text{ axis:} \quad c = \frac{(3k-2)\sqrt{3}}{6} t$$

(i) Above  $x$  axis

We observe that for  $k = 1, 3, 5, \dots$ , we have holes on  $y$  axis and for  $k = 2, 4, 6, \dots$ , we have holes on  $y_1$  axis. So we work separately if  $k$  is an odd or even number.

– For  $k = 1, 3, 5, \dots$ , we use (36) and divide  $b/2$  by  $t$ . Then we take the integer part from the result. We duplicate the integer and add one unit for the central hole. The number  $z_k$  of the holes on a particular row  $k$  can be written as:

$$z_k = 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - \frac{1}{3}(3k-1)^2}}{2} \right], \quad k = 1, 3, 5, \dots \quad (37)$$

– For  $k = 2, 4, 6, \dots$ , we use (36) and divide  $b/2+t/2$  by  $t$  to obtain a number and the integer part of it. We duplicate the integer and then we have  $z_k$ , as:

$$z_k = 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - \frac{1}{3}(3k-1)^2}}{2} \right], \quad k = 2, 4, 6, \dots \quad (38)$$

(ii) Below  $x$  axis

We observe that we have holes on  $y_1$  axis for  $k = 1, 3, 5, \dots$ , and on  $y$  axis for  $k = 2, 4, 6, \dots$ , and therefore we must work separately if  $k$  is an odd or even number.

– For  $k = 1, 3, 5, \dots$ , we use (36) and divide  $b/2+t/2$  by  $t$ . After that we take the integer part from the result. We duplicate it and the number  $z_k$  of the holes on a particular row  $k$  is:

$$z_k = 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - \frac{1}{3}(3k-2)^2}}{2} \right], \quad k = 1, 3, 5, \dots \quad (39)$$

– For  $k = 2, 4, 6, \dots$ , we use (36) and divide  $b/2$  by  $t$ , and then we take the integer part from the result. We duplicate it, and add one unit for the central hole, so the number  $z_k$  is:

$$z_k = 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - \frac{1}{3}(3k-2)^2}}{2} \right], \quad k = 2, 4, 6, \dots \quad (40)$$

#### F. Number of holes in a circular segment

The number of holes which are included in a segment of a circle can be calculated according to the previous analysis.

We usually need to know this number if we are looking for the number of tubes passing through a baffle window of a heat exchanger. If  $B_c$  is the height of the baffle cut, the remaining distance above  $x$  axis is  $R-B_c$ . The geometry of a segmental baffle is shown in Fig. 5, where the shaded area is the cut. The concept is to find how many horizontal rows are fitted along the distance  $R-B_c$  first and then to find the number of rows in the baffle segment by subtraction from the total number  $m_1$  of rows above  $x$  axis. Since this number is already determined, we work separately for each pattern, as:

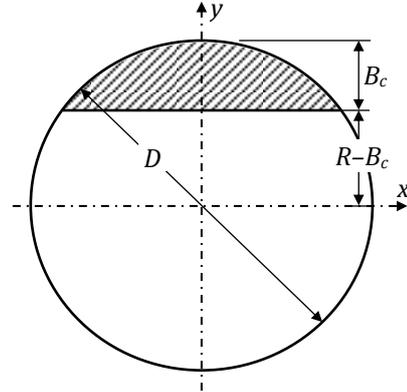


Fig. 5: Segmental baffle geometry

**Square pattern SQ-1.** We divide the distance  $R-B_c$  by  $t$  and take the integer part from the result to obtain the number  $j$  of rows parallel to  $x$  axis that can be fitted along the distance  $R-B_c$ . For  $j$  the following equation can be written:

$$j = \text{Int} \left[ \frac{D-2B_c}{2t} \right] \quad (41)$$

The total number of rows above  $x$  axis is known by (11) as  $m_1$  and consequently the rows from  $j+1$  up to  $m_1$  are included in the baffle window. Each of them has a particular number of holes given by (28). The sum gives the number of holes included in the circular segment, say  $N_c$ , as:

$$N_c = \sum_k^{m_1} \left[ 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - 4k^2}}{2} \right] \right] \quad (42)$$

where  $k = j+1, j+2, j+3, \dots, m_1$

If the baffle cut is in the opposite, i.e. below  $x$  axis, the equation (42) for  $N_c$  remains the same, due to symmetry of the arrangement of holes with respect to  $x$  axis.

**Square pattern SQ-2.** For this case we divide  $R-B_c+t/2$  by  $t$  and after taking the integer part we find the number  $j$  of rows parallel to  $x$  axis, and along the distance  $R-B_c$ , as:

$$j = \text{Int} \left[ \frac{D-2B_c+t}{2t} \right] \quad (43)$$

The total number of rows above  $x$  axis is already known by (14) as  $m_1$ . Then the rows from  $j+1$  up to  $m_1$  are included in the baffle window and each of them has a particular number of holes given by (31). The sum gives the number of holes included in the circular segment, say  $N_c$ , as:

$$N_c = \sum_k^{m_1} 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - (2k-1)^2}}{2} \right] \quad (44)$$

where  $k = j+1, j+2, j+3, \dots, m_1$

If the baffle cut is in the opposite, i.e. below  $x$  axis, the equation (44) for  $N_c$  is also valid, due to symmetry of the arrangement of holes with respect to  $x$  axis.

**Triangular pattern TR-1.** Following the same procedure as before, we divide for this pattern the distance  $R-B_c$  by the height of the equilateral triangle  $h = t\sqrt{3}/2$  and take the integer part. The number  $j$  of rows parallel to  $x$  axis is:

$$j = \text{Int} \left[ \frac{\sqrt{3}(D-2B_c)}{3t} \right] \quad (45)$$

The total number of rows above  $x$  axis is already determined by (18) as  $m_1$ , and consequently the rows from  $j+1$  up to  $m_1$  are included in the baffle window. Each of them has a particular number of holes given by (34) and (35). The sum gives the number of holes included in the circular segment, say  $N_c$ , as:

$$N_c = \begin{cases} \sum_k^{m_1} \left[ 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - 3k^2}}{2} \right] \right] & \text{for } k = 1, 3, 5, \dots \\ + \sum_k^{m_1} 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - 3k^2}}{2} \right] & \text{for } k = 2, 4, 6, \dots \end{cases} \quad (46)$$

where  $k = j+1, j+2, j+3, \dots, m_1$

If the baffle cut is in the opposite, below  $x$  axis, the number  $N_c$  is also given by (46), due to symmetry of the hole layout with respect to  $x$  axis.

**Triangular pattern TR-2.** For the triangular pattern TR-2 there is a different number of holes in the circular segment if it is cut above or below  $x$  axis. For this reason we proceed separately for these two cases.

(i) Above  $x$  axis

The height of the equilateral triangle is  $h = t\sqrt{3}/2$ . We divide the distance  $R-B_c+h/3$  by  $h$ , take the integer part, and the number  $j$  of rows parallel to  $x$  axis can be written as:

$$j = \text{Int} \left[ \frac{\sqrt{3}(D-2B_c)+t}{3t} \right] \quad (47)$$

The total number of rows above  $x$  axis is given by (22) as  $m_1$ , and thus the rows from  $j+1$  up to  $m_1$  are included in the circular segment. Each of them has a particular number of holes given by (37) and (38) and therefore the sum gives the number of holes within the circular segment, say  $N_c$ , as:

$$N_c = \begin{cases} \sum_k^{m_1} \left[ 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - \frac{1}{3}(3k-1)^2}}{2} \right] \right] & \text{for } k = 1, 3, 5, \dots \\ + \sum_k^{m_1} 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - \frac{1}{3}(3k-1)^2}}{2} \right] & \text{for } k = 2, 4, 6, \dots \end{cases} \quad (48)$$

where  $k = j+1, j+2, j+3, \dots, m_1$

(ii) Below  $x$  axis

For this case we divide the distance  $R-B_c+2h/3$  by  $h$  and taking the integer part we have the number  $j$  of rows parallel to  $x$  axis, as:

$$j = \text{Int} \left[ \frac{\sqrt{3}(D-2B_c)+2t}{3t} \right] \quad (49)$$

The total number of rows below  $x$  axis is given by (25) as  $m_2$ . As before, the rows from  $j+1$  up to  $m_2$  are included in the baffle window and each of them has a particular number of holes given by (39) and (40). Consequently, the sum gives the holes included in the circular segment, say  $N_c$ , as:

$$N_c = \begin{cases} \sum_k^{m_2} 2 \text{Int} \left[ \frac{1 + \sqrt{f^2 - \frac{1}{3}(3k-2)^2}}{2} \right] & \text{for } k = 1, 3, 5, \dots \\ + \sum_k^{m_2} \left[ 1 + 2 \text{Int} \left[ \frac{\sqrt{f^2 - \frac{1}{3}(3k-2)^2}}{2} \right] \right] & \text{for } k = 2, 4, 6, \dots \end{cases} \quad (50)$$

where  $k = j+1, j+2, j+3, \dots, m_2$

### III. NUMERICAL EXAMPLES AND DISCUSSION

**Example 1.** A heat exchanger should have  $n=347$  tubes in order to satisfy the heat transfer requirements. The tubes are made from seamless steel and according to ISO standards they have an outer diameter  $d_0=21.3$  mm and thickness  $s=2$  mm. The clearance between tubes is taken  $\delta=10$  mm and the allowed diametric clearance between tube bundle and shell must be  $C=15$  mm. The height of the baffle window is 25% of the diameter. The tube pitch is:  $t=d_0+\delta=21.3+10=31.3$  mm.

By using the previous analysis we calculate for SQ-1 and TR-1 the layout characteristics, namely: Shell inside diameter  $D_i$ , diameter  $D_b$  of the tube bundle, total number  $m$  of rows of the tube-sheet, number  $z_0$  of tubes on the central row, number  $N_c$  of tubes in the baffle window.

(a) Square pattern SQ-1:

From Table A.1:  $f=20.88061$  for  $n=349$ , which is just greater than given  $n$ .

$$\text{From (10): } D_i = 20.88061 \times 31.3 + 21.3 + 2 \times 15 = 704.86 \text{ mm}$$

$$\text{From (10): } D_b = 20.88061 \times 31.3 + 21.3 = 674.86 \text{ mm}$$

$$\text{From (12): } m = 1 + 2 \times \text{Int} \left[ \frac{20.88061}{2} \right] = 21$$

$$\text{From (29): } z_0 = m = 21$$

The height of the baffle window is:

$$B_c = 25\% D_b = 0.25 \times 674.86 = 168.72 \text{ mm}$$

$$\text{From (41): } j = \text{Int} \left[ \frac{674.86 - 2 \times 168.72}{2 \times 31.3} \right] = 5$$

$$\text{From (11): } m_1 = \text{Int} \left[ \frac{20.88061}{2} \right] = 10$$

Thus the calculation of  $N_c$  is performed for  $k=6, 7, 8, 9, 10$ .

$$\text{From (42): } N_c = \sum_{k=6}^{10} \left[ 1 + 2 \times \text{Int} \left[ \frac{\sqrt{20.88061^2 - 4k^2}}{2} \right] \right] \Rightarrow$$

$$\Rightarrow N_c = 17 + 15 + 13 + 11 + 7 = 63$$

(b) Triangular pattern TR-1:

From Table A.3:  $f=19.28730$  for  $n=349$ , which is just greater than given  $n$ .

$$\text{From (10): } D_i = 19.28730 \times 31.3 + 21.3 + 2 \times 15 = 654.99 \text{ mm}$$

$$\text{From (10): } D_b = 19.28730 \times 31.3 + 21.3 = 624.99 \text{ mm}$$

$$\text{From (18): } m_1 = \text{Int} \left[ \frac{\sqrt{3} \times 19.28730}{6} \right] + \text{Int} \left[ \frac{3 + \sqrt{3} \sqrt{19.28730^2 - 1}}{6} \right] = 11$$

$$\text{From (19): } m = 1 + 2m_1 = 1 + 2 \times 11 = 23$$

$$\text{From (35) and for } k=0: z_0 = 1 + 2 \times \text{Int} \left[ \frac{19.28730}{2} \right] = 19$$

The height of the baffle window is:

$$B_c = 25\% D_b = 0.25 \times 624.99 = 156.25 \text{ mm}$$

$$\text{From (45): } j = \text{Int} \left[ \frac{\sqrt{3}(624.99 - 2 \times 156.25)}{3 \times 31.3} \right] = 5$$

Thus the calculation of  $N_c$  must be performed for  $k=j+1$  to  $m_1$ , which means  $k=6, 7, 8, \dots, 11$ .

From (46):

$$N_c = \begin{cases} \sum_{k=7}^{11} \left[ 1 + 2 \text{Int} \left[ \frac{\sqrt{19.28730^2 - 3k^2}}{2} \right] \right] & \text{for } k = 1, 3, 5, \dots \\ + \sum_{k=6}^{11} 2 \text{Int} \left[ \frac{1 + \sqrt{19.28730^2 - 3k^2}}{2} \right] & \text{for } k = 2, 4, 6, \dots \end{cases}$$

which gives the result:

$$N_c = 15 + 11 + 3 + 2 \cdot (8 + 7 + 4) = 67$$

We observe that the arrangement according to TR-1 pattern gives the smallest shell inside diameter and can accommodate two more tubes than 347, namely 349 tubes.

**Example 2.** To compare our results with the values taken from available tables of tube counts or with the results extracted from approximate empirical equations, we consider a heat exchanger with adapted parameters, as: Shell inside diameter  $D_i=13.25''=336.55$  mm, triangular pattern TR-1, tubes of outer diameter  $d_0=3/4''=19.05$  mm, pitch  $t=1''=25.4$  mm and clearance  $C=0$ . From (10) we have:  $D_b=D_i$ .

The characteristic parameter  $f$  which corresponds to the data of the heat exchanger can be calculated by applying (10) for  $C=0$ , as:

$$f = \frac{D_i - d_0}{t} = \frac{336.55 - 19.05}{25.4} = 12.5$$

From the appended Table A.3 and for  $f=12.49 < 12.5$  our method gives  $n=151$  tubes.

(a) Using the tables of tube counts by Serth and Lestina [8], and for the same data, we find  $n=136$ . There is a deviation of about 10% or 15 tubes less.

(b) The application of an empirical equation cited in the Handbook by Sinnott [10], for bundle diameter  $D_b=336.55$  mm,  $d_0=19.05$  mm, and the given factors  $K_1=0.319$  and  $n_1=2.142$  for the triangular pattern with a pitch  $t=1.25d_0$ , gives the following result:

$$n = K_1 \left( \frac{D_b}{d_0} \right)^{n_1} = 0.319 \left( \frac{336.55}{19.05} \right)^{2.142} = 149.69 \approx 150$$

To compare this result with our method the pitch  $t$  must be adapted, because in this case it is slightly different from the aforementioned value, which is:  $t=1.25 \times 19.05=23.81$  mm. From (10) we find the new parameter  $f$ :

$$f = \frac{D_i - d_0}{t} = \frac{336.55 - 19.05}{23.81} = 13.33$$

From Table A.3 and for  $f=13.11488 < 13.33$  our method results to  $n=163$  tubes. We observe that the empirical equation gave 13 tubes less, which corresponds to a deviation of about 8%.

(c) The comparison with the approximate equation in Perry's Handbook [9], for the triangular pattern with a pitch  $t=1.25d_0$ , requires the value of a parameter  $C_0$ :

$$C_0 = 0.75 \frac{D_b}{d_0} - 36 = 0.75 \frac{336.55}{19.05} - 36 = -22.75$$

The range of accuracy is  $-24 \leq C_0 \leq 24$ , which is valid. The number of tubes is given by the following relationship:

$$n = 1298 + 74.86C_0 + 1.283C_0^2 - 0.0078C_0^3 - 0.0006C_0^4$$

Applying  $C_0=-22.75$  we find  $n=190$  tubes. The result is greater than the above of our method, which gave 163 tubes, and much greater than 150 tubes of the previous empirical equation in [10]. The deviation from our prediction is about 16.6% or 27 tubes more and from the empirical equation 26.7% or 40 tubes more, respectively.

(d) The empirical equation by Guo *et al.* [19] gives the shell inside diameter  $D_i$ . To compare it with our method, we apply the initial data of the heat exchanger:  $d_0=19.05$  mm,  $t=25.4$  mm. Using the same number of tubes, i.e.  $n=151$ , we obtain:

$$\begin{aligned} D_i &= (1.1 \sqrt{n} - 1) t + 3 d_0 = \\ &= (1.1 \sqrt{151} - 1) \times 25.4 + 3 \times 19.05 = 375.08 \text{ mm} \end{aligned}$$

The calculated diameter is too larger compared with the value  $D_i=336.55$  mm of the heat exchanger, which means a deviation of about 11.4%.

All the above comparisons showed that there are remarkable deviations from the exact solution when using available tables for tube counts or empirical equations.

#### IV. CONCLUSION

The investigation of geometric characteristics of shell-and-tube heat exchangers was successfully performed and the solution was presented explicitly in a simplified form for many tube arrangements and tube dimensions. A simple working equation for shell inside diameter was given and a characteristic parameter was introduced to calculate the geometric factors of the heat exchanger. The values of the characteristic parameter were calculated by a computer program and tabulated. The comparison of the results with those obtained through approximate equations available in the literature, showed noticeable deviations from the exact solution. The values of tube counts that were taken from tables of Handbooks also compared and appear to be somewhat lower than the exact solution of the present study.

APPENDIX

**Table A.1:** Tube counts and parameter  $f$  for pattern SQ-1

Tube counts	Parameter	Tube counts	Parameter	Tube counts	Parameter
$n$	$f$	$n$	$f$	$n$	$f$
1	0	277	18.43909	609	27.85678
5	2	285	18.86796	613	28
9	2.82843	293	18.97367	621	28.07134
13	4	301	19.69772	633	28.28427
21	4.47214	305	19.79899	641	28.42534
25	5.65685	317	20	657	28.63564
29	6	325	20.09975	665	28.84441
37	6.32456	333	20.39608	673	29.12044
45	7.21110	341	20.59126	681	29.52965
49	8	349	20.88061	697	29.73214
57	8.24621	357	21.26029	709	30
61	8.48528	365	21.54066	717	30.06659
69	8.94427	373	21.63331	725	30.26549
81	10	377	22	733	30.46309
89	10.19804	385	22.09072	741	30.52867
97	10.77033	401	22.36068	749	30.59412
101	11.31371	405	22.62742	757	31.04835
109	11.66190	421	22.80351	761	31.11270
113	12	429	23.32381	769	31.24100
121	12.16552	437	23.40940	777	31.30495
129	12.64911	441	24	793	31.62278
137	12.80625	457	24.08319	797	32
145	13.41641	465	24.16609	805	32.06244
149	14	473	24.33105	821	32.24903
161	14.14214	481	24.41311	829	32.31099
169	14.42220	489	24.73863	845	32.55764
177	14.56022	497	25.05993	853	32.80244
185	15.23155	505	25.29822	861	32.98484
193	15.62050	509	25.45584	869	33.10589
197	16	517	25.61250	877	33.28663
213	16.12452	529	26	885	33.52611
221	16.49242	545	26.07681	889	33.94112
225	16.97056	553	26.30589	901	34
233	17.08801	561	26.68333	917	34.05877
241	17.20465	569	26.83282	925	34.17601
249	17.88854	577	26.90725	933	34.23449
253	18	593	27.20294	941	34.40930
261	18.11077	601	27.78489	949	34.52535

**Table A.2:** Tube counts and parameter  $f$  for pattern SQ-2

Tube counts	Parameter	Tube counts	Parameter	Tube counts	Parameter
$n$	$f$	$n$	$f$	$n$	$f$
4	1.41421	360	21.21320	732	30.36445
12	3.16228	368	21.40093	740	30.88689
16	4.24264	376	21.58703	756	31.01612
24	5.09902	384	21.95450	772	31.14482
32	5.83095	392	22.13594	788	31.40064
44	7.07107	400	22.67157	804	31.78050
52	7.61577	408	22.84732	812	31.90611
60	8.60233	424	23.02173	820	32.28003
68	9.05539	432	23.19483	824	32.52691
76	9.48683	440	23.53720	840	32.64965
80	9.89950	448	23.70654	848	32.89377
88	10.29563	460	24.04163	864	33.01515
96	11.04536	468	24.20744	872	33.13608
112	11.40175	484	24.69818	880	33.37664
120	12.08305	492	25.01999	896	33.61547
124	12.72792	500	25.17936	904	33.73426
140	13.03840	524	25.49510	912	33.97058
148	13.34166	532	25.80698	928	34.20526
156	13.92839	540	25.96151	936	34.43835
164	14.21267	548	26.41969	944	34.66987
172	14.76482	556	26.57066	952	34.78505
180	15.03330	560	26.87006	960	35.01428
188	15.29706	576	27.01851	968	35.12834
192	15.55635	584	27.16615	988	35.35534
208	15.81139	592	27.31300	1004	35.46830
216	16.55295	608	27.45906	1012	35.69314
232	17.02939	616	27.89265	1020	35.80503
240	17.26268	624	28.17801	1028	36.13862
248	17.49286	632	28.31960	1036	36.24914
256	17.72005	640	28.46050	1044	36.35932
268	18.38478	648	28.60070	1052	36.68787
276	18.60108	656	29.01724	1060	36.79674
284	19.02630	680	29.15476	1076	37.01351
300	19.23538	688	29.42788	1092	37.12142
308	19.64688	692	29.69848	1108	37.33631
316	19.84943	708	29.83287	1116	37.44329
332	20.24846	716	29.96665	1124	37.65634
348	21.02380	724	30.23243	1148	38.07887

NOMENCLATURE

$a$  = length of a circle chord parallel to  $y$  axis (mm)  
 $A$  = integer number  
 $b$  = length of a circle chord parallel to  $x$  axis (mm)  
 $B$  = integer number  
 $B_c$  = height of the baffle cut (mm)  
 $c$  = distance of a horizontal chord from  $x$  axis (mm)  
 $C$  = clearance between shell and tube bundle (mm)  
 $C_0$  = parameter of empirical equation  
 $d_0$  = tube outer diameter (mm)  
 $D$  = circle diameter (mm)  
 $D_b$  = tube bundle diameter (mm)  
 $D_i$  = shell inside diameter (mm)  
 $f$  = characteristic parameter  
 $h$  = height of the equilateral triangle (mm)  
 $i$  = number of holes at the circumference of a circle  
 $j$  = number of rows  
 $k$  = row numbering  
 $k_1$  = factor of empirical equation  
 $m, m_1, m_2$  = number of rows  
 $m_{1(y)}, m_{1(y1)}, m_{2(y)}, m_{2(y1)}$  = number of rows  
 $n$  = number of holes included in a circle

$n_1$  = factor of empirical equation  
 $N_c$  = number of holes in the baffle window  
 $R$  = circle radius (mm)  
 $s$  = tube thickness (mm)  
 $t$  = tube pitch (mm)  
 $x, y, y_1$  = orthogonal axes  
 $x', y'$  = inclined axes  
 $z_k, z_0, z_1$  = number of holes on a particular row

Greek Symbols

$\delta$  = clearance between adjacent tubes (mm)

Abbreviations

Int = integer part of a number  
 SQ-1 = square tube pattern with a hole at the center  
 SQ-2 = square tube pattern without a hole at the center  
 TR-1 = triangular tube pattern with a hole at the center  
 TR-2 = triangular tube pattern without a hole at the center

**Table A.3:** Tube counts and parameter  $f$  for pattern TR-1

Tube counts	Parameter	Tube counts	Parameter	Tube counts	Parameter
$n$	$f$	$n$	$f$	$n$	$f$
1	0	385	20.78460	859	30.78960
7	2	397	20.88062	871	31.04834
13	3.46410	409	21.07130	877	31.17692
19	4	421	21.16602	889	31.24100
31	5.29150	433	21.63330	913	31.43246
37	6	439	22	925	31.74902
43	6.92820	451	22.27106	931	32
55	7.21110	463	22.53886	955	32.18696
61	8	475	22.71564	967	32.74142
73	8.71780	499	23.06512	979	32.92416
85	9.16516	511	23.57966	1003	33.04542
91	10	517	24	1015	33.28664
97	10.39230	535	24.24872	1027	33.40658
109	10.58300	547	24.33104	1039	33.64520
121	11.13552	559	24.57642	1045	34
127	12	571	24.98000	1057	34.11744
139	12.16552	583	25.05992	1069	34.17602
151	12.49	595	25.53430	1075	34.64102
163	13.11488	613	26	1099	34.69870
169	13.85640	625	26.15340	1111	34.87120
187	14	637	26.22976	1123	35.04284
199	14.42220	649	26.45752	1135	35.15680
211	15.09966	661	26.90724	1147	35.38362
223	15.62050	673	27.05550	1159	35.55278
235	15.87450	685	27.49546	1165	36
241	16	691	27.71282	1177	36.05552
253	16.37070	703	27.78488	1189	36.16628
265	17.08800	721	28	1201	36.38682
271	17.32050	733	28.21348	1213	36.49658
283	17.43560	745	28.35490	1225	36.66060
295	17.77638	757	28.84442	1237	36.71512
301	18	769	29.05168	1261	37.04052
313	18.33030	793	29.46184	1273	37.36308
337	19.07878	805	29.59730	1285	37.46998
349	19.28730	817	29.86636	1303	38
361	19.69772	823	30	1309	38.10512
367	20	835	30.19934	1333	38.15756
379	20.29778	847	30.26550	1345	38.31448

**Table A.4:** Tube counts and parameter  $f$  for pattern TR-2

Tube counts	Parameter	Tube counts	Parameter	Tube counts	Parameter
$n$	$f$	$n$	$f$	$n$	$f$
3	1.15470	231	16.04161	498	23.35237
6	2.30940	240	16.16581	504	23.43786
12	3.05505	246	16.28906	510	23.69247
18	4.16333	252	16.65333	522	23.86071
21	4.61880	258	16.77299	528	24.02776
27	5.03322	270	17.00980	534	24.11086
30	5.77350	276	17.24336	540	24.19366
36	6.11010	282	17.47379	546	24.44040
42	6.42910	288	17.92577	552	24.68468
48	7.02377	294	18.03700	558	24.84620
54	7.57188	306	18.14754	570	25.00667
63	8.08290	309	18.47521	576	25.16611
69	8.32666	321	18.58315	588	25.32456
75	9.01850	327	18.90326	591	25.40341
78	9.23760	333	19.00877	597	25.48202
84	9.45163	339	19.21805	603	25.71640
90	9.86577	345	19.42507	609	25.79406
96	10.06645	348	19.62991	615	26.02563
102	10.26320	354	19.73153	627	26.10236
114	11.01514	366	20.03331	633	26.40707
120	11.37248	372	20.13289	636	26.55811
123	11.54701	378	20.23199	648	26.63331
129	11.71893	384	20.42874	654	26.85765
135	12.05543	390	20.52641	660	27.00617
141	12.22020	396	20.81666	672	27.15388
144	12.70171	402	21.00794	678	27.22744
150	12.85820	408	21.19748	690	27.30079
156	13.01281	420	21.38535	696	27.59227
168	13.31666	426	21.57159	702	27.73686
174	13.61372	435	21.93931	714	28.02380
180	14.04754	447	22.03028	720	28.09508
186	14.18920	453	22.12088	726	28.30783
192	14.46836	459	22.30097	732	28.37840
198	14.74223	465	22.47962	738	28.44879
207	15.01111	471	22.74496	744	28.58904
213	15.14376	477	23.00725	750	28.72862
219	15.27525	480	23.09401	753	28.86751
225	15.53491	492	23.18045	759	28.93671

REFERENCES

[1] D. Q. Kern, *Process Heat Transfer*. McGraw-Hill, New York, 1950.

[2] K. J. Bell, *Final report of the cooperative research program on shell and tube heat exchangers*. University of Delaware, Engineering Experimental Station, Bulletin No. 5, Newark, Delaware, 1963.

[3] M. Serna, and A. Jiménez, A compact formulation of the Bell-Delaware method for heat exchanger design and optimization. *Chem. Eng. Res. Des.*, Vol. 83 (A5): pp. 539–550, 2005.

[4] J. Taborek, *Shell-and-Tube Heat Exchangers*. In: Schlünder, E.U. (ed), *Heat Exchanger Design Handbook*, Section 3.3. Hemisphere Publishing Corp., USA, 1983.

[5] J. R. Thome, *Wolverine Engineering Data Book III*. Wolverine Tube Inc., USA, 2006.

[6] S. Kakac, and H. Liu, *Heat Exchangers*, 2<sup>nd</sup> ed. CRC Press, New York, 2002.

[7] R. K. Shah, and D. P. Sekulic, *Fundamentals of Heat Exchanger Design*. John Wiley & Sons, Inc., Hoboken, New Jersey, 2003.

[8] R. W. Serth, and T. G. Lestina, *Process Heat Transfer*, 2<sup>nd</sup> ed. Elsevier Inc., USA, 2014.

[9] D. W. Green, and R. H. Perry, *Perry's Chemical Engineers' Handbook*, 8<sup>th</sup> ed. McGraw-Hill, Inc., USA, 2008.

[10] R. K. Sinnott, *Chemical Engineering Design*. Coulson & Richardson's, Chemical Engineering series, Vol. 6, 4<sup>th</sup> ed. Elsevier Butterworth-Heinemann, 2005.

[11] P. S. Phadke, "Determining tube counts for shell and tube exchangers". *Chem. Eng. J.*, September 3, Vol. 91, pp. 65–68, 1984.

[12] K. Muralikrishna, and U. V. Shenoy, "Heat exchanger design targets for minimum area and cost". *Trans. IChemE, Part A, Chem. Eng. Res. Des.*, Vol 78, No. 2, pp. 161-167, 2000.

[13] M. Serna-Gonzalez, J. M. Ponce-Ortega, A. J. Castro-Montoya, and A. Jimenez-Gutierrez, "Feasible design space for shell-and-tube heat exchangers using the Bell-Delaware method". *Ind. Eng. Chem. Res.*, Vol. 46, No. 1, pp. 143-155, 2007.

[14] A. S. Sahin, B. Kilic, and U. Kilic, "Design and economic optimization of shell and tube heat exchangers using Artificial Bee Colony (ABC) algorithm". *Energy Convers. Manage.*, Vol. 52, pp. 3356–3362, 2011.

[15] S. Fettaka, J. Thibault, and Y. Gupta, "Design of shell-and-tube heat exchangers using multiobjective optimization". *Int. J. Heat Mass Transfer*, Vol. 60, pp. 343-354, 2013.

[16] TEMA, *Standards of the Tubular Exchange Manufacturers Association*. J. Harrison (ed.), New York, 1999.

[17] F. Tan, and S. Fok, "An educational tool for heat exchanger design". *Comput. Appl. Eng. Educ.*, Vol. 14, No. 2, pp. 77–89, 2006.

[18] S. T. M. Than, K. A. Lin, and M. S. Mon, "Heat Exchanger Design". *Int. J. of Mechanical and Mechatronics Engineering*, Vol. 2, No. 10, 2008.

[19] J. Guo, L. Cheng, and M. Xu, "Optimization design of shell-and-tube heat exchanger by entropy generation minimization and genetic algorithm". *Appl. Therm. Eng.*, Vol. 29, pp. 2954–2960, 2009.