Numerical Solution of MEW Equation with Splitting Technique

Melike Karta

Abstract— **In this study MEW equation of form**
$U_t + \epsilon U^2 U_x - \mu U_{xxt} = 0$
is splitted into two sub-equations such that one is linear and the other nonlinear by following form
$U_t - \mu U_{xxt} = 0$
$U_t + \epsilon U^2 U_x - \mu U_{xxt} = 0$
Sub-equations with initial and boundary conditions are numericallay solved with finite difference method by help linearization technique using strang siplitting techniques. MEW equation are calculated error norms L_2, L_∞ and invariants. Calculated values are compared with those available in the literature.

Index Terms— **Finite difference method, MEW equation, Splitting technique.**

I. INTRODUCTION

The one- dimensional generalized equal width (GEW) equation is of the form

$$U_t + \epsilon U^p U_x - \mu U_{xxt} = 0 \qquad (1)$$

With the physical boundary conditions $U \to 0$ as $x \to \pm\infty$, respectively ,where t is time x is space coordinate.p, ϵ and μ are positive parameters.In equation (1), when p=1 we obtain equal width (EW) equation introduced by Morrison and et al.[10] when $p = 2$, $\epsilon = 3$,, we obtain modified equal width (MEW) equation of the form

$$U_t + \epsilon U^2 U_x - \mu U_{xxt} = 0$$

The MEW equation with a limited set of boundary and conditions have an analytical solution as the EW equation. Thus,numerical solution of the MEW equation has been calculated by many author. Esen and Kutluay [1] used a linearized numerical scheme based on finite difference method to obtain solitary wave solutains of one- dimensional generalized equal width (MEW) equation.Evans and R.Raslan [2] studied the generalized GEW equation by using collocation method with help quadratic B-splines. .Hamdi and et al [4].derived exact solitary wave solutions of the (GEW) equation. Karakoç and et al. [5] solved numerically MEW equation by giving two different linearization techniques based on collocation finite element method using cubic b-splines approximate functions.Karakoç and Geyikli [6] a numerical solution of the modified equal width wave (MEW) equation solved with subdomain method using sextic B-spline. Raslan K. R ,A.Ramadan, M, Ameen, I. [7] solved modified equal width (MEW) equation by using the finite diffrence method. Keskin and Irk [8] presented a numerical study of the modified equal width (MEW) equation by using the finite diffrence method. Saka [12] proposed quntic b-spline collocation algorithms for numerical solution of the modified equal width (MEW) equation.Wazwaz [16] investigated the MEW equation and two of its variants by asine-cosine ansatz and thetanh method. Zaki [18] used a quintic b- spline collocation method to investigate the motion of asingle solitary wave interaction of two solitary waves and birth of solitons for the MEW equation.

This article is organized as follows: In section 2, strang splitting technique is briefly introduced. In section 3,Equation (2) is split into two sub-equations and sub-equations with initial -boundary conditions (5)-(6) are numerically solved using Crank- Nicolson finite difference approximation and a linearization technique. In section 4, the error norms L_2 and L_∞ are calculated and compared with those available in the literatüre. In section 5, a brief conclision is given.

II. SPLITTING TECHNIQUE

The second order symmetric [9] or more widely known strang splitting technique [15] is obtained and its scheme is given by

$$\frac{dU^*(t)}{dt} = AU^*(t), U^*(t_n) = U_n^0, \quad t \in [t_n, t_{n+\frac{1}{2}}]$$

$$\frac{dU^{**}(t)}{dt} = BU^{**}(t), U^{**}(t_n) = U^*(t_{n+\frac{1}{2}}), \quad t \in [t_n, t_{n+1}]$$

$$\frac{dU^{***}(t)}{dt} = AU^{***}(t), U^{***}(t_{n+\frac{1}{2}}) = U^{**}(t_{n+1}), \quad t \in [t_{n+\frac{1}{2}}, t_{n+1}]$$

Where $t_{n+\frac{1}{2}} = t_n + \frac{\Delta t}{2}$ and the desired solution is obtained by $U(t_{n+1}) = U^{***}(t_{n+1})$. The splitting error of scheme is [3]

$$E = \frac{(\Delta t)^2}{24}\left([A,[B,A]] - 2[B,[A,B]]\right)U(t_n) + O(\Delta t)^3$$

$$[A,B] = AB - BA$$

is defined as the commutator of two operators and the brackets are known as ''Lie bracket ''in the literatüre [17].Thus, the strang splitting has the second order accuracy.

III. APPLCATION OF THE METHOD

In this paper, we have split the one-dimensional modified equal width MEW equation by the following form

$$U_t + \epsilon u^2 U_x - \mu U_{xxt} = 0 \qquad (2)$$

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two sub-equations such that one is linear and the other nonlinear of form

$$U_{t-}\mu U_{xxt} = 0 \qquad (3)$$

and

$$U_t + \epsilon u^2 U_x - \mu U_{xxt} = 0 \qquad (4)$$

with the boundary and initial condition

$$U(a,t) = 0, U(b,t) = 0 \qquad (5)$$
$$U(x,0) = Asech(k(x - x_0)) \qquad (6)$$

using strang splitting techniques by help a linearization technique.
Procedure for Paper Submission

Where $A = \sqrt{\frac{c(p+1)(p+2)}{2\epsilon}}$ is amplitude of dipendent solitary wave $v = {}^{A}/_{h^2}$ is velocity of wave, $k = {}^{1}/_{\mu}$ is number-wave .Further, $c = {}^{1}/_{32}, p = 2, \quad \epsilon = 3, x_0 = 30$. Analitical solution of the MEW equation (2)

$$U(x,t) = A(k[x - x_0 - vt])$$

Let use assume that the solution domain of the problem given by equation (2) with condition boundary-initial (5) and (6) is bounded by region $a \leq x \leq b$. The interval $[a, b]$ is divided into Nequal suninterval such that $a < x_0 < x_1 < \dots < x_N = b$ for $m = 0,1,\dots,N$ at the nodal points x_m by selecting the space step size as $h = \frac{b-a}{N} = (x_{m+1} - x_m)$.

Throught paper, we have used the forward difference approximation for U_t, U_{xxt}, central difference for U_x and the Crank-nicolsan difference approximation for $U^2 U_x$ in equation (2) lead to

$$\frac{U_m^{n+1} - U_m^n}{\Delta t} + \frac{\epsilon}{2}[(U^2 U_x{}^{n+1}) + (U^2 U_x{}^{n})] = \frac{\mu}{\Delta t}[U_{xx}^{n+1} - U_{xx}^n]$$

implementing Rubin and Graves linearization technique [14] to equation (2)

$$(U^2 U_x)^{n+1} = U^{n+1} U^n U_x^n + U^n U^{n+1} U_x^n + U^n U^n U_x^{n+1} - 2U^n U^n U_x^n$$

We obtain

$$\left[\frac{\mu}{\Delta t h^2}\right] U_{m-1}^{n+1} + \left[\frac{2\mu}{\Delta t h^2} + \frac{1}{\Delta t}\right] U_m^{n+1} + [-\frac{\mu}{\Delta t h^2}] U_{m+1}^{n+1}$$
$$= \left[-\frac{\mu}{\Delta t h^2}\right] U_{m-1}^{n} + \left[\frac{2\mu}{\Delta t h^2} + \frac{1}{\Delta t}\right] U_m^{n} + [-\frac{\mu}{\Delta t h^2}] U_{m+1}^{n}$$

and

$$\left[-\frac{\epsilon}{4h}(U_m^n)^2 - \frac{\mu}{\Delta t h^2}\right] U_{m+1}^{n+1} + \left[\frac{1}{\Delta t} + \frac{\epsilon}{2h} U_m^n (U_{m+1}^n - U_{m-1}^n) + \frac{2\mu}{\Delta t h^2}\right] U_m^{n+1} +$$

$$\left[\frac{\epsilon}{4h}(U_m^n)^2 - \frac{\mu}{\Delta t h^2}\right] U_{m+1}^{n+1} = \left[-\frac{\mu}{\Delta t h^2}\right] U_{m-1}^n + \left[\frac{2\mu}{\Delta t h^2} + \frac{1}{\Delta t}\right] U_m^n + +[-\frac{\mu}{\Delta t h^2}] U_{m+1}^n$$

for m=1,2,…,N

IV. NUMERICAL EXAMPLES AND RESULTS

The MEW equation (2) has three invariant conditions to be mass, momentum, and energy respectively [11]

$$I_1 = \int_a^b U dx \cong h \sum_{j=1}^{N} (U_j^n)$$

$$I_2 = \int_a^b U^2 + \mu (U_x)^2 dx \cong h \sum_{j=1}^{N} (U_j^n)^2 + \mu ((U_x)_j^n)$$

$$I_3 = \int_a^b U^4 dx \cong h \sum_{j=1}^{N} (U_j^n)^4$$

to show the performence of the method, error norms L_2 and L_α are calculated

$$L_2 = \|U^{exact} - U_N\|_2 \cong \sqrt{h \sum_{j=0}^{N} \left|U_j^{exact} - (U_N)_j\right|^2}$$

$$L_\infty = \left\|U^{exact} - U_N\right\|_\infty \cong max \left|U^{exact} - (U_N)_j\right|$$

all computations have been done using matlab program.

h=0.1 ,k=0.05 , , tf=20, 0≤x≤80

	present method		Ref.[12]		Ref.[5]	
T	L_2 x 10^3	L_∞ x 10^3	L_2 x 10^3	L_∞ x 10^3	L_2x10^3	L_∞x10^3
5	0.00089	0.00055	0.00007	0.00008	0.0447267	0.0423438
10	0.00177	0.00111	0.00014	0.00016	0.0890842	0.0867198
15	0.00266	0.00168	0.00021	0.00024	0.1327126	0.1316924
20	0.00354	0.00225	0.00027	0.00032	0.1752706	0.1764596

h=0.1 ,k=0.2 , , tf=80, 0≤x≤80

	present method			Ref.[1]		
T	I_1	I_2	I_3	I_1	I_2	I_3
10	4.71234	3.3289	1.4161	4.7124	3.3295	1.4167
20	4.7123	3.3283	1.4155	4.7124	3.3289	1.4161
30	4.7118	3.3256	1.4115	4.7124	3.3284	1.4155
40	4.7122	3.3264	1.4128	4.7124	3.3271	1.4141
50	4.7126	3.3275	1.4145	4.7124	3.3266	1.4143
60	4.7125	3.3269	1.4139	4.7124	3.3262	1.4138
70	4.7123	3.3262	1.4133	4.7124	3.3259	1.4132
80	4.7121	3.3255	1.4127	4.7124	3.3254	1.4127

V. CONCLUSION

In this study, the numerical solutions of the MEW equation with the appropriate initial and boundary conditions have been obtained by the Strang splitting techniques with help a linearized technique .To show the accurary and efficiency of the proposed numerical schemes,error norms L_2 and L_α and I_1, I_2 and I_3 invariants have been calculated. when error norms norms L_2, L_α and invariant values calculated have been compared with study available in the literatüre, we have find some approximate values in different time steps. It is seen that the present method produce some good results.

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