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Mathematical analysis of predator-prey model with two preys and one predator

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Abstract— **The Mathematics of ecology involves the study of populations that interact, thereby affecting each other's growth rates. This paper investigates a special case of such interaction. To simplify the model, the paper make some assumptions that simplify the complication of the model. It specifically investigates the predator-prey model with two preys and one predator where the interaction between the species is analsysed both in two and three dimensions. Three dimensional Lotka Volterra model have been examined where a new assumption is added and the solutions of the model have been categorised into three stages representing the three coordinates system. These stages highlight the behaviour and the relationship of the preys with their individual predators. The relationship between the species is obtained in terms of mathematics equations where the equilibrium points of 3D Lotka Volterra model are obtained. Different interpretations arise from those equations. It have been found among other results that the prey population- x(t) will grows exponentially in the absence of predators-y(t) under the assumption that there is no threat to the prey other than the specific predator. This unbounded growth of x(t) is what biologically expected in the absence of the middle-level population y(t). However, this stage is the best for the prey population because it is free from predation and the z(t) population, which is the second predator, is left without source of food. In general, the population of all the systems become extinct in the absence of x(t). Moreover, the stability analysis is examined by finding the eigenvalues of the Jacobean matrix. The relationship between the species is presented in plots using MAPLE software.**

Index Terms— **Predator-prey system, 3D Lotka-Volterra, Stability, equilibrium points**

I. INTRODUCTION

The analysis of the relationship between two species came into existence in 1921 when Lotka model paired first order nonlinear differential equations showing the effects of a parasite on its prey and VioVolterra equated the relationship between the two species independent to Lotka (Egerton, 2015). In 1939, Leslie contributed population modelling when he starts his studies in biomathematics. Moreover, Richards F.J and Hopkins proposed a more general model formulation in 1959, which leads to the proper formulation of the Lotka-Volterra predator-prey model (Lambert .M. *et al*, 2010). As well as Lotka-Volterra model, Kolmogorov also investigated Mendel's laws and gene spreading where he came up with some hypotheses based on differential equations to explain the predator–prey model for small populations in particular (Livi, R. & Vulpiani,A, 2013).

The aim of the paper is to study the predator-prey models with at least one predator and two preys and to investigate the relationship that exists between these preys and the predator using Lotka-Volterra model and differential equations. To describe the relationship between the two species, study the interactions that occur between predators and their preys and propose a differential viewpoint on describing these interactions. The relationship between the species will be described in mathematical equations where the plots showing those relationships will also be presented.

II. LITERATURE REVIEW

Heberman,R. (2007) developed mathematical models describing the relationship between plants, animals and their environment (Ecology). His model formulation was based on the observed population data of the various species such as human population in a specific area or the population of fish and algae in a lake. In addition, he also highlights that to formulate a population model effectively some factors need to be considered for instance, the population of sharks depends on some factors including the number of fish, harmful bacteria, water temperature, salinity and many more. Moreover, Lambert (Lambert .M. *et al*, 2010) highlighted that population modelling is an application of differential equations in studying and understanding the changes in population of species in an ecosystem. They further stated that ecological population modelling is concerned with the changes in a population as a result of interaction between the organism and the physical environment. It is also concerned with interaction between the organisms and individuals of either the same or different species. Moreover, they further investigated the Lotka-Volterra model also known as predator-prey model as a pair of first order non-linear differential equations that are used to describe the dynamics of a system where two or more species interact.

Moreover, Li (Li, H., 2011) studied the Lotka-Volterra model with two species where he used the Matlab function to approximate numerically the solution of the Lotka-Volterra model. He also analysed the asymptotic behaviour of the solutions. On the other hand, Vaidyanathan (Vaidyanathan, S., 2010) investigated the nonlinear observer design for Lotka-Volterra models where he employed Sundarapandian's theorem (2002) to solve the problem of observer design for Lotka-Volterra model. He studied two species and three species predator-prey models to construct the nonlinear observers for these Lotka-Volterra models. Consequently, Dayar (Dayar, T.,&Mikeev,L.,2010) employed the analysis of Lotka-Volterra model where they emphasised that it is highly challenging because it shows strong fluctuation and hence becomes unstable at the end. Thus, due to this flexibility, traditional numerical techniques are not appropriate enough to solve the model. Therefore, they have suggested the adaptations and combinations of the traditional technique in order to have fast, effective, reliable and accurate results for some parameter ranges of the Lotka-Volterra model.

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Moreover, Korobeinikove A., & Wake G.C (1999) also observed the model of two animals living in the same territory. These two animals are foxes and rabbits where they depend on each other for survival but in different ways and manners. In the absence of the predators (foxes), the preys (rabbits grows indefinitely with a constant growth rate.

Meanwhile, the predator population (foxes) dies out without preys and the rate of change is therefore assumed to be constant. In addition, the model also assumed that increase in the number of predators will lead to decrease in the number of prey.

Analytic solutions of Lotka-Volterra model for sustained chemical oscillation was clearly investigated by (Evan, C. M., & Findley G. L.2000) where they presented a simple transformation of the model from two-dimensional to one-dimensional lotka-Volterra model using a second order non-linear differential equation. Meanwhile, the analysis of the solution leads to the development of the Lotka-Volterra related family of dynamical system.

A. Predator-Prey Model

The predator-prey model is a representation of the interaction between two species of animals that live in the same ecosystem whereby the quantity of each group of these species depends on the birth or death rate and the successful meetings with the individuals of the other species Restrepo, J., & Sánchez, C. (2010).

The predator-prey model is based on the following assumptions;

- Prey population has innumerable quantity of food
- Both predators and prey populations have meetings that are proportional to the product between the two populations.
- Predators depend on prey for survival.

α- Is the birth rate of the prey

β- Death rate of predators

γ- Growth rate of each population

δ – Decrease of each of the populations.

Meanwhile, the Lotka-Volterra model assumes two inputs which adds and subtracts prey or predators in some intervals.

B. Lotka-Volterra Equations

Differential equations have been modelled to study the relationships that exist between predator and prey population by the well-known model of mathematical ecology-Lotkt-Volterra. The Lotka-Volterra model was the first multi-species model that describes interactions between two species living in the same ecosystem, a predator and a prey. The model involves two species and can be transformed into one, two and three dimensional systems. Its idea involves two main equations one which describes how the prey population changes and the second which describes how the predator population changes. The three dimensions (3D) is to cover a three-species food chain where the dynamical competitive systems are more restricted but the analytic and geometric classification becomes more complex.

In 3D Lotka-Volterra system, the new species z is the predators which depend exclusively on the middle-level populations (y) for their survival.

Chauvet *et al* (2002) Investigated modelling the ecosystem of the three dimensional Lotka-Volterra model with the linear three-species food chain where the prey (x) is consumed by a middle level species (y). Meanwhile, this species-y is then prey upon by a top level predator z. However, the examples of such three-species ecosystems include mouse-snake-owl.

Korobeinikove A., & Wake G.C .(1999) also investigated the global properties of the classical three-dimensional Lotka-Volterra with two prey one predator and one prey two predator systems. The investigation was made assuming that there is no competition. In the three-species lotka-Volterra model, they also showed that, except for a pathological case, one species is always driven to elimination thereby making the system behaves like a two-dimensional predator-prey Lotka-Volterra system.

III. Interaction between the Species

The investigation of the relationship between the two species can be obtained by evaluating the integrals of the individual populations using the method of separation of variables.

$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} = -cy + dxy$$

$$\frac{dx}{dt} = x(a - by)$$

Integrating and simplifying yeilds $x(t) = Ce^{(a-by)t}$ where $a = b = c = y = 1$. Therefore $x(t) \to \infty$ as $t \to \infty$ that is the prey population increase with time.

Similarly considering the predator population: $\frac{dy}{dt} = -cy + dxy$ simplifying gives

$$y(t) = K + e^{-(c+dx)t} \quad (3.1)$$

Therefore, (3.1) is the population of one the preys. Similarly, the second prey is given as

$\frac{dx}{dt} = ax - bxy$. If $x' = \frac{dx}{dt}$ and $y' = \frac{dy}{dt}$ Then $x' = ax - bxy,\ y' = -cy + dxy$.

$$\frac{x'}{y'} = \frac{ax - bxy}{-cy + dxy} = \frac{x(a - by)}{y(-c + dx)}$$

Separating the variables gives $\int_{y_0}^{y} \frac{(a-by)}{y}\, dy = \int_{x_0}^{x} \frac{(-c+dx)}{x}\, dx$ and simplifying yield $-cln(x - x_0) + d(x - x_0) = aln(y - y_0) - b(y - y_0)$

. Therefore,

$$R = -c\log\frac{x}{x_0} + d(x - x_0) = a\log\frac{y}{y_0} - b(y - y_0) \quad (3.2)$$

Therefore, solving for x_o and y_0 give $x_o = {}^{c}/_{d}$ and $y_0 = {}^{a}/_{b}$. These solutions are periodic for $x_o, y_0 > 0$. Implies that the equilibrium points are $({}^{c}/_{d}, {}^{a}/_{b}, 0)$.

Now, investigating the initial condition for $x_o, y_0 > 0$, in the absence of y (t). That is the predator population is zero $(y = 0)$, $\frac{dx}{dt} = ax$. Evaluating yields $x(t) = ke^{at}$

However, there exist three solutions at this initial condition and these can be simplified as

- $(x, y) = (0,0)$ meaning they are all zeros on both axis
- $(x - \text{axis is not zero})\ \text{and} y = 0$

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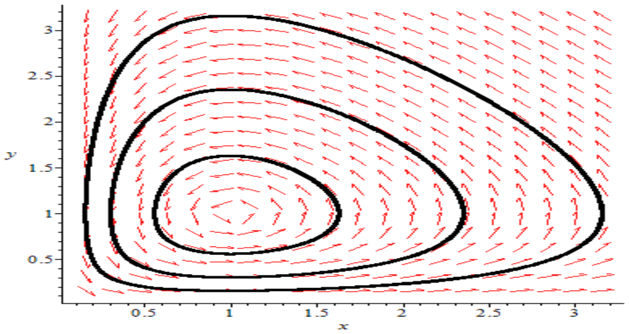
- $(y-\text{axis is not zero})\text{and } x=0$

The phase plot of the 2D predator model is given by the graph below;



Figurer3.1: Phase plot of 2D predator-prey system for initial conditions

$(x(0),y(0)) = (.3,.3),(.45,.45),(.67,.67)$, for $a = b = c = e = 3$

Meanwhile, the relationship between the two species is represented in the figure below

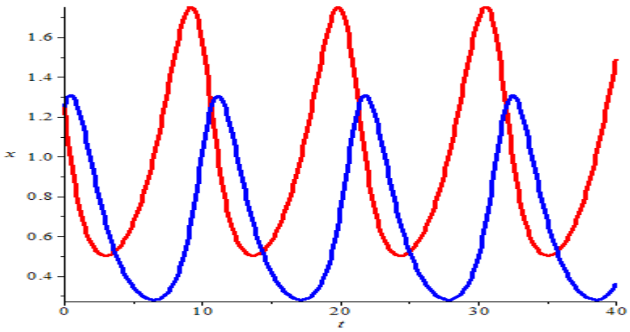


Figure 3.2: 2D plot of Lotka-Volterra model

From the graph above, it is indicated that there is an interaction between the two species. Therefore, the relationship that exist between x and y is directly proportional. Thus increase in the prey population will also increase the predator population and vice-versa.

However, the function R obtained above which is differentiable with respect to t indicates that the solution of the 2D Lotak-volterra system lies within the coordinates of the plane of the function.

IV. Three Dimensional (3D) Lotka-Volterra

The three dimensional (3D) Lotka-Volterra model can be written as;

$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -cy + dxy - eyz \\ \frac{dz}{dt} = -fz + gyz \end{cases} \qquad (4.1)$$

where

a Is the growth rate of the prey in the absence of predators

b Represent the effects of the predation on the preys

c Referred to the natural death rate of the predator in the absence of prey

e denotes the effect of predation on species y by species z

f denotes the natural death rate of the predator z in the absence of prey,

g is the efficiency and propagation rate of the predator z in the presence of prey.

It is obvious that the population can never be negative, we therefore restrict the variables, $x, y, z \geq 0$. Similarly, a,b,c,d,e,f,g> 0.

In addition to the assumptions under the 2D system, the 3D system has the following additional assumptions;

- In the absence of the middle-level population (y)-which is the prey to the new species (z), the population of the new species decreases exponentially.
- The predator population (y) decreases relative to the new species (z). Thus, y decreases on meeting with the members of the new populations due to predation.
- The new population (z) increases relative to the original predator population (y) which is its preys.
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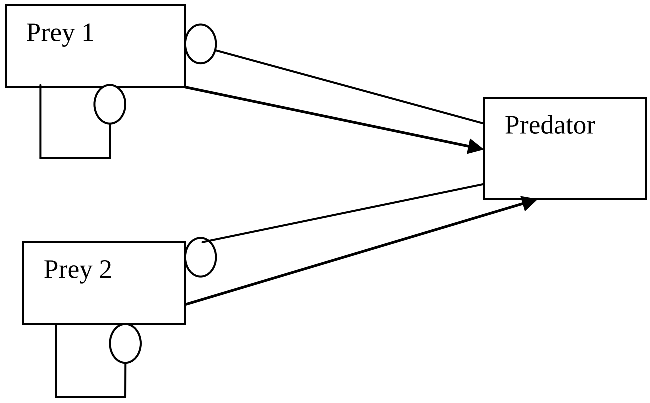

Figure 3.3: Graph of predator-prey model with 2 preys and one predator

Figure 3.3 above presents the preys and the predator.

Table 3.1: Interaction between preys and predator

Effect on	Prey 1	Prey 2	Predator
Prey 1	+	-	+
Prey 2	-	+	+
Predator	-	-	+

Table 3.1 above shows the relationship that exists between the preys and the predator. It also explains the effects of each prey on the individual preys and predators. It is clearly indicated from the table that the relationship between the same species is positive whereas, it is negative otherwise.

It is very important to understand the basic assumptions under the 3D Lotka-Volterra as well; the 3D system has the following additional assumptions

i. In the absence of the middle-level population (y)-which is the prey to the new species (z), the population of the new species decreases exponentially.

ii. The predator population (y) decreases relative to the new species (z). Thus, y decreases on meeting with the members of the new populations due to predation.

iii. The new population (z) increases relative to the original predator population (y) which is its preys.
iv.

From equation (2.1) above, $-fz$ term models the first assumption, $-eyz$ term models the second assumption and Gzy term models the third assumption.
However, to get the equilibrium points, we equate (4.1) to zero and simplify to get the equilibrium points. however, in the absence of the top level-predator (Z),that $z = 0$, the 3D model reduces to the classical Lotka-Volterra model with the following equilibrium points.

$$x(a - by) = 0$$
$$y(-c + dx) = 0$$
$$z(-f + gy) = 0$$

where $x > 0, y > 0$ and $z > 0$.

Therefore, in the absence of the top-level predator (z), the equilibrium points are;
$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c/d \\ a/b \\ 0 \end{pmatrix}$ and $y = f/g$. Therefore, y can be written as either $y = a/b$ or $y = f/g$

A. Solutions of the 3D system

The solutions of this system can be categorised into three stages representing the three coordinates system .These stages highlight the behaviour and the relationship of the preys with their individual predators. These categories are shown below

i. Category I (Plane $x = 0$) In the absence of the first prey, (4.1) is reduced to the classical Lotka-Volterra. The change of population of y with time is then represented as $\frac{dy}{dt} = -cy - eyz$. Separating of variables and integrating yield

$$y(t) = Ke^{-(c+ez)t} \qquad (4.1.1)$$

From (4.2) it can be clearly seen that $if\ k = c = e = z = 1,\ y(t) \to 0$ as $t \to \infty$. Meaning that as time goes on, the population of y will exponentially reduce to zero which will definitely cause the population of x also to decrease exponentially, i.e $z(t) \to 0$ as $t \to \infty$.

Similarly, $\frac{dz}{dt} = -fz + gyz$. Separating the variables yields $\frac{dz}{z(-f+gy)} = \frac{dy}{y(-c-ez)}$

We now integrate $\int \frac{dz}{z(-f+gy)} = \int \frac{dy}{y(-c-ez)}$

$$C = \text{gy} + c\ln z + \text{ez} - f\ln y. \qquad (4.1.2)$$

Similarly from (4.3) it can be deduce that in the absence of the first prey population ($x = 0$), the middle –level population (y) decreases to zero because it has no source of food for feeding which causes the z-population also to vanish as time increases as shown in the graph below

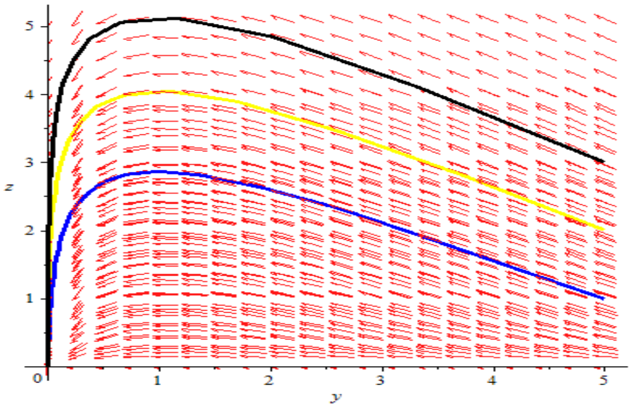

Figure4.1: phase plot for yz- plane for the 3D lotka-volterra.

ii. Second stage (Plane $y = 0$): meaning that the population of second preys, the system reduced to

$$\begin{cases} \frac{dx}{dt} = ax \\ \frac{dy}{dt} = 0 \\ \frac{dz}{dt} = -fz \end{cases} \qquad (4.1.3)$$

Therefore, from $\frac{dz}{dt} = -fz$ separating the variables and integrating gives

$z(t) = ke^{-ft}$. If $k = f = 1 \Rightarrow z(t) = e^{-t}z(t) \to 0$ as $t \to \infty$.
Similarly, $\frac{dx}{dt} = ax$ gives $x(t) = at.$ Therefore, $x(t) = ke^{at}$. Meanwhile from $\frac{dz}{dt} = -fz$. Equating $\frac{dx}{ax}$ and $\frac{dz}{-fz}$ and integrating we have $Z = CX^{(-\frac{f}{a})}$.

The graphs representing the relationship between the species when y=0, is shown in the figure below.

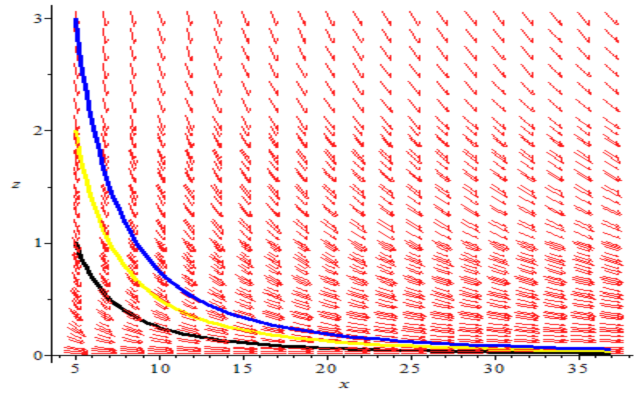

Figure 4.2: Plot of the relationship between species in the absence of the second prey.

From Figure 4.2 above, it is clearly seen that the line representing the relationship are parallel therefore, the integral exist. The contour plot also plays a vital role in explaining the relationship of any species in an ecosystem. Therefore, the contour plot for the system when y=0 is shown in Figure 4.3 below

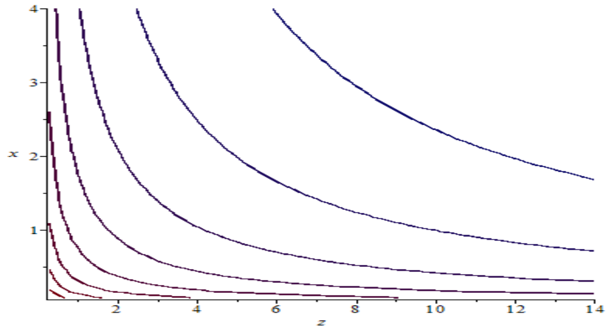

Figure 4.3: Graph of the contour plots of XZ-plane (y=0)

Meaning that, the prey population $x(t)$ will grows exponentially in the absence of $y(t)$.

This unbounded growth of $x(t)$ is what biologically expected in the absence of the middle-level population $y(t)$. However, this stage is the best for the prey population because it is free from predation and the $z(t)$ population is left without source of food. In general, the population of all the systems become extinct in the absence of $x(t)$.

iii. Third stage (Plane z=0): When $z(t) = 0$, the system is represented as

$$\frac{dx}{dt} = ax - bxy$$
$$\frac{dy}{dt} = -cy + dxy$$
$$\frac{dz}{dt} = 0$$

If $x' = \frac{dx}{dt}$ and $y' = \frac{dy}{dt}$ Then $x' = ax - bxy$ and $y' = -cy + dxy$

To obtain the solution we divide the two equations $\frac{x'}{y'} = \frac{ax-bxy}{-cy+dxy} = \frac{x(a-by)}{y(-c+dx)}$

Separating the variables and integrating gives $= -c\log\frac{x}{x_0} + d(x - x_0) = a\log\frac{y}{y_0} - b(y - y_0)$

Therefore, solving for x_o and y_0 give $x_o = {}^{c}/_{d}$ and $y_0 = {}^{a}/_{b}$. These solutions are periodic for $x_o, y_0 > 0$. This implies that the equilibrium points are $({}^{c}/_{d}, {}^{a}/_{b}, 0)$

Now, investigating the initial condition for $x_o, y_0 > 0$, in the absence of y(t). That is the predator population is zero $(y = 0)$ $\frac{dx}{dt} = ax$. Simplifying gives $x(t) = ke^{at}$. If $= 0$, $\frac{dy}{dt} = -cy$, $\ln y = ke^{-ct}$.

However, there exist three solutions at this initial condition and these can be simplified as

- $(x, y) = (0,0)$ meaning they are all zeros on both axis
- $(x - axis\ is\ not\ zero)\ and\ y = 0$
- $(y - axis\ is\ not\ zero) and\ x = 0$

B. *Analysis of coordinates*

To analyse the coordinates effectively, it is highly important to observe the behaviour of the system in all the planes thereby investigating the interactions and effects of each plane on the individual species.

From the plane $x = 0$ (the population of mouse is zero) the systems then reduced to the classical Lotka-Volterra and as we verified in the above integrals, if the $x = 0$, the population of $y(t)$ dies out because of the absence of its prey. Consequently, the populations of $z(t)$ is therefore left without the source of food and hence will also go extinct. Thus, without $x(t)$, the two (2) predator populations will both die out.

However, in the plane $y = 0$, the prey population grows exponentially due to the absence of predation and the population of $z(t)$ is left without food resource and therefore will die out. Meanwhile, opposite phenomena is applied to prey population $x(t)$ because there is no interaction between $x(t)$ and $Z(t)$.

Whereas, in the plane $z(t) = 0$, the system is totally reduce to the classical Lotka-Volterra where the population of predator increases with prey and decreases in the absence of the prey.

V. Equilibrium State of 3D Lotka-Volterra

In the analysis of predator-prey model or differential equations in general, it is very important to consider the solutions that do not change with time. These solutions are referred to as the equilibrium points. Thus, the equilibrium steady-state is obtained when the system is equated to zero. Thus, for the 3D Lotka-Volterra system, there are two (2) equilibria points located at (0, 0, 0) and $({}^{c}/_{d}, {}^{a}/_{b}, 0)$. These points stands for x, y and z respectively. These equilibrium points can be asymptotic and liner.

If the 3D Lotka-Volterra system is liner then the stability of the equilibrium points (x_0, y_0, z_0) can be obtained by stabilising these points in a linearized system represented below $\begin{cases} \frac{dx}{dt} = \frac{df}{dx} + \frac{df}{dy} + \frac{df}{dz} \\ \frac{dy}{dt} = \frac{dg}{dx} + \frac{dg}{dy} + \frac{dg}{dz} \\ \frac{dz}{dt} = \frac{dh}{dx} + \frac{dh}{dy} + \frac{dh}{dz} \end{cases}$ where the functions f, g and h are defined as follows

$$\begin{cases} \frac{dx}{dt} = f(x,y,z) = ax - bxy \\ \frac{dy}{dt} = g(x,y,z) = -cy + bxy - eyz \\ \frac{dz}{dt} = h(x,y,z) = -fz + gyz \end{cases}$$

The behaviour of the linearized system can now be evaluated by determining the eigenvalues of the Jacobin matrix given by

$$J(x,y,z) = \begin{bmatrix} \frac{df}{dx} & \frac{df}{dy} & \frac{df}{dz} \\ \frac{dg}{dx} & \frac{dg}{dy} & \frac{dg}{dz} \\ \frac{dh}{dx} & \frac{dh}{dy} & \frac{dh}{dz} \end{bmatrix}$$

The Jacobean matrix given above presents the linearized system of the model. However, by substituting the functions (f, g and h) into the Jacobean and simplifying we get

$$J(x,y,z) = \begin{bmatrix} a - by & -bx & 0 \\ yd & -c + dx - ez & -ey \\ 0 & gz & -f + gy \end{bmatrix}$$

A. *Stability Analysis*

The stability analysis of the system will be explained by determining the eigenvalues of the above Jacobean matrix. If the eigenvalues have negative real part, the system is said to be asymptotically stable, whereas, if the eigenvalues of the Jacobean have positive real part; the system is said to be asymptotically unstable. Therefore, different techniques will be applied in determining the eigenvalues of the Jacobean.

To analyse the stability, the equilibrium points (0, 0, 0) and ($^c/_d$, $^a/_b$, 0) need to be substituted into the Jacobean to investigate whether the eigenvalues have negative or positive real parts. The principle of linearization is employed to investigate the stabilising of these points Wuhaib, S. A. &Abu Hasan, Y. (2012). Therefore, substituting (0, 0, 0) gives

$$J(0,0,0) = \begin{bmatrix} a & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & -f \end{bmatrix} \qquad (5.1.1)$$

The eigenvalues (λ_i) of the resulting matrix are

$$\lambda_i = \begin{bmatrix} a \\ -c \\ -f \end{bmatrix}$$

It is clearly seen that, only one of the eigenvalues is positive and real with the other two (2) negatives but real indicating that the equilibrium points (0, 0, 0) is not stable.

However, the instability of this point is obvious because of the assumptions of the unbounded growth of the prey population $x(t)$ in the absence of predators. In addition, this point is of little interest because in reality, there is no population that exist in (0, 0, 0) indicating that all the three (3) species does not exist entirely.

Similarly, substituting the second equilibrium point ($^c/_d$, $^a/_b$, 0) into the Jacobean gives

$$J\left(\frac{c}{d},\frac{a}{b},0\right) = \begin{bmatrix} 0 & -{}^{bc}/_{d} & 0 \\ \frac{ad}{b} & -c & -{}^{ea}/_{b} \\ 0 & 0 & -f + \frac{ga}{b} \end{bmatrix} \qquad (5.1.2)$$

Finding the eigenvalues yields ($\frac{ga}{b}$-f) and$(\pm i\sqrt{ac})$. The real part of the eigenvalues ($\frac{ga}{b}$-f) may be positive, negative or even zero (0) depending on the value of the constant. If the eigenvalue is negative, then the point ($^c/_d$, $^a/_b$, 0) is stable otherwise it is unstable. The plot below represents the behaviour of the species with time.

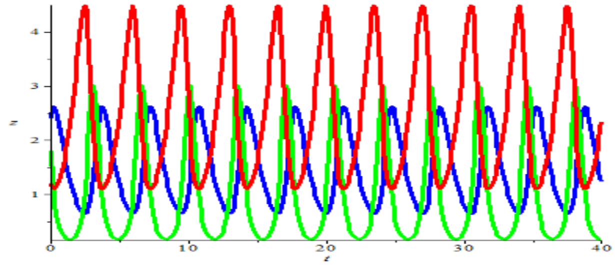

Figure 5.1: Plots of the three populations over time.

In the graph above, the prey population (x) is represented by RED line, the middle level predator (y) is represented in Green and the top level predator population (z) is represented in blue. However, the initial conditions are $x_0 = 1.2, y_o = 1.8, z_o = 2.4$ and all the constants are equal to 1. It is also indicated that each of these populations reached their maximum points and starts dropping with the prey population having the highest peak level followed by y and finally with the z-population. Meanwhile, when the middle-level population (y) is zero, the plot reduces thereby showing only the relationship between x and z as shown in the figure below

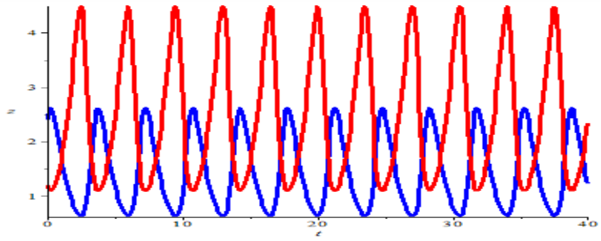

Figure 5.2 Plots of the two populations (x and z) over time

The graph showed that there is no interaction between the two (2) populations over time and hence the relationship between them is inversely proportional unlike the case for absence of z-population where there is interaction between x and y. Therefore, the relationship between the two populations is directly proportional as indicated in the graph below

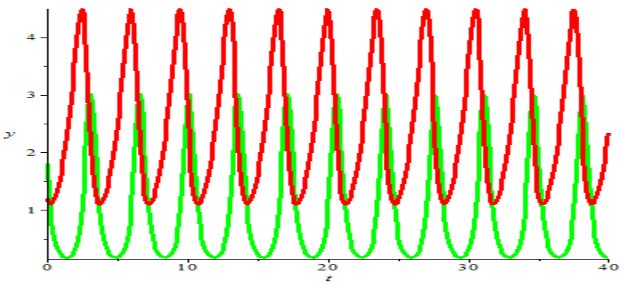

Figure 5.3: Plots of the two populations (x and y) over time

VI. Conclusion

The interaction between two or more spices living in the same ecosystem where the interaction between one prey and predator was examined. The study is further extend to the concept to three Dimensional system of the Lotka-Volterra where separation of variables was employed to evaluate the relationship between the species in the absence of the others. To clearly understand the interaction, each of the three species population is set to zero and the resulting relationship is then investigated using integration using technique of

separation of variables. Furthermore, MAPLE codes were implemented to analyse these interactions graphically.
In addition, the stability analysis of the system was also observed by finding the equilibrium points and the eigenvalues of the corresponding Jacobean matrices.

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