

# Firefly Optimization Design And Simulation Of A Single-Axis Helmholtz Coils For Spacecraft Components Testing

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**Abstract**— Geomagnetic field affects the attitude of spacecraft. Thus, in order to prevent magnetic interference in spacecraft, there is need to measure the magnetic moment of spacecraft, eliminate the residual magnetism, and verify the magnetic characteristics of the subsystems and components for attitude control. This paper presents the design of a single-axis Helmholtz coils using firefly, a multi-objective metaheuristic optimization algorithm. The ultimate goal of optimization design in this research is to investigate the optimum coil geometry that will produce the maximum homogeneous magnetic field inside the coils. The modified Firefly Optimization Algorithm (FOA) was written using MATLAB program to obtain the optimum parameters for the Helmholtz coils. The parameters obtained were used to simulate magnetic field inside the coils. Simulations were carried out on three different coils geometries which served to investigate the coil geometry capable of producing maximum homogenous magnetic field inside the coils. The three coils geometries investigated are two rectangular and a square shapes. The first of the three designs is a square geometry where side “a = b”, the second is a rectangular geometry, where side “a = 2b”, while the third is also a rectangular geometry with side “a = 1/2 b”. It was observed that optimum magnetic field was achieved when the geometry of Helmholtz coils is a square shape. We can therefore conclude from the results obtained that the magnetic field is very homogeneous as long as the shape of the Helmholtz coils is approximately a square.

**Index Terms**— Helmholtz coil design optimization; Firefly; Spacecraft, Short circuit impedance

## I. INTRODUCTION

Helmholtz coils is a magnetic field generating device with a parallel pair of two similar circular coils, spaced one radius apart with the two magnetic field generating coils connected in series, such that electric current passes in the same direction in each coil (Abbott, 2015). In Helmholtz coils, magnetic fields are created to null the earth’s magnetic field, calibrate magnetic sensors, and used for other experiments in which a controllable amount of uniform magnetic field is required (Abbott, 2015). Helmholtz coils was constructed for the characterization of spacecraft magnetic field in the 1960’s at the Goddard Space Flight Center for the purpose of simulating geomagnetic and interplanetary magnetic field environments (Vernier *et al*, 2005). The schematic describing the configuration of Helmholtz coils is shown in Figure 1.

The basic principle of Helmholtz coil is that, the coils are carefully arranged to produce uniform magnetic field at the center (Abbott, 2015; Daron *et al.*, 2015). This arrangement

of coil was invented by German physicist, Hermann von Helmholtz, over a century ago (Bhatt *et al*, 2010).

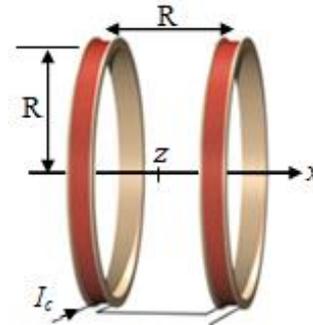


Figure 1: The Schematic Diagram Describing Helmholtz Coils Arrangement (Bhatt *et al*, 2010)

The intensity of magnetic field inside the Helmholtz coils is directly proportional to the number of turns and the current through the coils. As shown in Figure 1, the magnetic field at a point along the axis of a circular coil is expressed as (Abbott, 2015).

$$B(z) = N \cdot \frac{\mu_0 \cdot I_C}{2} \cdot \frac{r^2}{(z^2 + r^2)^{\frac{3}{2}}} \quad (1)$$

Where:  $z$  is the distance from the centre of each coil to the axial field point,  $N$  is the number of turns,  $r$  is the radius of each turn of a coil,  $I_C$  is the coil current, and  $\mu_0$  is the permeability of air. The field uniformity along the axial distance  $z$  can be iteratively computed by using the closed form solution for the magnetic flux density of two identical coils as (Reitz, 1967):

$$\text{Minimize } B(z) = 6.284 \times 10^{-7} \left( \frac{1}{(z^2 + r^2)^{1.5}} + \frac{1}{((x-z)^2 + r^2)^{1.5}} \right) \cdot N \cdot I_c \cdot r^2 \quad (2)$$

Where:  $N$  is the number of turns,  $r$  is the radius of each coil,  $I_C$  is the coil current in Amperes,  $B(z)$  is the magnetic flux density in Tesla,  $x$  is the distance between the coils in meters, and  $z$  is the distance along common axis in meters.

The magnetic field of two circular coils separated by the radius of the coil is relatively uniform throughout the cylindrical volume whose radius is approximately half of the coil radius (Edward and Theodore, 1998). Although, magnetic field is a vector, but the direction of the axial points is always determined by the rotational symmetry of the coil along the axis. For two coils placed a distance  $R$  (equal to the radius of each coil) apart, and both coils connected in series while carrying a current  $I_c$ , the magnetic field strength  $B$  can

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be obtained at the midpoint of the two coils according to the following equation (Bhatt *et al.*, 2010):

$$B = \frac{8.99 \times 10^{-7} \cdot N \cdot I_C}{r} \quad (3)$$

Where:  $N$  is the number of turns,  $r$  is the radius of each coil, and  $I_C$  is the coil current.

Each coil used in Helmholtz coils produces uniform magnetic field parallel to the axis of the coils (Bhatt *et al.*, 2010), and the two coils form a system such that a controllable amount of uniform magnetic field can be accomplished by controlling the applied electric current (Abbott, 2015). The uniform field within the coils is as a result of the summation of fields parallel to the coils axis and the difference of vertical component fields (Abbott, 2015). Helmholtz coils needs a large area with uniform magnetic field so as to accommodate a large spacecraft or UAV (Butta *et al.*, 2010). The advantage of having a large area is to ensure easy access of the specimen under testing (Butta *et al.*, 2010). However, a laboratory design and characterization of Helmholtz coil is a better choice when the commercially available Helmholtz coils with larger area are very expensive. The power required for driving the coils must be taken into consideration when calculating the diameter of wire to be used, so as to ensure optimal current needed to generate the required magnetic field (Bhatt *et al.*, 2010) and also to avoid overheating of the coils.

When a coil is driven by current, the wire of the coil becomes hot as a result of Joule-heating. In order to ensure that the wire used for the coils does not melt under the applied current, there is need to investigate the maximum current as a function of the increase in temperature per hour. This is calculated as:

$$I_{max} = \frac{\pi \cdot d_w^2}{4} \cdot \left( \sqrt{\frac{\Delta T \cdot c \cdot \rho_m}{\rho_r}} \right) \quad (4)$$

Where:  $c$  is the specific heat capacity of the wire (J/kg.K);  $T$  is the increase in temperature per hour (K/hr);  $I$  is the applied coil current (A);  $\rho_m$  is wire density (kg/m<sup>3</sup>);  $\rho_r$  is the wire resistivity (Ω.m); and  $d_w$  is diameter of the wire (mm). It is assumed in this research that there is no heat exchange and that the heating is uniform.

In this research, optimization of the entire Helmholtz coils was carried out based on the analytical model equations, which includes the wire diameter, coil radius, and number of winding turns. Analytical model equations were numerically solved by using modifying multi-objective FOA to simultaneously finding the optimum dimensions and geometry of the coils subject to a set of parameters (number of winding turns) as the optimization constraints.

## II. MATERIALS AND METHODS

The modified FOA was written in MATLAB program to perform the optimization design of the proposed Helmholtz coils. The coils optimization problem involves few number of design variables (number of coil turns, wire diameter, and coils radius). Most of these variables have influence either on the objective function (magnetic field) or on the specific

constraints (geometric dimensions). The goal of the optimization in this research is to simultaneously optimize the dimensions and geometry of the coils that maximize the homogeneous magnetic field with minimum excitation current.

### A. Firefly Optimization Design of the Helmholtz Coils

The algorithm starts by placing the fireflies in random locations (Yang, 2013). The location of a firefly corresponds to the values of the parameters (geometry and dimensions of the coils) for the objective function (magnetic field) to be solved (Yang, 2013). Then from each firefly's newly acquired position, the objective function is evaluated, and the firefly's light intensity is set as the inverse evaluation (Yang, 2013). The inverse is used since the goal is to maximize the objective function. Thus a lower function evaluation will result in a higher light intensity (Yang, 2013). In order to ease the problem of high dimensionality, four design variables were identified and are presented in Table 1. The geometric constraints and operating limits are shown in Table 2.

Table 1: Helmholtz Coils Design Variables

Variables	Range	Unit
Coil width turns	5 – 100	-
Coil height turns	5 – 100	-
Wire diameter	0.135 – 2.0	mm
Coil radius	0.05 – 0.15	m

Table 2: Helmholtz Coils Design Constraints

Constraints	Range	Unit
Total winding turns	≤ 1000	-
Magnetic field	≤ 5.00	mT

The design variables are written as:

$$X = [x_1, x_2, \dots, x_4] = \begin{bmatrix} \text{Coil width turns} \\ \text{Coil height turns} \\ \text{Wire diameter} \\ \text{Coil radius} \end{bmatrix}^T \quad (5)$$

The geometric constraints and operating limits were represented as:

$$g(x) \leq 0 \Leftrightarrow \begin{cases} \text{Total winding turns} \\ \text{Total magnetic field} \end{cases} \quad (6)$$

The optimization design problem of the Helmholtz coils was formulated by defining the homogeneous magnetic field inside the coils as the objective function:

$$\text{Maximize } B(z) = \frac{8.99 \times 10^{-7} \cdot N \cdot I_C}{r} \quad (7)$$

The multiple objectives in this research using modified FOA, a nature-inspired metaheuristic were implemented based on the following steps (Yang, 2013):

1. Initialize the number of fireflies,  $n$ , biggest attraction  $\beta_0$ , absorption coefficient of light intensity  $\gamma$ , step size factor  $\alpha$ , and maximum number of iterations or generations  $t_{max}$ .
2. Initialize the positions of fireflies randomly, namely initializing design variables of the coils parameters (coil width turns, coil height turns, wire diameter, coil radius, coil current), the values of objective function (homogeneous magnetic field) of fireflies are set as their maximum brightness of fluorescence  $I_0$ .
3. Calculate relative brightness and attractiveness of fireflies, which belong to the population. The direction of movement depends on the relative brightness of fireflies. Here (Yang, 2013):

$$I = I_0 \times e^{-\gamma r_{ij}} \quad (8)$$

$$\beta = \beta_0 \times e^{-\gamma r_{ij}} \quad (9)$$

where  $I_0$  is the maximum fluorescence brightness of the firefly, namely the fluorescence brightness itself ( $r = 0$ ), which depends on the value of the objective function.  $\beta_0$  is the maximum attractiveness, namely the attractiveness of the light source ( $r = 0$ ).  $\gamma$  is the absorption coefficient of the light intensity. The fluorescence will gradually weaken according to the increasing distance and the absorption of media. The absorption coefficient of light intensity is set to reflect this feature.  $r_{ij}$  is the spatial distance between firefly  $i$  and  $j$ .

4. Update the spatial positions of fireflies. Random perturbations are injected to the firefly with the best position. The updated equation is:

$$x_i = x_i + \beta \times (x_j - x_i) + \alpha \times (rand - 0.5) \quad (10)$$

where  $x_i, x_j$  represent the spatial positions of firefly  $i$  and  $j$ , respectively.  $\alpha$  is the step size factor.  $rand$  is random factor distributed uniformly in  $[0,1]$ .

5. Recalculate the brightness of fireflies according to the updated positions.
6. Return to Step 3 until the search precision is met or the maximum number of generations is achieved.

### III. RESULTS AND DISCUSSION

The need to investigate the effect of geometrical winding on the performance of Helmholtz coils motivated us to simultaneously finding the dimensions of the coil winding turns (number of coil layers and coil width), coil wire diameter, and coil radius (design variables). Hence, we have optimized the design of the Helmholtz coils using modified FOA to obtain the optimal coils parameters. The modified FOA was written in MatLab environment and the results were obtained by running the developed program on a 1.50 GHz Intel® core™ Duo CPU Windows 7 Ultimate 32-bit Personal Computer.

#### A. Helmholtz Coils Optimization Design Results

When all the contributions (design variables and constraints) have been added up in the modified FOA, the program returns the total magnetic field, which is the main objective function of the optimization in this research. In the MatLab-program, the formula for the magnetic field (Equation 7) at the

midpoint of the two circular coils of Helmholtz coils was employed. This formula enables us to obtain the total magnetic field (contribution of the two coils) in the middle of the two coils, radius of the coil form, the total number of turns in the coil (number of turns that form the coil width and layers), size (diameter) of the wire required. Table 3 shows the trend of the design variables when displayed on MatLab command window. The oscillatory trend in the iterative process observed was as a result of the randomization parameter of the modified FOA (Yang, 2013).

Table 3: Firefly Optimization Results for the Helmholtz Coils Parameters

Iterations	Number of Turns	Number of Layers	Wire size (mm)	Coil Radius (m)	Magnetic Flux Density (mT)
1	21.9153	21.6101	0.556095	0.0906376	2.7935
2	19.8990	19.7701	0.571099	0.0800906	2.7697
3	19.8990	19.7701	0.571099	0.0800906	2.7697
4	19.8990	19.7701	0.571099	0.0800906	2.7697
5	19.1452	19.0503	0.535038	0.0808358	2.2233
6	19.6413	19.3517	0.582357	0.1000000	2.2285
7	19.6821	19.2712	0.580761	0.0996980	2.2184
8	19.2667	19.1120	0.531642	0.0812005	2.2159
9	19.6556	19.2650	0.578865	0.0999984	2.1937
10	19.6556	19.2650	0.578865	0.0999984	2.1937
11	19.6525	19.2639	0.578387	0.0999255	2.1912
12	19.6525	19.2639	0.578387	0.0999255	2.1912
13	19.6451	19.2638	0.578299	0.0999732	2.1886
14	19.6479	19.2684	0.578227	0.1000000	2.1883
15	19.6506	19.2684	0.578192	0.1000000	2.1881
16	19.6477	19.2681	0.578164	0.1000000	2.1878
17	19.6477	19.2620	0.578080	0.0999428	2.1874
18	19.6477	19.2620	0.578080	0.0999428	2.1874
19	19.6445	19.2619	0.578088	0.0999449	2.1874
20	19.6444	19.2619	0.578074	0.0999434	2.1873

As shown in Table 3 above, in an effort to optimize the magnetic field inside the Helmholtz coils, consequently, there was no significant increase in the dimensions and geometry of the coils between the runs. Table 4 shows the summary of the design parameters obtained for the coils design.

Table 4: Summary of FOA Optimum Parameters for the Helmholtz Coils Design.

Input variables				Output Variable
Coil width turns	Coil height turns	Wire diameter (mm)	Coil radius (mm)	Magnetic field (mT)
19	19	0.578	100	2.187

As seen in Table 4, the predicted optimal magnetic field of the Helmholtz coils is 2.187mT, with total number of each coil winding as 361 turns. Based on the optimization results obtained, the maximum current required to generate the predicted maximum magnetic field in the coils is found by using Equation (4). In order to prevent the coils from melting, increase in temperature was chosen to be 100 K/hr ( $2.77 \times 10^{-2}$  K/s) while other relevant parameters of the coils wire considered are shown in Table 5.

Table 5: The Electrical Properties of the Helmholtz Coils Wire.

Wire Density, $\rho_m$ (Kg/m <sup>3</sup> )	Specific Heat Capacity, $c$ (J/Kg.K)	Wire diameter, $d_w$ (mm)	Resistivity, $\rho_r$ ( $\Omega$ .m)
8960	385	0.578	$1.6 \times 10^{-8}$

The diameter of wire obtained from modified FOA enables us to calculate the required current for driven the coils in series. By using the values presented in Table 5, the optimum maximum current required by the coils is calculated as:

$$I_{max} = \frac{3.142 \cdot (0.578 \times 10^{-3})^2}{4} \cdot \left( \frac{2.77 \times 10^{-2} \cdot 385 \cdot 8960}{1.6 \times 10^{-8}} \right)$$

Hence, maximum current needed by the coils is 0.642A.

*B. Simulation of Magnetic Field inside the Helmholtz Coils*

In order to investigate the effect of geometric dimensions of the coils on the performance of the proposed Helmholtz coils, the optimization results were validated by using MatLab program to test three different geometrical designs shown in Figure 2 (with “a” as the coil width and “b” as the coil height) and compare the magnetic field of the three geometries. The first design is a square geometry, where side “a = b”. The second design is a rectangular geometry with side “b = 2a”, while the third design is a rectangular geometry, where side “a = 2b”.

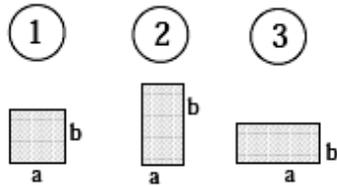


Figure 2: Three Geometries Considered for the Proposed Helmholtz Coils Design.

The magnetic fields arising from the modified FOA results for the square geometry and other two geometries calculated with MatLab Program are plotted in Figure 3 to Figure 5. The three graphs correspond to different relations between “a” (coil width) and “b” (coil height).

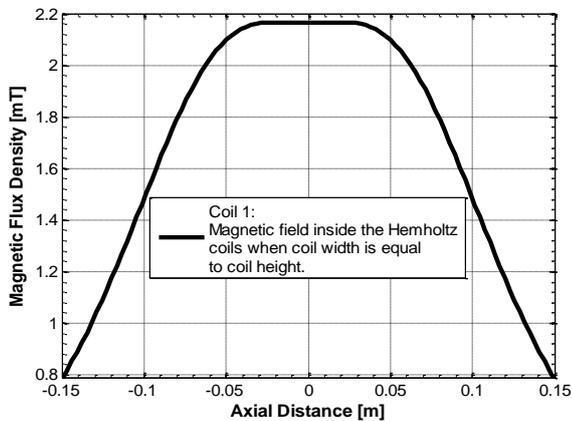


Figure 3: Axial Magnetic Field of Helmholtz Coils when a = b.

As shown in Figure 3, the magnetic field obtained in the middle (within 0.06 m axial distance) of the Helmholtz coils is

2.16mT and is uniformly distributed (axially) between -0.03m (Left coil) and +0.03m (Right coil). This implies that this square geometry produced uniform magnetic field that covers a range of about 60% of the predicted coil radius (0.1m) and 99% of the predicted field (2.18mT) from FOA design.

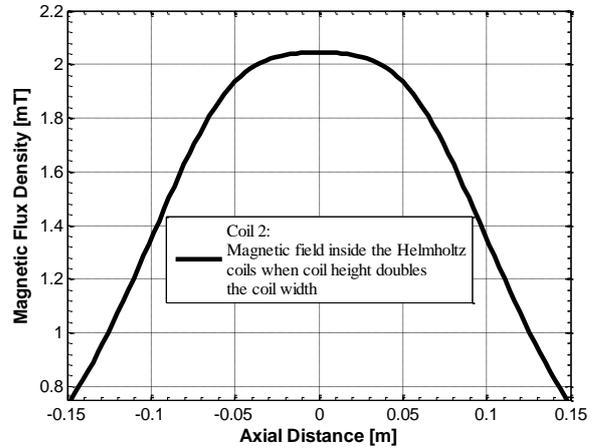


Figure 4: Axial Magnetic Field of Helmholtz Coils when b = 2a.

As seen in Figure 4, the reduction in the coil width into half while increasing the coil height resulted into 0.024m axial uniform magnetic field of about 2.04mT, which is lower than the predicted value (2.18mT) from FOA design. This implies that the uniform magnetic field inside the Helmholtz coil covers 24% of the axial distance between the coils radius of 0.1 and generates about 93.5% of the predicted field (2.18mT) from FOA design. This coil geometry is seen to exhibit minimum magnetic field inside the coils.

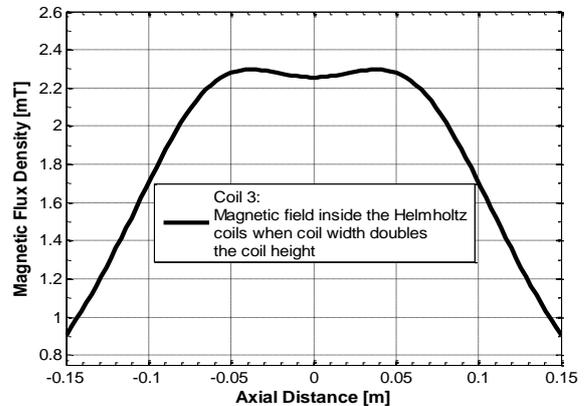


Figure 5: Axial Magnetic Field of Helmholtz Coils when a = 2b.

Figure 5 shows the third coil geometry when the coil width doubles the coil height. This coil geometry is seen to exhibit maximum magnetic field inside the coils as compared to that of Figure 3 and Figure 4. However, the magnetic field is not uniform, as the field is maximum (2.3mT) at -0.04m distance, which gradually decreases towards the midpoint of the two coils and later increases and becomes maximum (2.3mT) at +0.04.

IV. CONCLUSION

In this research, the design of a single-axis Helmholtz coil has been achieved by using firefly optimization algorithm, a multi-objective metaheuristic optimization algorithm. The ultimate goal of optimization design in this research is to

investigate the optimum coil geometry that will produce the most homogeneous and maximum magnetic field. The firefly optimization algorithm was written using Matlab program to obtain the optimum parameters of the coils and the magnetic field inside the coils. The parameters were used to compare the magnetic fields of three geometries, which are two rectangular and square shapes. The first of the three designs is a square geometry, where side  $a = b$ , the second design is a rectangular geometry, where the sides are  $b = 2a$ , while the third design is also a rectangular geometry with  $a = 2b$ . It was observed that homogeneous magnetic field was obtained, when the shape of the coils is a square geometry. We can therefore conclude from the results obtained that the magnetic field is very homogeneous as long as the shape of the coils is approximately a square. This was achieved by plotting Equation (2) as a function of the distance  $z$ , while the magnetic field value was obtained by Equation (7). The magnetic fields for the three coil geometries were simulated using MatLab program and analyzed. It was observed that the first of the three designs which is a square geometry, where side  $a = b$  shows good homogeneous magnetic field of 2.16 mT in the range of about  $\pm 0.03\text{m}$ , while the second design, which is a rectangular geometry, where the sides are  $b = 2a$ , exhibit maximum homogeneous magnetic field of 2.04 mT in the range of about  $\pm 0.012\text{m}$ , and finally the third design, which is also a rectangular geometry with  $a = 2b$  produced the magnetic field value of about 2.3mT in the range of  $\pm 0.04\text{m}$ , which is higher than the predicted value (2.18mT) of FOA design. However, due to the nature of the shape of this coil, the magnetic is observed to be fluctuating and not uniform inside the coils. Hence, good sensitivity and low power consumption, as well as the possibility of generating magnetic field along the axial direction make the designed FOA-based Helmholtz coil suitable for spacecraft and portable sensor calibration and characterization.

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