Proportional Integral (PI) Control of a Single Joint System for Robotic Application

Daniel Effiong Oku, Ibrahim Ahmed Tunde, Asiya Effiom Asiya

***Abstract*— The aim of this work is to demonstrate how a Proportional Integral controller can be implemented in a systematic format by pole placement technique as against the conventional method of tuning a PI controller using Nichols Ziegler method and the controller is applied to test the dynamic response of a single joint system, consisting of a DC motor as the actuator and a coupled body that forms a single joint system with the motor acting as an actuator to cause rotation of the arm. The PI controller is designed and implemented on the system using pole placement technique. The results analysed shows that the controller is able to overcome steady state offset while maintaining the systems stability. In the time domain, a set-point of 1 radian was tracked at the output with a time constant of 0.775s and peak overshoot of 19%, while in the frequency analysis an infinite gain margin was realised and a phase margin of 81.5335o were both gotten from Bode and Nyquist plots respectively, The performance of the system was greatly improved by the implementation of the PI controller when compared to the open loop system whose step response was unstable with a gain margin of 0.1700 and a phase margin of 65.9822o respectively.**

***Index Terms*— Pole placement, PI Controller, DC Motor, Single Joint System, Robot arm, Steady State offset, Nichol Ziegler.**

I. INTRODUCTION

Most dynamic systems are open loop unstable and therefore need a considerable application of control strategies to track set point and attain stability [1]. The application of controllers will greatly depend on the kind of system considered. In this work we try to apply a PI controller on a dynamic model of a single joint system modelled by [8]. Generally PI controllers have been applied to dynamic systems in different ways. One of the method presented by [2] used Nichols Ziegler method of Tuning and more other papers in the use of PI controllers have also used Ziegler's method. Other methods applied in PI controller tuning are Cohen-coon tuning, matlab tuning and manual tuning [10]. It should also be noted that the actuator used for this system was a simple DC motor which has been used by [3][4][5][6][7][8] in their respective design. The choice of a PI controller is from the fact that it is mostly the kind of controller used in industrial applications or in combination with the derivative action to form PID controllers [2], this actuator find particular applications in the robotic industries in robot arm rotation [9]. The method employed here is a pole placement technique which has the advantage that the poles can be placed on the left hand side of the S-Plane which guarantees the system stability instead of tuning the controller. The advantage of applying the integral action will help to cancel steady state error and guaranteed the systems stability.

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II. METHODOLOGY

The single joint system can be represented below as an arm coupled by a DC motor to the horizontal plane:

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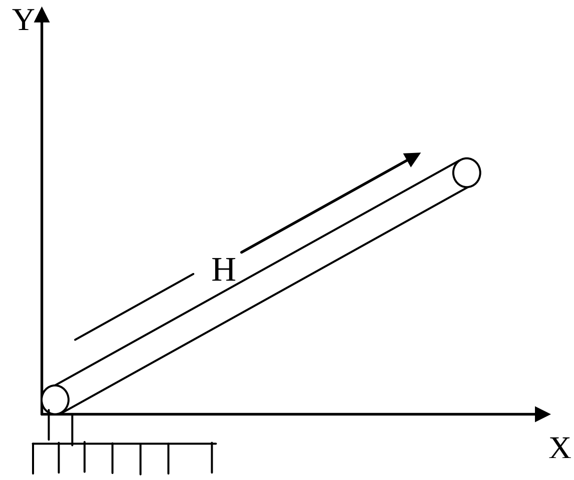

Fig.1: Single Joint System Coupled to the X-Y Axis

[9] modelled the system above and came out with a mathematical model of the single joint systems as represented in the equation below.

$$\frac{RJ}{K_2}\frac{d^2\theta}{dt^2}+\frac{RH}{2K_2}MgCos\theta+\frac{RT_L}{K_2}+\frac{LJ}{K_2}\frac{d^3\theta}{dt^3}-\frac{LH}{2K_2}MgSin\theta\frac{d\theta}{dt}+\frac{L}{K_2}\frac{dT_L}{dt}+K_1\frac{d\theta}{dt}=U(t) \quad (1)$$

The ratio of the Laplace transform of the output to the input can be represented in the equation below.

$$G(s)=\frac{\Delta\theta(s)}{\Delta U(s)}=\frac{K_2}{LJs^3+RJs^2+\left(K_1K_2-\frac{L}{2}HMgSin\theta_o\right)s-\frac{R}{2}HMgSin\theta_o} \quad (2)$$

The values of each parameter in equation 2 is substitued using the table of values below

Parameter	Value
Resistance of the resistor	0.1Ω
Inductance of the inductor	1.25mH
Motor Torque	0.1kg-m2
K2	0.5
K1	0.4
Weight (Mg)	2N

Table1: Parameter and their value chosen for the design

Thus the transfer function in polynomial form in given as;

$$G(s)=\frac{4000}{s^3+80s^2+1592s-680} \quad (3)$$

The general transfer function of the closed-loop system can be written as;

$$TF=\frac{GC}{1+GC} \quad (4)$$

Where G = Transfer function of the plant (system)
C = The controller function

Designing of the proportional integral controller (PD)
The general formula for a proportional integral controller is gotten from the PID controller and then setting the derivative action to zero leaving the proportional and the integral action. Using the formula for a PID controller, the closed loop system can be derived as follows.

$$PID=\frac{sK_p+K_I+s^2K_d}{s} \quad (5)$$

$$PID=K_p+\frac{K_I}{s}+sK_d \quad (6)$$

Thus the closed loop transfer function can be given as

$$G(s)=\frac{\Delta\theta(s)}{\Delta U(s)}=\frac{\dfrac{4000(sK_p+K_I+s^2K_d}{s(s^3+80s^2+1592s-680}}{1+\dfrac{4000(sK_p+K_I+s^2K_d)}{s(s^3+80s^2+1592s-680}} \quad (7)$$

$$G(s)=\frac{4000\ (sK_p+K_I+s^2K_d)}{s^4+80s^3+(4000K_d+1592)s^2+(4000K_p-680)s+4000K_I} \quad (8)$$

Therefore the characteristic polynomial which determines the stability of the closed-loop system is

$$P_X=s^4+80s^3+(4000K_d+1592)s^2+(4000K_p-680)s+4000K_I \quad (9)$$

It is desired for this design to ensure that the poles of the characteristic polynomial Px are all located in the left half side of the S-plane for stability to be acquired. Thus the following poles were chosen for the pole placement which produces coefficient comparable to the characteristics equation. This is done by considering poles, whose product after multiplication gives the coefficient of both S3 and S2, thus making it possible for the characteristic polynomial and the desired polynomial to be compared.

$$P_Y=(S+50)(S+20)(S+10)(S+1) \quad (10)$$

$$P_Y=s^4+80s^3+1709s^2+10630s+9{,}000 \quad (11)$$

Comparing (9) and (11)

$$4000K_p-680=10{,}630 \quad (12)$$

$$K_p=\frac{10630+680}{4000}=\frac{11310}{4000}=2.8275$$

$$4000K_I=9000 \quad (13)$$

$$K_I=\frac{9000}{4000}=2.25$$

In order to make the PI controller, we are going to ignore the Derivative action (Kd) and consider the proportional (Kp) and the Integral (KI) action to make the controller a PI controller.

The input is defined as;

$$U(t)=1 \quad for\ t\geq 0$$
$$=0 \quad t\angle 0$$

III. RESULTS AND DISCUSSION

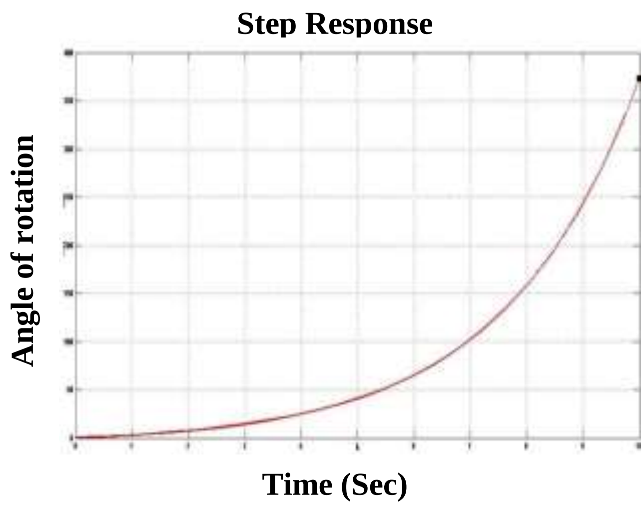

Fig 2: Open Loop Step Response of the Single Joint Robot Arm.

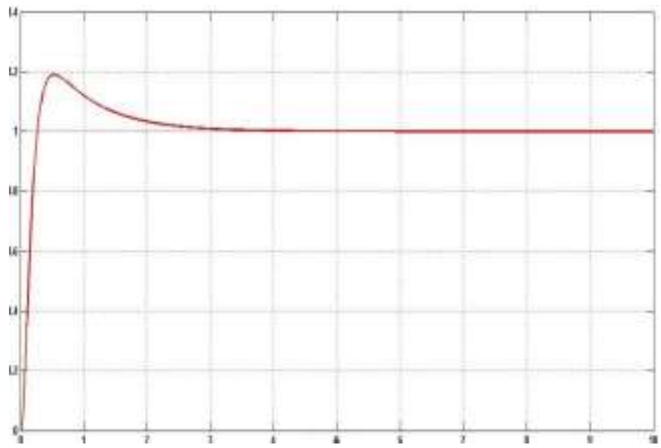

Fig 3: Step Response Using a Proportional Integral Controller

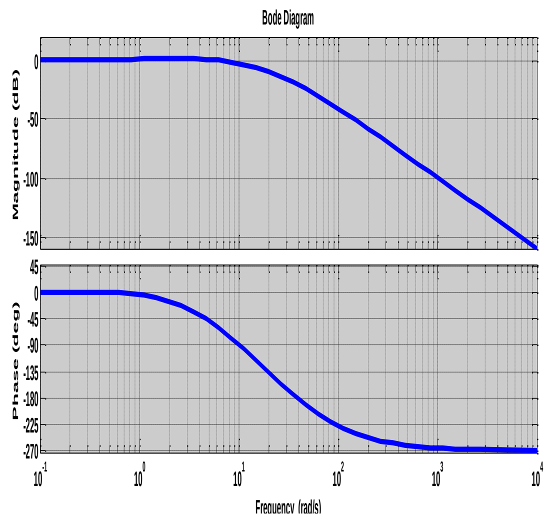

Fig.4: Bode plots using a PI Controller.

The time domain analysis was done first for the open loop step response, followed by the PI controller action. In Fig (3), the system's step response was unstable, even though it was meant to track and stabilize at 1radian. The location of the open loop poles for the fourth order system by a root-locus shows that the poles are located at -40.2+298i, -40.2-298i, -13.2 and 0.418, which confirms the system is unstable as one of the poles (0.418) is located on the right-half side of the S-plane. This necessitated the application of the PI controller to make the closed loop system track set-point and also be stable.

The introduction of a proportional integral controller produces a step response whose output is bounded. It is known that integral action removes steady state error and allows the system to stabilize at set point, thus after the introduction of a proportional integral action , as shown in fig(3), the system output converges to set-point at about 3.1s, with a maximum overshoot given as 1.19rad.

$$M_p = Y_{\max} - Y_{ss}$$

Where:
Mp = Peak overshoot
Ymax = Maximum overshoot
Yss = Steady State Value
From fig (16), Ymax = 1.19, Yss =1.00
Thus

$$M_P = 1.19 - 1.00$$
$$= 0.19 \; radians$$

$$Percentage \; peak \; overshoot = \frac{Y_{MAX} - Y_{SS}}{Y_{SS}} \times 100\%$$

$$Percentage \; peak \; overshoot = \frac{0.19}{1} \times 100\%$$
$$= 19\%$$

The settling time from Fig (3) was calculated as 3.1s which is the time taken for the signal to converge to a steady state value.

The Rise Time is the time taken for the response to rise from 10% to 90% of its steady state value. Thus from fig (3),

Rise Time = 0.3s-0.1s
= 0.2s.

$$\text{Time constant} = \frac{\text{Settling Time}}{4}$$
$$= \frac{3.1}{4}$$
$$= 0.775\text{s}$$

IV. CONCLUSION

After analysing the PI controller application, it is observed that both in the time domain and the frequency domain, the PI controller shows a good performance for both set-point tracking and stability. In the time domain the aims and the objectives were met as the arm tracked the set-point of 1radian, with a settling time of 3.1s, and a time constant of 0.17s, having an overshoot of 0.19 and percentage peak overshoot of 19%.

In the frequency domain, a gain margin of infinity and a phase margin of 81.5335o have been realized using the proportional integral controller which are both desirable in the frequency domain.

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