Analytic Hierarchy Process Improvement

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***Abstract*— This paper introduces the analytic hierarchy process in detail and the specific operation steps. An example in the analytic hierarchy process is used as the research object. Based on the traditional analytic hierarchy process, the harmonic mean is instead of the summation method. The consistency test of the matrix replaces the consistency test of the maximum eigenvalue. The example shows that the improved AHP method is more concise than the original method and is a feasible method.**

***Index Terms*— Analytic hierarchy process; Weights ; Consistency test; Harmonic average;**

I. INTRODUCTION

The analytic hierarchy process was proposed by T.L.satty.It refers to analysis methods that divide decision-making issues into target level, criterion level, and program level. This method deeply analyzes the essence, influencing factors, and internal relations of complex problems and then constructs a hierarchical model to quantify the complex decision-making process, thereby solving multi-criteria or non-structural characteristic decision problems. Analytic hierarchy process can effectively analyze the non-sequential relationships among the levels of the target criterion system, mathematically and systematically thinking process, so that the decision-making basis is easily accepted.

Prior to this, many experts [1-5] and scholars have made detailed discussions on the improvement and application of the AHP. Xia Ping [1] and others used the Delphi method to select experts to analyze the importance and feasibility of evaluation indicators and approached "real priorities" more than the traditional AHP. Li Qing [2] and others used the improved analytic hierarchy process to study the influence of common factors in the blasting process on the blasting effect. This method makes the impact of various factors on the blasting more explicit. Cao Yupeng [3] and others established the optimal transfer matrix and calculated the weight of each factor using the square root method, which can reasonably determine the overall quality of students, at the same time provide a basis for students to analyze their own disadvantages and advantages.Ji Yongqiang analyzed [4] the common calculation method of weight vector-----arithmetic mean, and introduced the calculation method and application of weight vector in detail. Li Zhan [5] and others. analyzed the practical problems in tourism decision-making by proving matrix consistency lemma and constructing a matrix that compliance conditions ,and provided practical evidence for the improvement of the analytic hierarchy process.

The above literature discuss several common methods of analysis---- arithmetic method and geometric method，and the improved analytic hierarchy process,and discuss the application and application effects of the analytic hierarchy process in different fields. This article mainly discusses the basic concepts of the AHP and the advantages of the improved AHP compared to traditional hierarchical distribution.

II. ANALYTIC HIERARCHY PROCESS INTRODUCTION

Analytic Hierarchy Process (AHP) uses arithmetic method and geometric method. The basic principle is to rank the affiliation relationships among various factors of the system from high to low, and to establish the mutual relations between different levels of elements, based on the judgment of certain objective reality. , determine the relative importance of each level, and then use mathematical methods to determine the relative importance weights of all elements of each layer, and make decisions based on the results of the ranking. The AHP can be roughly divided into the following four steps

2.1 Establish Hierarchical Hierarchy Model

Investigation and study of decision-making objects, analysis of the affiliation of the factors involved in the target system, and then divided into different levels, and build an orderly hierarchical structure model.

The level includes three types of goals, criteria and indicators. The target level is the summary of the objectives that the problem will ultimately achieve; the criterion level is the corresponding evaluation criteria for the target; and the indicator level is the specific detailed indicator of each element of the criterion level.

2.2 Construction of the judgment matrix

According to the hierarchical model, the judgment matrix is constructed layer by layer from top to bottom. Each layer of elements is based on the criteria of each element in the next level. The judgment matrix is constructed by comparing the two by two in the 1-9 scale method. The specific scale meaning are as follows:

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scale	definition	meaning
1	Equally important	Two elements are equally important to a criterion
3	Slightly important	Two elements are slightly more important to a criterion than the latter
5	Obviously important	Two elements are significantly more important to a criterion than the latter
7	Strong importance	Two elements are more important to a criterion than the latter
9	Extremely important	Two elements are extremely important to a criterion, the former being more important than the latter.
2, 4,6,8	Adjacent scale median	A scale representing a compromise between two adjacent scales.

2.3 Hierarchical single ordering and consistency check

The maximum eigenvalues of the judgment matrix and the corresponding eigenvectors are solved. After normalization the hierarchically sorted eigenvectors are obtained. Since the result of the judgment matrix has a certain degree of objectivity, it is necessary to carry out consistency test analysis. The judgment matrix must be modified until a satisfactory consistency criterion is met.

2.4 Level total sort

Calculation of the combined weight of index layer factors relative to the total objective of the system from top to bottom. Finally, the ranking results of the impact of each factor on the overall target are obtained.

In summary, when we are making decision analysis, the first is that the judgment matrix is difficult to determine, Second, there is no scientific basis for the consistency of the results. For example,the accuracy of $CR \le 0.1$ is not known. This paper effectively avoids the test of the maximum eigenvalues by checking the consistency of the matrix, and this article has changed the commonly used arithmetic method to increase the sorting difference of attributes. This is useful for the practicality and accuracy of the analytic hierarchy process. It is a great improvement.

III. Preliminary Knowledge

3.1 Definition: In general, if a positive reciprocal matrix A is satisfied,

$$a_{ij} \cdot a_{jk} = a_{ik}, i, j, k = 1,2,\Lambda\ ,n,$$

then A is called a consistency matrix.Be called for short uniform matrix

3.2 Fundamental model

(1)Create a transfer matrix B， The elements in the transfer matrix are satisfied

$$b_{ij} = \lg a_{ij}, (i, j = 1,2\Lambda\ ,n)$$

(2) Establish an optimal transfer matrix C ,The elements in the optimal transfer matrix are satisfied

$$c_{ij} = \frac{1}{n}\sum_{k=1}^{n}(b_{ik} - b_{jk})$$

(3) Creating a quasi-optimal consistency matrix D， The elements in the quasi-conformity matrix are satisfied

$$d_{ij} = 10^{c_{ij}}$$

(4) Calculate the weight of each factor. This article uses the harmonic average to calculate the weight of each factor

$$W_i = \frac{\overline{w_i}}{\sum_{i=1}^{n}\overline{w_i}},$$

Where

$$\overline{w_i} = \frac{n}{\sum_{j=1}^{n}\frac{1}{d_{ij}}}$$

IV. Case Analysis

A college sends one candidate from the three candidates (A; B; C) as the middle-level leader. The merits of the candidates are measured by six attributes. The six attributes are: 1. Health status; 2. professional knowledge; 3. Written expression ability; 4, eloquence; 5, ethical standards; 6. Work style; the attribute importance matrix set by relevant departments is A

$$A = \begin{pmatrix} 1 & 1 & 1 & 4 & 1 & 1/2 \\ 1 & 1 & 2 & 4 & 1 & 1/2 \\ 1 & 1/2 & 1 & 5 & 3 & 1/2 \\ 1/4 & 1/4 & 1/5 & 1 & 1/3 & 1/3 \\ 1 & 1 & 1/3 & 3 & 1 & 1 \\ 2 & 2 & 2 & 3 & 1 & 1 \end{pmatrix}$$

Transfer matrix is B

$$B = \begin{pmatrix} 0 & 0 & 0 & 0.602 & 0 & -0.301 \\ 0 & 0 & 0.301 & 0.602 & 0.477 & -0.301 \\ 0 & -0.301 & 0 & 0.699 & -0.477 & -0.301 \\ -0.602 & -0.602 & -0.699 & 0 & -0.477 & -0.477 \\ 0 & 0 & -0.477 & 0.477 & 0 & 0 \\ 0.301 & 0.301 & 0.301 & 0.477 & 0 & 0 \end{pmatrix}$$

Optimal transfer matrix is C

$$C = \begin{pmatrix} 0 & -0.050 & -0.046 & 0.526 & 0.050 & -0.180 \\ 0.050 & 0 & 0.005 & 0.577 & 0.100 & -0.130 \\ 0.046 & -0.005 & 0 & 0.572 & 0.096 & -0.134 \\ -0.526 & -0.577 & -0.572 & 0 & -0.476 & -0.706 \\ -0.050 & -0.100 & -0.100 & 0.476 & 0 & -0.230 \\ 0.180 & 0.130 & 0.134 & 0.706 & 0.230 & 0 \end{pmatrix}$$

Quasi-optimal transfer matrix is D

$$D=\begin{pmatrix} 1 & 0.891 & 0.901 & 3.360 & 1.122 & 0.661 \\ 1.122 & 1 & 1.011 & 3.772 & 1.260 & 0.742 \\ 1.110 & 0.989 & 1 & 3.732 & 1.247 & 0.734 \\ 0.298 & 0.265 & 0.268 & 1 & 0.334 & 0.197 \\ 0.891 & 0.794 & 0.802 & 2.994 & 1 & 0.589 \\ 1.513 & 1.348 & 1.363 & 5.085 & 1.698 & 1 \end{pmatrix} W$$

eight vector

$$\omega^{A}=(0.169 \quad 0.189 \quad 0.187 \quad 0.050 \quad 0.150 \quad 0.255)$$

Comparison between three participants

$$A_1=\begin{pmatrix} 1 & 1/4 & 1/2 \\ 4 & 1 & 3 \\ 2 & 1/3 & 1 \end{pmatrix}, \quad A_2=\begin{pmatrix} 1 & 1/4 & 1/5 \\ 4 & 1 & 1/2 \\ 5 & 2 & 1 \end{pmatrix},$$

$$A_3=\begin{pmatrix} 1 & 3 & 1/3 \\ 1/3 & 1 & 1/5 \\ 3 & 5 & 1 \end{pmatrix}, \quad A_4=\begin{pmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{pmatrix},$$

$$A_5=\begin{pmatrix} 1 & 1 & 7 \\ 1 & 1 & 7 \\ 1/7 & 1/7 & 1 \end{pmatrix}, \quad A_6=\begin{pmatrix} 1 & 7 & 9 \\ 1/7 & 1 & 2 \\ 1/9 & 1/2 & 1 \end{pmatrix},$$

Calculated according to the same method

$$\omega_1=\begin{pmatrix} 0.136 \\ 0.625 \\ 0.238 \end{pmatrix}, \quad \omega_2=\begin{pmatrix} 0.097 \\ 0.333 \\ 0.570 \end{pmatrix}, \quad \omega_3=\begin{pmatrix} 0.258 \\ 0.105 \\ 0.637 \end{pmatrix},$$

$$\omega_4=\begin{pmatrix} 0.279 \\ 0.649 \\ 0.072 \end{pmatrix}, \quad \omega_5=\begin{pmatrix} 0.467 \\ 0.467 \\ 0.067 \end{pmatrix}, \quad \omega_6=\begin{pmatrix} 0.793 \\ 0.131 \\ 0.076 \end{pmatrix},$$

Decision matrix made up of eigenvectors

$$\omega^{*}=\begin{pmatrix} 0.136 & 0.097 & 0.258 & 0.278 & 0.467 & 0.793 \\ 0.625 & 0.333 & 0.104 & 0.649 & 0.467 & 0.131 \\ 0.239 & 0.570 & 0.637 & 0.072 & 0.067 & 0.076 \end{pmatrix}$$

Calculated by $\omega=\omega_{A}\omega^{*}$ obtain $\omega=\begin{pmatrix} 0.376 \\ 0.324 \\ 0.300 \end{pmatrix}$

It can be seen that among these three people we should give priority to A, followed by B, and finally to C.

V. CONCLUSION

In summary, the improved AHP method not only has higher accuracy than the traditional AHP method,And there is no need to check the consistency of the maximum eigenvalues and avoids inaccuracies $CR \le 0.1$

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