An Estimation Method of the Range of Weighting Coefficients where the Solution Prefered by an Operator is Optimal in Multi-Objective Optimization

Hirokazu Kobayashi, Ryosuke Tachi, Hisashi Tamaki

Abstract— **In the weight method, which is known as one of solutions to the multi-objective optimization problem, it is possible to obtain Pareto optimal solutions by repeating the different weighting coefficients from previous ones and solving by using them. In this research, as a reverse procedure to the above, when a solution preferred by an operator is given, a method of estimating the range of the weighting coefficients where this solution becomes optimal is proposed. We regard a target problem as a mathematical programming problem and estimate the range of weighting coefficients for each purpose based on the basic idea of simplex method. Through some examples, the effectiveness of the proposed approach is examined, and it is confirmed that we can estimate the range of weighting coefficients where the solution preferred by the operator becomes optimal, when this solution exists in feasible region.**

Index Terms— **multi-object, weight method, parameter tuning, optimize, weighting coefficient.**

I. INTRODUCTION

Combined with the increase of demand for industrial products, high quality and diversification of demand, the scale of production and logistics has become larger and more complicated. Under such circumstances, production plans and distribution plans that can realize high efficiency and high productivity are required to carry out efficient procurement, transportation, production and shipment etc [1][2][3]. In response to this request, the creation of production plan and logistics plan is considered as "optimally achieving object under given constraints for the mathematical model", that is, as solving the mathematical programming problem. And then many techniques to solve the problem have been developed [4][5][6]. In the mathematical programming problem, it is important to accurately describe what kind of object it is and what solution is required. With these descriptions, the constraints and objectives of the target problem will be clarified.

Here, in the industry, target plans need to be able to deal with the problems, where there are plural purposes as evaluation indicators, in many cases. These purposes are, for example, to make the procurement cost as low as possible, shorten the transportation time as much as possible, shorten the construction period as much as possible, and keep the delivery schedule as much as possible. But it is common that these objectives are contradictory and the units are also different. The optimization problems with multiple objectives are called multi-objective optimization problems and it is generally difficult to obtain a solution as compared with a case where the purpose is single [7][8][9][10].

To solve such a problem, for example, a weight method, where weighting coefficients are set for each purpose and the problem is solved by solving the sum total, is used [11][12][13]. Depending on the weighting coefficients, which mean importance degree of these objectives, the planned results are significantly different. Therefore, this setting is very important in order to obtain a satisfactory result for operators.

However, in many cases, operators, who are actually making plans, do not regard the target multi-objective problems as mathematical programming problems, create solutions independently, and select preference solutions according to their own judgments. In such cases, it is difficult to quantitatively evaluate which purposes operators put emphasis on and which purposes operators select the solution preferred .

Therefore, in this paper, we assume that the process where operators optimize the plan with plural purposes and make a preference solution without considering the process as solving mathematical programming problem, is the process where operators solve the single-objective optimization problem with the weighted sum of the each purpose sum on the base of the formulation of the mathematical programming problem, especially under the linear programming problem. Under this assumption, we propose a method to estimate the range of weighting coefficients of each purpose from the solution actually preferred by the operator. First, the formulation of an optimization model of the problem is described. Then, we propose a method of estimating the range of the weighting coefficients where this solution becomes optimal when a solution preferred by the operator is given. Finally, through some experiments with examples of practical size, it is confirmed that the proposed method works correctly.

II. DESCRIPTION OF THE PROBLEM

In this paper, we deal with a multi-objective linear programming problem which minimizes objective function using weight method as the target problem. Here we define the decision variable as $\boldsymbol{x} \in \Re^{m}$, the coefficient matrix for the constraints as $A \in \Re^{m \times n}$, the right hand side constant vector for the constraints as $\boldsymbol{b} \in \Re^{m}$, and the cost coefficient matrix as $C \in \Re^{p \times n}$. Then using these definitions, we describe the objective function and constraints of the multi-objective

Hirokazu Kobayashi, Process Research Laboratories, Nippon Steel & Sumitomo Metal Corporation, 20-1, Shintomi, Futtsu, Chiba 293-8511, Japan

Ryosuke Tachi, Department of Computer Science and Systems Engineering, Kobe University, Rokkodai-cho 1-1, Nada-ku, Kobe 657-8501, Japan

Hisashi Tamaki, Graduate School of System informatics, Kobe University, Rokkodai-cho 1-1, Nada-ku, Kobe 657-8501, Japan,

linear programming problem.

A. The objective function

Using the decision variable $\boldsymbol{x}$ and the cost coefficient matrix C, the objective function of the multi-objective linear programming problem is described below.

$$\min\ C\boldsymbol{x} \tag{1}$$

Here, the objective function of the single-objective optimization problem obtained by introducing the weighting coefficients $w \in \Re^p$ for each objective of the equation (1) is given by the equation (2).

$$\min Z = \boldsymbol{w}^{\mathrm{T}} C\boldsymbol{x} \tag{2}$$

And then, the equation (3), and (4) are obtained from the preconditions of the weight method as below.

$$\boldsymbol{w} \geq 0 \tag{3}$$

$$\sum_{k=1}^{p} w_k = 1\ ,\ \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_p \end{pmatrix} \tag{4}$$

B. The constraints

Using the decision variable $\boldsymbol{x}$, the coefficient matrix A and the right hand side constant vector $\boldsymbol{b}$, the constraints of the multi-objective linear programming problem are described below.

$$A\boldsymbol{x} \geq \boldsymbol{b} \tag{5}$$

$$\boldsymbol{x} \geq 0 \tag{6}$$

Here, to simplify of the problem, it is assumed that all constraints are independent, that is, $\mathrm{rank}\ A = m$. And also, it is assumed that there is no degeneracy in the basic solutions obtained under the constraints.

III. The Range of Weighting Coefficients where the Preference Solution is Optimal in Multi-Objective Optimization

In this section, we describe the framework concerning the idea of the estimation of the range of weighting coefficients by the proposed approach which takes reverse procedure from the normal procedure. The purpose here is that when a solution $\boldsymbol{x}^D$ preferred by an operator is given, we estimate the range of weighting coefficients $\boldsymbol{w}$ by which this solution $\boldsymbol{x}^D$ is the optimal solution of the problem with the above scalarized objective function which is described as the equation (2), or it is to find the range of the weighting coefficients $\boldsymbol{w}$ where the distance between the preference solution $\boldsymbol{x}^D$and the optimal solutions of the problem obtained, when changing the weighting coefficients $\boldsymbol{w}$ within the range of the equation (5), is closest.

A. The scope of the preference solutions to be handled

Preference solutions $\boldsymbol{x}^D$ to be handled are classified into the following three categories.

(a) The preference solution $\boldsymbol{x}^D$ is an infeasible solution.

(b) The preference solution $\boldsymbol{x}^D$ is a feasible solution, but not Pareto optimal or not a basic solution.

(c) The preference solution $\boldsymbol{x}^D$ is a basic solution which is Pareto optimal.

In this paper, we consider except when preference solutions $\boldsymbol{x}^D$ are infeasible solutions categorized in (a). Since the examination in the case (a), where preference solutions $\boldsymbol{x}^D$ are infeasible solutions, is difficult to examine as compared with other cases described above, we will not deal with it here and it will be given to the next study. In this study, considering these (b) and (c) cases, estimation of the range of the weighting coefficients is investigated in two stages. At first, we will examine the case (b). In this examination, we convert the preference solution $\boldsymbol{x}^D$ that is a feasible solution but not a Pareto optimal or not a basis solution, to a basic solution which is Pareto optimal in some way. If we can transfer the preference solution $\boldsymbol{x}^D$, which is a feasible solution but not a Pareto optimal or not a basis solution, to the basic solution which is Pareto optimal in this way, it can be considered as the case (c). As a second step, when the preference solution $\boldsymbol{x}^D$ that is a basic solution which is a Pareto optimal is given, that is, for the case (c), we estimate the range of weighting coefficients $\boldsymbol{w}$ where this solution is an optimal solution of the linear programing problem with the scalarized objective function. By following the above procedure, the case (b) and the case (c) can be handled. We will discuss these procedures in detail at the next chapter.

B. Obtaining the basic solution which is Pareto optimal

In this section, when the preference solution $\boldsymbol{x}^D$ is a feasible solution, but not Pareto optimal or it is not a basic solution, we investigate the method to convert it to a basic solution which is Pareto optimal. As these methods, the methods described like below exist.

- Transferring the preference solution $\boldsymbol{x}^D$.
- Modifying the linear programing problem.

In this study, we consider the case where we modify the linear programing problem. For this, the methods described like below exist.

- Modifying the cost coefficient matrix C for the objective function.
- Modifying the coefficient matrix A for the constraints.
- Modifying the right hand side constant vector $\boldsymbol{b}$ for the constraints.

In this study, we consider the case where we modify the right hand side constant vector $\boldsymbol{b}$ for the constraints. In this method, we consider this modification using the concept of distance. At this time there are two ways to consider distance as follows.

- Distance in solution space.
- Distance in objective function space.

In this study, we consider the case where we use distance in solution space.

We define the constraints including non-negative constraints as the expanded constraints. Accord to expansion of the constraints, the coefficient matrix of the expanded constraints is defined as the expanded coefficient matrix A', and the constant vector is expanded to the expanded constant vector $\boldsymbol{b}'$. The set of subscripts representing the number of row in the expanded coefficient matrix of the expanded constraints, on which there is at least one Pareto optimal solution, is defined as $\mathcal{J}^P$. Here this set is assumed to be obtained naturally, and $\mathcal{J}^P$ is described as the equation (7) below. The expanded constraints having elements of this set $\mathcal{J}^P$ as subscripts are defined as Pareto optimal constraints.

$$\mathcal{J}^P = \{i_1^P, i_2^P, \cdots, i_h^P\} \tag{7}$$

Based on the above, when a preference solution that is not a Pareto optimal or is not a basis solution is given, we calculate the distance between the preference solution and each constraint in the solution space respectively, and the expanded constant vector $\boldsymbol{b}'$ of the Pareto optimal constraints is modified in order from the closest distance constraint. By doing this, we propose a method to convert the preference solution to the basic solution which is Pareto optimal.

At first, we calculate the distance between the preference solution $\boldsymbol{x}^D$ and each Pareto optimal constraint in the solution space respectively. In defining A_i' as the row vector of the i-th row of the expanded coefficient matrix $\boldsymbol{A}'$, the boundary of the Pareto optimal constraint can be expressed by the following equation (8).

$$A_i'\boldsymbol{x} = b_i' \quad (i = i_1^P, i_2^P, \cdots, i_h^P) \tag{8}$$

In this case, the distance $d_1, d_2, \dots, d_h$ between the preference solution and the hyperplane formed by each Pareto optimal constraint can be expressed by the following equation (9).

$$d_i = \frac{|A_i'x^D - b_i'|}{\|A_i'\|} \quad (i = i_1^P, i_2^P, \cdots, i_h^P) \tag{9}$$

Then, the expanded constant vector $\boldsymbol{b}'$ of the Pareto optimal constraints is modified in order from the closest distance constraint. Then, we modify n elements of the expanded constant vector $\boldsymbol{b}'$ in order from the closest distance so that the preference solution passes on the boundary of the Pareto optimal constraints corresponding to the modified elements. Letting $\boldsymbol{b}^R$ be the expanded constant vector after modified, $\boldsymbol{b}^R$ is expressed by the following equation (10).

$$b_i^R = \begin{cases} A_i'\boldsymbol{x}^D & (i = j_1, j_2, \cdots, j_n) \\ b_i' & (otherwize) \end{cases} \tag{10}$$

After modification of expanded constant vector $\boldsymbol{b}'$ to $\boldsymbol{b}^R$, the preference solution $\boldsymbol{x}$ becomes a basis solution which is Pareto optimal.

The update of the right hand side constant vector $\boldsymbol{b}$ (parallel movement) for the constraints based on this idea is summarized as follows.

1° Calculate the distances $d_{i_1^P}, \cdots, d_{i_h^P}$ between the preference solution $\boldsymbol{x}^D$ and the each boundary of each Pareto optimal constraint in the solution space respectively.

$$d_i = \frac{|A_i'\boldsymbol{x}^D - b_i'|}{\|A_i'\|} \quad (i = i_1^P, i_2^P, \cdots, i_h^P)$$

2° Define the set in which subscripts of Pareto optimal constraint are rearranged in descending order of distance.

$$\mathcal{J} = \{j_1, j_2, \cdots, j_h | j_1, j_2, \cdots, j_h \in \mathcal{J}^P, d_{j1} \le d_{j2} \le \cdots \le d_{jh}\}$$

3° Modify n elements of the expanded constant vector $\boldsymbol{b}'$ in order from the closest distance so that the preference solution $\boldsymbol{x}^D$ passes on the boundary of the Pareto optimal constraints corresponding to the modified elements, and get $\boldsymbol{b}^R$.

$$b_i^R = \begin{cases} A_i'\boldsymbol{x}^D & (i = j_1, j_2, \cdots, j_n) \\ b_i' & (otherwize) \end{cases}$$

C. Estimating the range of the weighting coefficients

In the previous section, the linear programming problem is modified, so that the preference solution $\boldsymbol{x}^D$ becomes a basic solution which is Pareto optimal, by modifying the constant vector $\boldsymbol{b}$ (parallel movement) of the constraint conditions. In this section, the idea and procedure of the estimation method to obtain the range of the weighting coefficients $\boldsymbol{w}$, which make the preference solution $\boldsymbol{x}^D$ the optimal solution for the modified linear programming problem, will be described in detail.

In this study, we use the concept of the relative cost coefficients of the simplex method to estimate the weighting coefficients. The simplex method is that at first the value of the objective function from one vertex of the feasible region is calculated, further the calculation target is moved to the adjacent vertex so that the value of the objective function decreases, and finally the optimal solution is obtained. When searching the solution so that the value of the objective function decreases, the search is performed based on the relatives cost coefficients. In the case of the problem in standard form, the solution is optimal if and only if the relative cost coefficients are non-negative [14]. For this reason, in case that the preference solution $\boldsymbol{x}^D$ which is the basic solution that is Pareto optimal is given, the procedure to obtain the range of the weighting coefficients $\boldsymbol{w}$, where the preference solution $\boldsymbol{x}^D$ is optimal, is equivalent to the procedure to calculate the relative cost coefficients for the preference solution $\boldsymbol{x}^D$ and to obtain the range of the weighting coefficients $\boldsymbol{w}$, where the relative cost coefficients become non-negative. First of all, in order to use Simplex method, it is necessary to formulate the target problem into the standard form by introducing slack variable $\boldsymbol{s}$. The problem that converted the target problem into the standard form using $\tilde{\boldsymbol{x}}, \tilde{A}, \tilde{C}$ is described as follows. At this time, the slack variable $\boldsymbol{s}$ is a n-dimensional vector, I_m is a m-dimensional identity matrix, and $O_{p,m}$ is a zero matrix of p rows and m columns. Therefore, $\tilde{\boldsymbol{x}}$ is the $(m+n)$-dimensional vector, $\tilde{A}$ is the $m \times (m+n)$ matrix, and $\tilde{C}$ is the $p \times (m+n)$ matrix.

$$\min Z = \boldsymbol{w}^{\mathrm{T}}\tilde{C}\tilde{\boldsymbol{x}} \tag{11}$$

subject to

$$\tilde{A}\tilde{\boldsymbol{x}} = \boldsymbol{b}^R, \ \boldsymbol{x} \ge 0 \tag{12}$$

$$\tilde{\boldsymbol{x}} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{s} \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} A & -I_m \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} C & O_{p,m} \end{pmatrix}$$

Here, substituting the preference solution $\boldsymbol{x}^D$ for $\boldsymbol{x}$ in the equation (11), the value of the slack variable $\boldsymbol{s}$ is expressed by the following equation (12).

$$\boldsymbol{s} = A\boldsymbol{x}^D - \boldsymbol{b}^R \tag{13}$$

By using the values of preference solution $\boldsymbol{x}^D$ and the slack variable $\boldsymbol{s}$, $\tilde{\boldsymbol{x}}$ is obtained as a constant vector. The variable whose value is positive among $\tilde{\boldsymbol{x}}$ is defined as the basic variable $\tilde{\boldsymbol{x}}^{\mathrm{B}}$, and the variable whose value is zero among $\tilde{\boldsymbol{x}}$ is defined as the nonbasic variable $\tilde{\boldsymbol{x}}^{\mathrm{N}}$. When the constraints (12) is transformed and represented by the basic variable $\tilde{\boldsymbol{x}}^{\mathrm{B}}$ and the nonbasic variable $\tilde{\boldsymbol{x}}^{\mathrm{N}}$, the following equation (14) is obtained. Here, the matrices $\tilde{A}^{\mathrm{B}}$ and $\tilde{A}^{\mathrm{N}}$ are obtained by dividing the matrix $\tilde{A}$ into columns corresponding to basic variables $\tilde{\boldsymbol{x}}^{\mathrm{B}}$ and columns corresponding to nonbasic variables $\tilde{\boldsymbol{x}}^{\mathrm{N}}$. These matrices are defined as the basic matrix $\tilde{A}^{\mathrm{B}}$ and the nonbasic matrix $\tilde{A}^{\mathrm{N}}$ respectively. Since it is assumed that each constraint is independent and there is no degeneration in the preference solution $\boldsymbol{x}^D$, $\tilde{A}^{\mathrm{B}}$ is the $m \times m$ regular matrix, and $\tilde{A}^{\mathrm{N}}$ is the $m \times n$ matrix.

$$\tilde{A}^{\mathrm{B}}\tilde{\boldsymbol{x}}^{\mathrm{B}} + \tilde{A}^{\mathrm{N}}\tilde{\boldsymbol{x}}^{\mathrm{N}} = \boldsymbol{b}^R \tag{14}$$

Similarly, when the objective function (11) is expressed using the basic variable $\tilde{\boldsymbol{x}}^{\mathrm{B}}$ and the nonbasic variable $\tilde{\boldsymbol{x}}^{\mathrm{N}}$, the following equation (15) is obtained. Here, the matrices $\tilde{C}^{\mathrm{B}}$ and $\tilde{C}^{\mathrm{N}}$ are obtained by dividing the matrix $\tilde{C}$ into columns corresponding to basic variable $\tilde{\boldsymbol{x}}^{\mathrm{B}}$ and columns corresponding to nonbasic variable $\tilde{\boldsymbol{x}}^{\mathrm{N}}$. These matrices are defined as the cost basic matrix $\tilde{C}^{\mathrm{B}}$ and the cost nonbasic matrix $\tilde{C}^{\mathrm{N}}$ respectively. And here $\tilde{C}^{\mathrm{B}}$ is the $p \times m$ matrix, and $\tilde{C}^{\mathrm{N}}$ is the $p \times n$ matrix.

$$Z = \boldsymbol{w}^{\mathrm{T}}\left(\tilde{C}^{\mathrm{B}}\tilde{\boldsymbol{x}}^{\mathrm{B}} + \tilde{C}^{\mathrm{N}}\tilde{\boldsymbol{x}}^{\mathrm{N}}\right) \tag{15}$$

By eliminating $\tilde{\boldsymbol{x}}^{\mathrm{B}}$ from equation (15) using equation (14), the objective function is expressed by the following equation (16).

$$Z = \boldsymbol{w}^{\mathrm{T}}\tilde{C}^{\mathrm{B}}\left(\tilde{A}^{\mathrm{B}}\right)^{-1}\boldsymbol{b}^R + \boldsymbol{w}^{\mathrm{T}}\left\{\tilde{C}^{\mathrm{N}} - \tilde{C}^{\mathrm{B}}\left(\tilde{A}^{\mathrm{B}}\right)^{-1}\tilde{A}^{\mathrm{N}}\right\}\tilde{\boldsymbol{x}}^{\mathrm{N}} \tag{16}$$

The coefficients of $\tilde{\boldsymbol{x}}^{\mathrm{N}}$ in the equation (16) are the relative cost coefficients. In defining the relative cost coefficients as $\boldsymbol{\mu}(\boldsymbol{w})$, they can be expressed by the following equation (17).

$$\boldsymbol{\mu}(\boldsymbol{w}) = \boldsymbol{w}^{\mathrm{T}}\left\{\tilde{C}^{\mathrm{N}} - \tilde{C}^{\mathrm{B}}\left(\tilde{A}^{\mathrm{B}}\right)^{-1}\tilde{A}^{\mathrm{N}}\right\} \tag{17}$$

The preference solution $\boldsymbol{x}^D$ is an optimal solution if and only if the relative cost coefficients $\boldsymbol{\mu}(\boldsymbol{w})$ are nonnegative. That is, the range of the weighting coefficients $\boldsymbol{w}$, which make the preference solution $\boldsymbol{x}^D$ the optimal solution, are calculated by the previous equation (17), and the equation (3) , (4) which are preconditions of the weight method. In summary, it can be described as follows.

$$\boldsymbol{\mu}(\boldsymbol{w}) \geq 0$$

$$\boldsymbol{w} \geq 0$$

$$\sum_{k=1}^{p} w_k = 1$$

IV. Calculation Example

In this chapter, by using the estimation method proposed in the previous chapter, we estimate the range of the weighting coefficients $\boldsymbol{w}$ for an example. Also, we choose weighting coefficients $\boldsymbol{w}^D$ appropriately from the range of weighting coefficients obtained as the estimation result, and optimize the linear programing problem using the weighting coefficients $\boldsymbol{w}^D$ without giving a preference solution $\boldsymbol{x}^D$. By doing this, it is confirmed that the initially preference solution $\boldsymbol{x}^D$ is correctly obtained, and it is confirmed whether the proposed estimation method functions correctly.

A. Example

We consider the following single-objective optimization problem which is obtained by applying the weight method for the multi-objective optimization problem having 2 objects, 3 constraints and 2 decision variables.

$$\begin{aligned} \min \quad & \boldsymbol{w}^{\mathrm{T}} C\boldsymbol{x} \\ \text{subject to } & A\boldsymbol{x} \geq \boldsymbol{b} \\ & \boldsymbol{x} > 0 \end{aligned}$$

$$\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$C = \begin{pmatrix} 5 & 1 \\ 1 & 6 \end{pmatrix}, A = \begin{pmatrix} 3 & 1 \\ 1 & 4 \\ -4 & -5 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \\ -20 \end{pmatrix}$$

The solution space constructed by the constraints for the example is shown in Fig. 1.

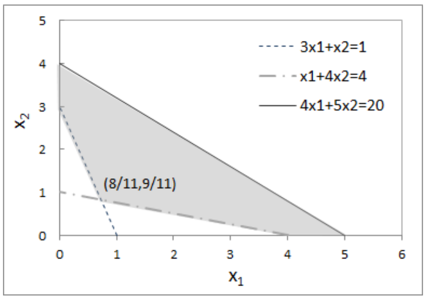

Fig. 1 The Solution Space for the Example

B. Calculation Result and Discussion

In this example, the case that the preference solution is feasible but is neither Pareto optimal nor a basic solution is considered. A case where we select the following preference solution $\boldsymbol{x}^D$ is considered.

$$\boldsymbol{x}^D = \begin{pmatrix} \frac{37}{44} \\ \frac{43}{44} \end{pmatrix}$$

Here, in order to modify the constant vector $\boldsymbol{b}$ (parallel movement) of the constraints, we consider the distance between the preference solution $\boldsymbol{x}^D$ and the hyperplane constructed by each Pareto optimal constraint. Therefore, we consider the expanded coefficient matrix A' and the expanded constant vector $\boldsymbol{b}'$. The expanded coefficient matrix A' and the extended constant vector $\boldsymbol{b}'$ obtained by expanding the coefficient matrix and constant vector to those including nonnegative conditions are as follows.

$$\begin{pmatrix} A'_1 \\ A'_2 \\ A'_3 \\ A'_4 \\ A'_5 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 4 \\ -4 & -5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -20 \\ 0 \\ 0 \end{pmatrix}$$

Here, the set $\mathcal{J}^P$ of subscripts representing the number of row in the expanded coefficient matrix of the expanded constraints where at least there is one Pareto optimal solution, is $\{1, 2, 4, 5\}$. Therefore, the expansion coefficient matrix A' and the expanded constant vector $\boldsymbol{b}'$ are as follows.

$$A' = \begin{pmatrix} A'_1 \\ A'_2 \\ A'_4 \\ A'_5 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 4 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{b}' = \begin{pmatrix} b'_1 \\ b'_2 \\ b'_4 \\ b'_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \\ 0 \end{pmatrix}$$

Next, according to the procedure for modifying the constant vector $\boldsymbol{b}$ (parallel movement) of the constraints, $\boldsymbol{b}^R$ is obtained.

1◦ Calculate the distances $d_{i_1^P}, \cdots, d_{i_h^P}$.

$$d_1 = \frac{|A'_1\boldsymbol{x}^D - b'_1|}{\|A'_1\|} = \frac{\left|3 \times \frac{37}{44} + 1 \times \frac{43}{44} - 3\right|}{\sqrt{3^2+1^2}} = 0.1581 \ldots$$

$$d_2 = \frac{|A'_2\boldsymbol{x}^D - b'_2|}{\|A'_2\|} = \frac{\left|1 \times \frac{37}{44} + 4 \times \frac{43}{44} - 4\right|}{\sqrt{1^2+4^2}} = 0.1819 \ldots$$

$$d_4 = \frac{|A'_4\boldsymbol{x}^D - b'_4|}{\|A'_4\|} = \frac{\left|1 \times \frac{37}{44} + 0 \times \frac{43}{44} - 0\right|}{\sqrt{1^2+0^2}} = 0.8409 \ldots$$

$$d_5 = \frac{|A'_5\boldsymbol{x}^D - b'_5|}{\|A'_5\|} = \frac{\left|0 \times \frac{37}{44} + 1 \times \frac{43}{44} - 0\right|}{\sqrt{0^2+1^2}} = 0.9773 \ldots$$

2◦ Define the set $\mathcal{J}$.

$$\mathcal{J} = \{1,2,4,5 \mid 1,2,4,5 \in \mathcal{J}^P, d_1 \le d_2 \le d_4 \le d_5\}$$

3◦ Get $\boldsymbol{b}^R$.

$$b_1^R = 3 \times \frac{37}{44} + 1 \times \frac{43}{44} = \frac{7}{2}$$

$$b_2^R = 1 \times \frac{37}{44} + 4 \times \frac{43}{44} = \frac{19}{4}$$

$$\boldsymbol{b}^R = \begin{pmatrix} \frac{7}{2} \\ \frac{19}{4} \\ -20 \\ 0 \\ 0 \end{pmatrix}$$

Since $\boldsymbol{b}^R$ is the constant vector after modifying the constant vector $\boldsymbol{b}$ (parallel movement) of the constraints, the constraint after modifying is as follows.

$$A\boldsymbol{x} \ge \boldsymbol{b}^R$$

The solution space formed by the modified constraints is shown in Fig. 2.

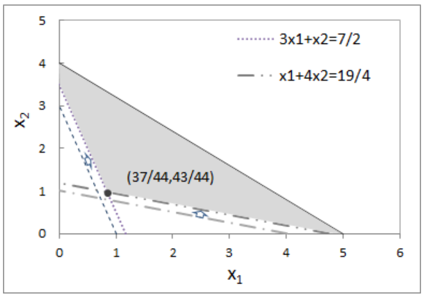

Fig. 2 THE SOLUTION SPACE FOR THE MODIFIED CONSTRAINTS

Using this modified constraint, we can proceed to the next procedure to estimate the range of the weighting coefficients as follows.

1◦ We introduce the slack variables $\boldsymbol{s}$. Along with this, expand $\boldsymbol{x}$, A, C to $\tilde{\boldsymbol{x}}$, $\tilde{A}$, $\tilde{C}$ and make the problem into a standard form.

$$\tilde{\boldsymbol{x}} = \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}, \tilde{C} = \begin{pmatrix} 5 & 1 & 0 & 0 & 0 \\ 1 & 6 & 0 & 0 & 0 \end{pmatrix}, \tilde{A} = \begin{pmatrix} 3 & 1 & -1 & 0 & 0 \\ 1 & 4 & 0 & -1 & 0 \\ -4 & -5 & 0 & 0 & -1 \end{pmatrix}$$

2◦ Calculate the value of the slack variable $\boldsymbol{s}$.

$$\boldsymbol{s} = A\boldsymbol{x}^D - \boldsymbol{b} = \begin{pmatrix} 3 & 1 \\ 1 & 4 \\ -4 & -5 \end{pmatrix} \begin{pmatrix} \frac{37}{44} \\ \frac{43}{44} \end{pmatrix} - \begin{pmatrix} \frac{7}{2} \\ \frac{19}{4} \\ -12 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{15}{4} \end{pmatrix}$$

3◦ Obtain $\tilde{\boldsymbol{x}}^{\mathrm{B}}$, $\tilde{\boldsymbol{x}}^{\mathrm{N}}$, $\tilde{A}^{\mathrm{B}}$, A^{N}, $\tilde{A}^{\mathrm{N}}$, $\tilde{C}^{\mathrm{B}}$, $\tilde{C}^{\mathrm{N}}$.

$$\tilde{\boldsymbol{x}}^{\mathrm{B}} = \begin{pmatrix} x_1 \\ x_2 \\ s_3 \end{pmatrix}, \tilde{\boldsymbol{x}}^{\mathrm{N}} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

$$\tilde{A}^{\mathrm{B}} = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 4 & 0 \\ -4 & -5 & -1 \end{pmatrix}, \tilde{A}^{\mathrm{N}} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\tilde{C}^{\mathrm{B}} = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 6 & 0 \end{pmatrix}, \tilde{C}^{\mathrm{N}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

4◦ Calculate the relative cost coefficients $\boldsymbol{\mu}(\boldsymbol{w})$

$$\boldsymbol{\mu}(\boldsymbol{w}) = \begin{pmatrix} \frac{19}{11}w_1 - \frac{2}{11}w_2 & -\frac{2}{11}w_1 + \frac{17}{11}w_2 \end{pmatrix}$$

5◦ Calculate the range of the weighting coefficients $\boldsymbol{w}$

$$\left.\begin{array}{l} \frac{19}{11}w_1 - \frac{2}{11}w_2 \ge 0 \\ -\frac{2}{11}w_1 + \frac{17}{11}w_2 \ge 0 \\ w_1 \ge 0 \\ w_2 \ge 0 \\ w_1 + w_2 = 1 \end{array}\right. \Rightarrow \frac{2}{21} \le w_1 \le \frac{17}{19}$$

Here, to verify whether the proposed estimation method works correctly for the modified constraints, optimizations for the cases when the weighting coefficients are set within the obtained range and other cases when the weighting coefficients are set outside the range are examined.

- Case1 : $w_1 = 0.05,\ w_2 = 0.95$ (out of the range)

  Optimal value $\boldsymbol{x}^* = \begin{pmatrix} \frac{19}{4} \\ 0 \end{pmatrix}$

- Case2 : $w_1 = 0.10,\ w_2 = 0.90$ (within the range)

  Optimal value $\boldsymbol{x}^* = \begin{pmatrix} \frac{37}{44} \\ \frac{43}{44} \end{pmatrix}$

- Case3 : $w_1 = 0.85,\ w_2 = 0.15$ (within the range)

  Optimal value $\boldsymbol{x}^* = \begin{pmatrix} \frac{37}{44} \\ \frac{43}{44} \end{pmatrix}$

- Case4 : $w_1 = 0.90,\ w_2 = 0.10$ (out of the range)

  Optimal value $\boldsymbol{x}^* = \begin{pmatrix} 0 \\ \frac{7}{2} \end{pmatrix}$

As results of the above verification, using the proposed estimation method, it was confirmed that the range of the weighting coefficients can be correctly estimated when the preference solution is Pareto optimal and non-degenerate basic solution.

V. Conclusion

In this paper, when a solution preferred by an operator is given, a method of estimating the range of the weighting coefficients where this solution becomes optimal is proposed. To extend to consider the cases where the preference solution is an feasible solution but not Pareto optimal or it is not a basic solution, we proposed the method which is constructed by two stages of carrying out acquisition of the basis solution which is Pareto optimal and then estimating the weighting coefficients.

In the proposed estimation method of the range of weighting coefficients, it was confirmed that we can estimate the range of weighting coefficients that the preference solution is optimal to the following cases.

- The preference solution is a basic solution which is Pareto optimal.
- The preference solution is not Pareto optimal or not a basic solution, but Pareto optimal constraints is given.

We also confirmed that the proposed estimation method works correctly by applying some cases of appropriate weighting coefficients from the range of weighting coefficients obtained by estimating weighting coefficients and by being optimized for examples.

The following issues have been left for further studies.

- The preference solution is not Pareto optimal or not a basis solution and Pareto optimal constraints are not given.
- The preference solution is an infeasible solution.

References

[1] F. Schiavone and S. Sprenger, "Operations management and digital technologies," Production Planning & Control, vol. 28, 2017, pp. 1281-1283.

[2] B. Tjahjono, C. Esplugues, E. Ares, and G. Pelaez, "What does Industry 4.0 mean to Supply Chain? ," Procedia Manufacturing, vol. 13, 2017, pp. 1175-1182.

[3] H. Hvolby, and K. Steger-Jensen, "Technical and industrial issues of Advanced Planning and Scheduling (APS) systems," Computers in Industry, vol. 61, no. 9, 2010, pp. 845-851.

[4] J. T. Linderoth, and A. Lodi, MILP Software, Wiley Encyclopedia of Operations Research and Management Science (Cochran, J. ed.), Wiley, New York, vol. 5, 2011, pp. 3239-3248.

[5] Y. Xu, T. K. Ralphs, L. Ladányi, and M. J. Saltzman, "Computational Experience with a Software Framework for Parallel Integer Programming," INFORMS Journal on Computing, vol. 21, no. 3, 2009, pp. 383-397.

[6] A.A.G. Bruzzone, D. Anghinolfi, M. Paolucci, and F. Tonelli, "Energy-aware scheduling for improving manufacturing process sustainability: A mathematical model for flexible flow shops," CIRP Annals, vol. 61, 2012, pp. 459-462.

[7] K. Deb, A. Pratap, S. Agarwal, and T.Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-Ⅱ," IEEE Transactions on Evolutionary Computation, vol. 6, no. 2, 2002, pp. 182-197.

[8] J. J. Durillo1, J. Garc´ıa-Nieto1, A. J. Nebro1, C. A. C. Coello2, F. Luna1, and E. Alba, "Multi-Objective Particle Swarm Optimizers," An Experimental Comparison; EMO '09, Springer-Verlag, 2009, pp. 495-509.

[9] M. Ehrgott, Multicriteria Optimization (Second Edition), Springer, Berlin Heidelberg , 2005.

[10] W. Xia, and Z. Wu, "An effective hybrid optimization approach for multi-objective flexible job-shop scheduling problems," Computers & Industrial Engineering, vol. 48, 2005, pp. 409-425.

[11] C. L. Hwang, and A. S. M. Masud, Multiple objective decision making, methods and applications - methods and applications, Springer-Verlag, 1979.

[12] R. T. Marler and J. S. Arora, "The weighted sum method for multi-objective optimization: new insights," Structural and Multidisciplinary Optimization, Springer, vol. 41, no. 6, 2010, pp. 853–862.

[13] K. Deb, "Multi-objective Optimization," Search Methodologies, Springer, 2013, pp. 304-449.

[14] E. D. Nering, and A. W. Tucker, Linear Programs and Related Problems, Academic Press, 1993.

Hirokazu Kobayashi received his B.E. and M.E. degrees in applied mathematics and physics from Kyoto University in 1990 and 1992, respectively. He joined Nippon Steel & Sumitomo Metal Corporation at the same year, and it reaches the present. He is engaged in research on production planning, logistics optimization..

Ryosuke Tachi received his B.E. degree in Computer Science and Systems Engineering from Kobe University in 2017. He is currently pursuing the M.E. degree at Kobe University. His research interests include mathematical planning model of scheduling problems.

Hisashi Tamaki received his B.E., M.E. and Ph.D. degrees in Electrical Engineering from Kyoto University in 1985, 1987 and 1993, respectively. From 1990 until 1995, he was with Kyoto University, Japan. Since 1995, he has been with Kobe University, Japan, and currently he is a Professor at the Graduate School of System Informatics. His research interests include system optimization, emergent computation and agent-based simulation. He is a member of IEEE, IEEJ, SICE, ISCIE and ISIJ