Using Constructivist Approach to Enhance Students' Understanding of Logarithmic Functions: A Case Study of Kalpohin Senior High School, Tamale-Ghana

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***Abstract*— This paper deals with using constructivist approach to enhance students' understanding of logarithmic function in Kalpohin Senior High School. Purposive sampling technique was employed to obtain a sample (n = 60) of male and female second year Agricultural Science and Home Economics students from Kalpohin Senior High School in Tamale, the Northern Region of Ghana. A constructivist-based instructional (CBI) approach was employed and evaluated using three instruments- pre-test, post-test and unstructured interviews to measure students' understanding of logarithmic function, their attitudes and views about Constructivist approach.**

***Index Terms*— Constructivist theory, epistemology, logarithmic function**

I. INTRODUCTION

In Ghanaian schools a teacher writes peculiarities on the whiteboard and then goes on solving the problem related to it. Students prepare for examinations by memorizing concepts and formulae and by solving related problems on topics taught in class. However, some of the students cannot comprehend the concepts; some others are not interested in the subject as they think it is not useful to them and others are like spectators, while few students go to the whiteboard and solve problems. Teachers only expect students to write, memorize and solve questions.

Recently, researchers (Makgakga and Sepeng, 2013; Thompson, 2014; Bhattacharjee, 2015) have shown an increased interest in the used of various approaches in the learning and teaching of logarithmic functions. All these good approaches seem effective for the understanding of logarithmic functions, but the constructivist approach is the best among them because the National Council of Curriculum and Assessment in Ghana (NCCA), in its intended curriculum emphasized the constructivist approach as an intervention for the understanding of mathematical concepts including logarithm concepts and functions. Mathematics teachers are to use the implemented curriculum by making sure that their schemes of work; lesson notes are prepared very well based on the intended curriculum in order to enhance the understanding of mathematical concepts like logarithmic functions. My argument for constructivist approach is based on recent research on the use of constructivist approach as a theory of knowledge and learning which can be easily applied to the understanding of logarithmic function by students.

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Constructivist Theory

Learning theories are called social constructivist in which knowledge construction is through social interactions. It highlights the role of social interactions meaning making, especially the support of more knowledgeable others in knowledge construction. Social constructivists focus on social and cultural mathematical and pedagogical practices and attend to individuals' internalization of them. They conceive of learners in social settings, concentrating, to various degrees on learner's participation in them (Thompson, 2013).

According to Bhattacharjee (2015), "constructivism is an epistemology or a theory used to explain how people know what they know". A constructivist view of learning suggest an approach to teaching that gives learners the opportunity for concrete, contextually meaningful experience through which they can search for patterns; raise questions; and model, interpret and defend their strategies and ideas.

The classroom in this constructivist model is seen as a community of learners engaged in activity, discourse, interpretation, justification and reflection. The traditional hierarchy of teacher as autocratic knower and learner as unknowing, controlled subject studying and practicing what the teacher knows begins to dissipate as teachers assume more as facilitator's role and learners take on more ownership of the ideas. Indeed, autonomy, mutual reciprocity of social relations and empowerment become the goals. This theory hypothesizes that individuals will try to make sense of all information that they perceive and that each individual will therefore construct their own meaning from that information.

Constructivist Based Instruction in Mathematics Education

Public Senior High Schools Curricula, in many countries give less emphasis to mechanical symbol manipulation abilities, in part because this kind of mathematics can be done by computer and because of an increasing concern for more flexible problem-solving skills. New curriculum proposals also reject traditional teacher-centered pedagogy and favor student-centered approaches.

Simon, Goodchild and English, (2005), postulated that learners are responsible for constructing their own knowledge. Learning is dependent upon existing knowledge,

which has been constructed in the context of prior experience, and hence individuals are likely to construct differing knowledge in response to given experience (Glasersfeld, 1989).

Problem Statement

Our experience as mathematics teachers have made us identify that students are having conceptual problems in solving logarithms especially Form two students. We have chosen Constructivist approach because of its important in Mathematics Education and the relevant to the understanding of logarithmic functions. This is because at the basic level of education as pointed by Mereku (2001), the Ghanaian mathematics teacher is regarded as a demonstrator of process and transmitter of information and taught largely through lecturing and teacher-centered approaches. From class work, class test and end of term examination assessment, it came to light that most of the students' have difficulties in solving logarithmic functions.

Thus recurring student failures in logarithms problems test warrant the used of Constructivist approach to enhance students understanding of logarithmic functions.

Purpose of the Study

We aimed to determine possible misconceptions of Kalpohin Senior High School students in applying properties of logarithms so as to design meaningful and effective instructional activities to enhance their understanding. This paper came out with an intervention strategy using Constructivist Based Instructional Approach which enhanced the performance of Kalpohin SHS students' understanding of logarithmic functions.

Research Questions

The research sought answers to the following questions:

1. What is the effect of the use of constructivist based instruction on students understanding of concept of logarithmic functions?
2. To what extent can constructivist based instruction improve students' ability to solve problems on logarithmic functions?

Significance of the Study

This paper helped deepen students' understanding of logarithmic functions through the design of Constructivist Based Instruction and better equip those who plan curriculum and teach students to meet the needs of Senior High School students in Mathematics.

Finally, the paper would be of great importance to educational planners especially Ministry of Education, Ghana Education Service, Teacher Education Division, West African Examinations Council, other beneficiaries of education as well as organizations that do have roles to play in the promotion and development of mathematics education in Ghana.

Literature Review

This aspect deals with the theoretical framework of the paper and what other researchers have written on solving logarithmic functions.

Theoretical Framework

According to Umbarger (2010) Mathematics is the learning and understanding of ideas, theories and rules that stay with you for years and allow you to attack and solve problems that are not in the same style as the problems the teacher solved when teaching any Mathematical topic. The presence of programmable/ scientific calculators today means that even most teachers have not experienced the joys of working with logarithm tables. Doing so will replace the mystery of the study of logarithms with a deep appreciation and understanding of logarithms ideas and concepts that will stay with the student for a longer period of time.

Empirical evidence from research work of Heather (2011), points to the difficulties in teaching logarithms in schools. Students find logarithm related questions difficult and therefore see 'log' as a common factor in expression $\log x + \log y$ and write it as $\log(x+y)$. Students have difficulty in recognizing logarithm as a number (Berezovski and Zazkis, 2006).

Weber (2002b), for instance used the theory of Action, Process, Object and Schema or APOS theory to study students understanding of exponentiation and logarithm functions. He reported that most of the students involved in the study could only understand exponentiation as action but not do so as a process. While Berezovski & Zazkis (2006) split "understanding logarithms" into three categories: logarithms as numbers, logarithms as operations and logarithms as functions. Heather, (2011) argued that their framework was not complete because students found logarithms difficult; hence the framework was modified to include logarithm in contextual problem.

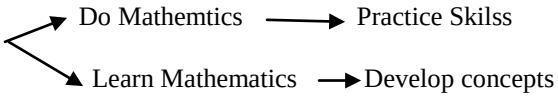
Weber (2002b) indicated that learners being able to view exponentiation as action and process are the ones who can compute b^x as b x times and when they repeat the action and reflect upon it, they interiorize that action as the process (Dubinsky, 1991). Terms such as 2^4 can be interpreted as an external prompt for the student to compute 2x2x2x2, which is a product of four factors of 2. Research indicates that students are not capable of viewing 2^4 in this way.

A student understanding of a mathematical concept is his or her collection of privately held beliefs about the concept. This definition does not imply that a student with a collection of beliefs about a concept understands it. Certainly, a student who believes the logarithmic function is a number such as e or π does not understand the logarithmic function. Instead, having beliefs implies that a student has an understanding of the concept. There is a great distinction between a declaration that a student understands and that he/she has an understanding. Mathematics teaching is meant to encourage the growth of a system of beliefs within the student that are consistent with culturally accepted beliefs. A student is said to understand a mathematical concept when based on an analysis of available evidence, the system of beliefs attributed to the student is consistent with culturally accepted beliefs about the concept. It is evidence of consistency that is used to decide whether or not a student understands a mathematical concept. Before we describe our framework, it should be explained what criteria made us to add a constructivist approach to our framework. The reason for considering this constructivist approach in our framework was that a lack of understanding of its application would lead to difficulties in solving certain

types of problems involving logarithmic functions. The other criterion for including constructivist approach in our framework was that it would apply to several kinds of problems involving the logarithms.

This approach is based on intervention strategies through well design activities where students might use to their advantage when solving many types of problems involving logarithms. For example, having seen a student try to solve an equation by factorizing "log" prompted us to include constructivist based instruction in understanding logarithmic notation and functions in our framework.

Dedactical Function of contructivism in Mathematics Education:



Constructivist Activities about Logarithm Concepts

Activity 1

We guided the students through the laws and properties of logarithm. Examples can be seen below:

i) The separation of two logarithmic functions by addition sign means "Multiply". That is $\log x^1 + \log 3x$

Answer: $\log 3x^3$

ii) The separation of two logarithmic functions by subtraction sign means "Divide". That is $\log 2x - \log 5$

Answer: $\log(\frac{1x}{5})$

iii) Logarithm of a function with an exponent, that is $\log x^1$

Answer: $2\log x$

Activity 2

Activity two was done to teach students how to change logarithmic functions to exponential functions and operations using constructivist approach of teaching.

i) Change $\log_1(x+1) + \log_1 3 = 0$ to an exponential function.

We realized that many of the students were able to write the correct answer. However, some still wrote $2^{3(x+1)}$ instead of $2^0 = 3(x+1)$.

We then explained;

Apply the addition law of logarithm, that is $\log_1 3(x+1) = 0$, then

Answer: $2^0 = 3(x+1)$

Activity 3

Before the Post-test, we guided the students through the following intervention process. In activity three, we guided the students to solve logarithm problems using constructivist approach of teaching.

a) Read the question carefully and analyze it based on the laws and properties of logarithms.
b) Equate the logarithmic function to a variable and solve for the variable.
c) Change the logarithmic function to exponential function and solve.
d) Determine the solution set of the given question by substituting final answers to the question and see whether the Left Hand Side (LHS) equals the Right Hand Side (RHS). Examples:

i) Solve $\log_{10}(10+9x) - \log_{10}(11-x) = 2$

Me: What are we trying to find?
Students' response: The variable x

Me: Which law are we applying?
Students' response: Divisibility law of logarithm.
Me: Write applying the law.
Students' response: $\log_{10}(\frac{10+9x}{11-x}) = 2$

Me: What are we to do?
Students' response: Change logarithm to exponential form.
Me: Apply the exponential form.
Students' response: $10^1 = (\frac{10+9x}{11-x})$

Me: What are we to do next?
Students' response: Simplify
Me: Solve the equation
Students' response: $10 + 9x = 100(11-x)$

$10 + 9x = 1100 - 100x$

$9x + 100x = 1100 - 10$ (Grouping of like terms)

$109x = 1090$ (Divide by 109)

$\frac{109x}{109} = \frac{1090}{109}$

$x = 10$

Me: Check the answer
Students' response: LHS=10+9(10) RHS=100(11-10)
= 10+90 = 100(1)
= 100 = 100
LHS = RHS

II. METHOD

Research Design

The research design for this paper was action research in which a constructivist based instructional approach was employed to measure students' understanding of logarithmic function, their attitudes and views about constructivist based instruction. It involves the application of appropriate intervention strategies aimed at finding solutions to problem(s) identified in teaching-learning situation in order to bring about a change.

Population

The target population for this paper was all the students of Kalpohin Senior High School in the Northern Region of Ghana. The school has a population of one thousand seven hundred and eighty one (1781) students. Five hundred and seventy five (575) of these students are in form two comprising of three hundred and sixty six (366) boys and two hundred and nine (209) girls. The research was conducted in form two classes of Agricultural Science and Home Economics Science. The average age of students in these classes was seventeen (17) years and they came from different regions of Ghana.

Sample and Sampling Technique

The sampling strategy employed in this paper was purposive sampling. The researchers used this strategy because two departments were involved, Agricultural Science and Home Economics Science department. All Form two Agricultural and Home Economics Science students of Kalpohin Senior High School constituted the subjects for the paper. The sample size was Sixty (60) and they were both boys and girls with an average age of 17 years.

Instrumentation

Data was collected by means of Logarithm (Pre-test), Logarithm (Post-test) and unstructured interview guide. Students were given Pre-test questions that required application of logarithmic concepts in solving logarithm problems, followed by converting logarithmic functions into exponential functions. The Post-test followed the same questions like the Pre-test. Makgakga and Sepeng (2013) used the same questions for both Pre-test and Post-test since it is an Action Research and in order to avoid contamination of results (Mcmillan & Schumacher, 2010).

Some few students were selected for interview after the Pre-test and Post-test to explain how they solve the logarithm problems.

According to Chua & Wood (2005), the knowledge or computation category comprised routine questions that require direct recall or application of the definition and laws of logarithms, as well as simple manipulation or computation with answers obtained within two to three steps. The items in the understanding category do not just simply involve recalling the definition or the application of logarithmic laws, but require some understanding of the underlying concepts of logarithms. The items may be familiar or textbook-like, but in them the student must decide not only what to do but how to do it. Finally, items in the application grouping require the students to develop their own techniques for solving problems that they probably have not met in a textbook.

Table 3.1: Evidence of Sample Test Items by Category

CATEGORY	ITEM
Knowledge or Computation	Given that $a^m = 36$, find $\log_a 36$, in terms of m.
Understanding	Given that $a^m = 36$, find $\log_6 a$, in terms of m.
Application	Find the greatest possible integral value of p and q given that $p < \log_{10} 500 < q$

In Ghana, the Ministry of Education Teaching Syllabus for Core Mathematics (Senior High School), CRDD-2010 Distribution of Examination Paper Weights and Marks is similar in terms of the above table as illustrated below:

Dimensions	Paper 1	Paper 2	SBA	Total Marks	Total Marks Scale to 100
Knowledge and Understanding	30	20	10	60	30
Application of Knowledge	10	80	50	140	70
Total Marks	40	100	60	200	
% Contribution of Examples Papers	20	50	30		100

Intervention

The findings from the Pre-test result necessitated the design and implementation of an intervention. This is because the active and the motivated nature of learning is not optimized in the "old" style of classroom where students sit quietly and passively receiving the words of wisdom being given to them by lonely instructor standing in front of the class (Robert, 1994).

At the intervention stage the researchers applied the constructivist based instruction approach of teaching to help address students' challenges in understanding of logarithmic functions. The researcher also adopted tasks in the form of practical activities to help enhance students understanding of logarithmic functions. This meant relating classroom activities to learner's life experience to enable them see the relationship between what is taught in school and what is done at home thereby facilitating transfer of teaching.

During the intervention stage, the students were given strategies, opportunities and responses to what they were doing. The students were taken through by activity one, in order for them to discover the laws and properties of logarithms. The researchers as facilitators based on their pedagogical knowledge content guided them through the conditions necessary for applying the laws and properties.

Data Analysis and Results

Descriptive and Inferential statistics were used as tables and data analyzed using Statistics Package for Social Sciences (SPSS).

III. RESULTS

Table 4.1: Students' scores on the key abilities and skills assessed in the pre- and post- tests

		Pre-Test		Post-Test	
Function or Skill	Students Ability	Number	Percent	Number	Percent
Logarithmic	Able to analyze	17	28	58	97
	Unable to analyze	43	72	2	3
Exponential	Able to solve	16	27	53	88
	Unable to solve	44	73	7	12
Solution set	Able to determine	5	8	52	87
	Unable to determine	55	92	8	13

From Table 4.1, the number of students who answered questions correctly based on analysis of laws and properties of logarithms in the post test were 58 representing 97% as against only 17 representing 28% in the pre-test. Many of the students were able to answer these questions very well because during the intervention stage they were exposed to better way of memorizing the laws and properties of logarithms using the constructivist approach. This indicated that if students are given tasks and guided they are able to come out with the correct solutions by themselves.

Research Question 1: What is the effect of the use of constructivist based instruction on students understanding of concept of logarithmic functions?

In finding an answer to the research question 1, paired sample t-test was applied using the Pre-test and Post-test scores. Tables 4.2 and table 4.3, below shows the comparative analysis of the pre-test and the post-test results of the whole 60 students of Kalpohin Senior High School form two.

Table 4.2: Descriptive Statistics of Students' Scores in the Pre-Test and Post-Test

	N	Mean	Std. Dev.	Std. Error Mean
Pre-test percent score	60	31.35	19.618	2.533
Post-test percent score	60	72.05	15.527	2.005

The results therefore show that the students had understood basic concepts and strategies involving logarithmic functions and for that matter their performance improved tremendously after the intervention.

Table 4.3 reports a paired sample test carried out to compare the means of the two groups and also test the null hypothesis that there is no significant difference between the means of the groups as the data met all the assumptions of pair t-test.

Table 4.3: Paired Sample Test Comparing the Means of each Group

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Dev.	Std. Error Mean	Lower	Upper	T	Df	Sig. (2-tailed)
Pretest – Posttest	-40.70	20.79	2.68	-46.07	-35.33	-15.17	59	.000

The results of the paired sample t-test conducted to verify mean difference between a pre-test and post-test scores indicated there is significant improvement of Posttest scores (M = 72.05, SD = 16.62) over Pretest scores (M = 31.35, SD = 15.53) at $\alpha < 0.05$ level, with conditions: [t (59) = -15.17, p = 0.000]. We therefore reject the null hypothesis (H_0) and conclude that there is significance difference between the Pre-test and Post-test scores of students when taken through the use of Constructivist approach (Table 4.3).The analysis also shows that there was a significant change in the students' understanding in solving logarithmic functions.

One major category in the students responses were about boosting interest in logarithm. Students frequently expressed how they enjoyed the activities. Almost all of the students indicated that they enjoyed logarithm lessons with Constructivist Approach.

Another category in students responses were about motivation. They frequently expressed that the activities increased their motivation toward mathematics lesson. There was one of the student said that "when you learn it yourself, you say I can do it, I am successful in logarithm. You can pay attention, you enjoy and your interest increases." One student replied that learning benefits of logarithm in daily life took our attention.

The third category in students responses were about retention. They frequently expressed that the activities was effective in terms of retention of the topic logarithm. One student said "I will never forget the formulas for $\log(a.b)$ and $\log(a/b)$." Similarly, another student replied that "instead of memorizing the formulas that the teacher gave to me, I understood it better by discovery it on my own.

IV. Discussion and Conclusions

The overall impact in using Constructivist Based Instructional approach was positive on students understanding and solving of logarithmic function. The statistical difference showed that the intervention strategy, Constructivist approach improved students' problem solving skills as seen in the Post-test. Using this approach, students were able to apply basic concepts and strategies in learning of logarithms (Mulqueeny, 2012). This indicates that teachers do teach logarithmic function without the use of any activity, but rather routine method. This makes students unable to interact with each other during the lesson. Students' interaction also plays a major role in their learning especially when they work in groups and criticize their own classmates work.

This paper revealed that students did not know that they could learn better from their classmates until the intervention stage. To investigate whether the Constructivist approach will result in improving performance among Form two Agricultural Science and Home Economics Science students of Kalpohin SHS, the paper was guided by two research questions:

Research question 1: What is the effect of the use of constructivist based instruction on students understanding of concept of logarithmic functions?

A paired sample t-test was conducted to verify mean difference between pre-test and post-test scores. The results indicated a statistical significantly difference of means of

post-test scores (M=72.05, SD=16.62) over pre-test scores (M=31.35, SD=15.53) at $\alpha < 0.05$ level of confidence with conditions: [t (59) = -15.16, P= .000].

We therefore concluded that the used of Constructivist based instruction approach indeed has enhanced students understanding of concept of logarithmic functions.

Research question 2: To what extent can constructivist based instruction improve students' ability to solve problems on logarithmic functions?

The interviewed results of students views and opinions toward the used of constructivist based instruction approach revealed higher frequencies of responses of students on increase interest and motivation, retention and self-confidence of their understanding and ability to solve more questions on logarithm functions.

In Constructivist instruction there were process of discovery, utilization of technology and applications of the properties of logarithms (Cetin, 2004). The interviews were made to describe their thoughts about logarithm concepts and their opinions about implementation of Constructivist Based Instruction (CBI) in Mathematics lessons.

The results after both Pre-test and Post-test was consisted with Makgakga and Sepeng (2013) that reported a significant mean difference of pre-test and post-test results.

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