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The Study on Group Self -Pooling and Annuitization Schemes Payment on Dynamic Motality Model

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Abstract— **As the mortality rate declines year by year and the increase of life expectancy of population brings tremendous pressure on our pension system, it is particularly important to accurately predict the mortality rate. Due to the limited population mortality data in China, this article based on the Lee-Carter model, using Co-integration theory to overcome the limitation of ARIMA model and construct a prediction model of male mortality in China. Meanwhile a Group Self-Annuitization (GSA) with higher payment was introduced and the predicted mortality rate was substituted into the GSA model. Finally we give the endowment insurance proposal that is suitable for our country's national conditions, in the real sense, we can achieve the goal that the citizens in our country should be "empowered and old".**

Index Terms— **Lee-Carter Model; Co-integration Theory; Longevity Risk;Group Self-Annuitization.**

I. INTRODUCTION

With the dramatic development of the social economy, the gradual improvement of the living environment, the rapid increase in the level of medical care and the improvement of people's health awareness, the mortality rate of the population in various countries is declining. The aging of the population is becoming increasingly serious. China has stepped into an aging population society. As a result, a series of problems such as the shortage of funds in the national social security system, the slow development of enterprise annuities, and personal retirement income are insufficient to ensure a better quality of life. This will become a major social risk in China. It is based on China's current population status. At present the phenomenon of large scale, rapid development of aging, and "not getting rich first" has become a constraint on the development of China's population. Mortality is an important factor in the design of bonds for longevity risks and the design of old-age pensions, so the accurate prediction of future mortality rates will change. It is much crucial.

In order to improve the reliability of mortality model predictions, scholars have continued to explore and research. The most influential model in the world is the Lee-Carter model proposed by Lee and Carter (1992)[1], which contains future trends. The Lee-Carter model is simple and easy to understand and relatively robust. Many subsequent studies are based on this model. However, the Lee-Carter model has some deficiencies, such as lack of robustness in parameter estimation, and there is controversy about the fit of k_tLi (2004)[2] proposed the method of predicting the Lee-Carter model under limited data. Broumns (2002a)[3] applied the Bootstrap method to the Poisson model to quantify the longevity risk. Renshaw (2006)[4] the mortality model of the birth year effect, Chen (2007) and Hainaut (2007)[5] studied the mortality model with jumps, reflecting the impact of warfare and sudden epidemics on mortality. Meng (2010)[6] aimed to model the time items in the Lee-Carter model through a double random process aiming at the small sample size and lack of data in China's mortality data, and found that the improved Lee-Carter model is more suitable Predicting the current population mortality rate in China. Wang Xiaojun (2012)[7] conducted an empirical analysis of the current number of months for the use of personal accounts for basic old-age insurance by constructing a forecast model of China's population mortality under limited data. It is recommended that the government Establish adjustment mechanisms for the number of months of calculation. Yang (2013)[8] based on the Lee-Carter model, and found that the mortality factors in many countries have a long-term equilibrium relationship by studying the correlation between the mortality rates in the United States and the United Kingdom. Yan (2015)[9] found that the male mortality rate of Chinese males and Chinese males has a long-term equilibrium relationship through co-integration analysis, and combined with the extreme value theory to predict the Chinese mortality rate.

However, with the increase of the elderly population and the prolonged life expectancy, the longevity risk has an increasingly significant impact on China's social security system. This requires a suitable method to achieve effective transfer. John Piggott (2005)[10] proposed Consumers bear the risk of system longevity, and the concept of Group Self-Annuitisation is the concept of group self-Annuitisation, and they do not provide a good overview of the characteristics of the GSA model, and The derived payoff recursive model is not perfect. Jonathan Barry Forman (2014) [11] studied the scope of application of the GSA model from the perspective of the size of the company personnel. Moshe A . (2015)[12] solved the Euler-Lagrand solution. The Japanese equation is based on the structure of the GSA model that is maximized for final effectiveness, and tests the structure's sensitivity to the number of annuities and the risk of inverse selection.

This paper uses the co-integration theory of economics proposed by Zhang Yi et al. to introduce mortality prediction. The Lee-Carter model of male population mortality data in China and Taiwan is used to estimate the parameters of Chinese male mortality, which makes up for the historical mortality in China. The errors caused by missing data. Then Monte Carlo simulation analysis of GSA pension payment model and the distribution of the annual payment, to provide a new idea for our government, the insurance industry to solve the longevity risk.

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II. MODEL INTRODUCTION

A. Lee-Carter model

Assume that the x year old's central death rate $m_x(t),(t = t_1,t_2,\cdots,t_n)$obeys the logistic bilinear model in the t year, ie

$$\ln m_x(t) = \alpha_x + \beta_x k_t + \varepsilon_{x,t}\,. \quad (1)$$

That is α_x represents the trend of death rate with age x, which is the average of the x-year log mortality rate; β_x is a specific age parameter, indicating the sensitivity of age x to the change of mortality; k_t indicates the mortality of each age group. The time factor of the logarithmic trend; $\varepsilon_{x,t}$ represents the impact of historical information not captured by the model, $\varepsilon_{x,t}$is independently and uniformly distributed and$\varepsilon_{x,t}\sim N(0,\sigma^2)$. It is generally assumed that $\sum_x \beta_x = 1, \sum_t k_t = 0$. It can be obtained by least square method:

$$\hat{\alpha}_x = \frac{1}{t_n - t_1 + 1}\sum_{t=t_1}^{t_n} ln\hat{m}_x(t)\,. \quad (2)$$

The singular value decomposition (SVD) of the matrix $\ln m_x(t) - \alpha_x$ is used to obtain the estimates of $\hat{\beta}_x$ and $\hat{k}_t$. The $\hat{k}_t$ is quadratic fitted according to the variation of the time series $\hat{k}_t$, and then the predicted year k is extrapolated$\acute{k}_t$. Finally, using the prediction values of $\acute{k}_t, \hat{\alpha}_x$, and$\hat{\beta}_x$, get the forecast year$\dot{m}_x$,we get

$$\ln\dot{m}_x(t) = ln\hat{\alpha}_x + \hat{\beta}_x \acute{k}_t\,. \quad (3)$$

B. Cointegration Relationship

Using the Lee-Carter model to predict mortality, many people establish an ARIMA model for k_t, which requires a large amount of historical data and requires k_t to be a stationary time series. However, in practice, it is found that k_t is usually a non-stationary time series. Therefore, the method of predicting k_t using ARIMA model tends to have large deviations in data missing China. Yang et al. (2013) found that although k_t in a country is not a stationary time series, there may be co-integration relations among k_t in many countries. Engle et al. proposed the concept of Co-integration. They found that some time series are non-stationary, but their linear combination is smooth or lower order.

In order to verify whether the two time series k_{t_1} and k_{t_2} are cointegrated, Engle and Granger (1987) [13] proposed a two-step test (EG test).

The first step: After ADF and PP inspection, if both k_{t_1} and k_{t_2} are monotonic, one variable can be used for regression of another variable, by k_{t_1}=a $+ b k_{t_2} + e_t$, the estimated value of the model residual e_t is $\hat{e}_t = k_{t_1} - \hat{a} - \hat{b}k_{t_2}$, where $\hat{a}$ *and* $\hat{b}$ is the estimated value of a, b.

Step 2: Check whether $\hat{e}_t$ is stable, and determine if there is a co-integration relationship betweenk_{t_1}andk_{t_2}. If $\hat{e}_t\sim I(0)$, ie. $\hat{e}_t$ is a stationary sequence, then k_{t_1} and k_{t_2} has a cointegration relationship and $k_{t_1} = a + bk_{t_2} + e_t$ is a cointegration regression equation; otherwise k_{t_1} and k_{t_2}have no cointegration relationship.

C. Derivation of mutual benefit pension payment model

The declining mortality rate leads to a continuous increase in the life expectancy of the population. This has brought great pressure on the pension system in China. The ordinary old-age pension has been unable to pay the huge expenditures caused by the longevity risk. John Piggott et al. derived the principle of fair actuarial per The calculation method of the adjustment factor obtains the recursive model under general conditions:

$$ {}_x^kB_{i,t}^{*} = {}_x^{k-1}B_{i,t-1}^{*} \times \frac{F_t^{*}}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1}}\sum_{A_t} {}_x^kF_{i,t}^{*}} \times \frac{\ddot{a}_{x+t,t-1}}{\ddot{a}_{x+t,t}} \times \frac{1+R_t}{1+R}$$

$$= {}_x^{k-1}B_{i,t-1}^{*} \times \mathrm{MEA}_t \times \mathrm{CEA}_t \times \mathrm{IRA}_t\,. \quad (4)$$

$$\text{namely } \mathrm{MEA}_T = \frac{F_t^{*}}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1}}\sum_{A_t} {}_x^kF_{i,t}^{*}},$$

$$\mathrm{CEA}_t = \frac{\ddot{a}_{x+t,t-1}}{\ddot{a}_{x+t,t}}, \mathrm{IRA}_T = \frac{1+R_t}{1+R}.$$

MEA_t is called the death rate adjustment factor, CEA_t is called the independent change adjustment factor, and IRA_t is called the interest rate adjustment factor. ${}_x^k\mathrm{B}_{i,t}^{*}$ indicates that x years old enters the mutual aid pension plan and by t Join the amount of pension received by the j member of K-year pension plan; p_x represents the expected survival probability of x~x+1 years; R_t represents the investment rate set at the initial time; R_t represents t~t+1 The actual investment rate of return; F_t^{*} indicates the total funds of all surviving annuity pools at time t; ${}_x^k\mathrm{F}_{i,t}$ indicates that the i-th member t has not yet received between t-1~t moments. The total amount of funds when the dead person's funds are allocated; ${}_x^k\mathrm{F}_{i,t}^{*}$ indicates the balance of the i member t in the annuity pool; A_t indicates the number of survivors at t, $\ddot{\mathrm{a}}_{x+t}$ represents the actuarial present value of an ordinary life annuity paid annually by an individual in year t.

For members added at t=0, the initial fund is ${}_x^0\hat{\mathrm{F}}_{i,0}^{*}$, and the amount of pension received ${}_x^0\mathrm{B}_{i,0}^{*} = \frac{{}_x^0\hat{\mathrm{F}}_{i,0}^{*}}{\ddot{\mathrm{a}}_x}$.This article assumes that the expected investment return rate is equal to the actual investment rate of return (R = 5%), which is equivalent to the force of interest:

$$\delta = \ln\frac{1}{1+R} = 0.04879064\,.$$

The old age pension received is the average amount calculated based on the expected present value factor of the ordinary life annuity:

$$ {}_x^kB_{i,t}^{*} = \frac{{}_x^k\hat{F}_{i,t}^{*}}{\ddot{a}_{x+t}} \quad (5)$$

According to the mortality credit, ${}_x^k\hat{\mathrm{F}}_{i,t}^{*}$ represents the total amount of funds when the i-th member t receives the funds allotted for death between t-1 and t.

John Piggott et al. believe that MEA_t, CEA_t are separate, the mortality adjustment factor allows to deviate from the expected death toll of the GSA fund, and the changing expectation adjustment has a permanent impact on the potential mortality. These two effects are difficult to distinguish, for example, the GSA plan. The occurrence of additional deaths may be a result of a random deviation from expectations, or it may be the result of a permanent increase in potential mortality. These two regulatory factors were included in the single adjustment factor by Chao Qiao(2013)[14], and their derivation process Improve and obtain a GSA payoff recursive model that is closer to the actual situation. The mortality rate predicted by the random dynamic model is denoted by$\mu_{x,t}$, and the probability that an x year old individual has survived s year in the tth year is:

$${}_sp_{x,t} = exp\left[-\int_x^{x+s}\mu_{z,t}dz\right].$$

The actuarial present value of the ordinary life annuity paid annually by the x-year-old individual in year t is: $\ddot{\mathrm{a}}_{x+t} = \sum_{s=0}^{\infty} \mathrm{e}^{-\delta s}\ {}_s\mathrm{p}_{x,t}$. The i member of the age of x joins the GSA plan at the time t=0 and receives the old age pension as ${}^k_x B^*_{i,t} = \frac{{}^k_x \hat{F}^*_{i,t}}{\ddot{a}_{x+t,k}}$.

Assuming that the annuity pool is not closed, new participants may be added at any time t:

$$B^*_t = \sum_{k\geq 1}\sum_x\sum_{A_t} {}^k_x B^*_{i,t},$$

$$F^*_t = \sum_{k\geq 1}\sum_x\sum_{A_t} {}^k_x \hat{F}^*_{i,t}.$$

The recurrence formulas for individual annuity pool balances and collection annuities are expressed as:

$${}^k_x F^*_{i,t} = \left({}^{k-1}_{x}\hat{F}^*_{i,t-1} - {}^{k-1}_{x}B^*_{i,t-1}\right)e^{\delta}, \qquad (6)$$

For the entire annuity pool: $F^*_t = \hat{F}^*_t$.

The relationship between old age pensions received over a continuous period of time is given by an adjustment factor:

$$\text{TEA}_t = \frac{F^*_t}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1,t-1}}\frac{\ddot{a}_{x+t,t}}{\ddot{a}_{x+t,t-1}}\sum_{A_t} {}^k_x F^*_{i,t}}.$$

TEA_t is the total adjustment factor. Note that TEA_t is the total adjustment factor. And TEA_t is not equal to $\text{MEA}_t \times \text{CEA}_t$. They are

$$\text{MEA}_t = \frac{F^*_t}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1,t-1}}\sum_{A_t} {}^k_x F^*_{i,t}},$$

$$\text{CEA}_t = \frac{\ddot{a}_{x+t,t-1}}{\ddot{a}_{x+t,t}}$$

This means that CEA_t depends on the queue in which the participants are located. It is necessary to subjectively judge how to convert the actual survival information into a queue-dependent permanent improvement factor and a queue independent fluctuation factor MEA_t. The single factor TEA_t of the queue can be Avoiding the uncertainty of subjective judgments, the whole queue will bear the longevity risk of the system. If each queue is improved in terms of receiving annuities and income fluctuations, then the sharing principle will benefit all members of the annuity pool.

The following formula is used to calculate the balance of funds for participants in multiple queues:

$$\sum_{k\geq 1}\sum_x\sum_{A_t} {}^k_x \hat{F}^*_{i,t} = \sum_{k\geq 1}\sum_x\sum_{A_t} \frac{{}^k_x F^*_{i,t-1}}{\ddot{a}_{x+t,t-1}}\ddot{a}_{x+k,t} \times \text{TEA}_t \times{}' a_{x+k,t}$$

$$= \sum_{k\geq 1}\sum_x\sum_{A_t} \frac{\frac{{}^k_x F^*_{i,t}}{p_{x+k-1,t-1}}}{\sum_{k\geq 1}\sum_x\sum_{A_t}\frac{{}^k_x F^*_{i,t}}{p_{x+k-1,t-1}}} \times F^*_t.$$

$${}^k_x \hat{F}^*_{i,t-1} = \frac{\frac{{}^k_x F^*_{i,t}}{p_{x+k-1,t-1}}}{\sum_{k\geq 1}\sum_x\sum_{A_t}\frac{{}^k_x F^*_{i,t}}{p_{x+k-1,t-1}}} \times F^*_t. \qquad (7)$$

This expression can simply be interpreted as the participant's inherited amount is a weighted distribution of the total balance available in the annuity pool, weighted by ${}^k_x F^*_{i,t}$ and $p_{x+k-1,t-1}$ common decision. ${}^k_x F^*_{i,t}$ according to formula (6), the old-age pension ${}^k_x B^*_{i,t}$ obtained by formula (5) is obtained out.

III. Empirical Analysis

A. Data Selection and Processing

Given that there are many similarities between the process of population aging in Taiwan and the fact that there are sufficient mortality data in Taiwan, this makes the Taiwan mortality rate data more valuable for China. Therefore, this paper chooses to use Taiwan's population mortality data for China. Population mortality data are co-integrated. The data on Taiwan's population mortality are derived from the Human Life Table Database, and China's population mortality data are derived from the "China Population and Employment Statistics Yearbook" and "China Demographic Yearbook." To maintain data consistency. This paper merges the 0-year-old and 1-4 years-old data of Taiwan's primary population into 0-4 years old, then adopts a method of grouping every five years old, and determines that the highest age group is 90 years old and above. The rate data is divided into nine groups of 0-4, 5-9, …, 84-89, and 90 years of age according to age. At the same time, the data is divided into two parts. The first part (1997-2012) is used to fit the model. The second part (2013~2014) is used to validate the model.

B. Comparison of parameters between China and Taiwan

The data of male population mortality in China and Taiwan from 1997 to 2012 was substituted into the matrix $\ln m_x(t) - \alpha_x$. The estimated values of the parameters α_x, β_x and k_t were obtained by the singular value decomposition method, as shown in Fig. 1 to Fig. 3, respectively.

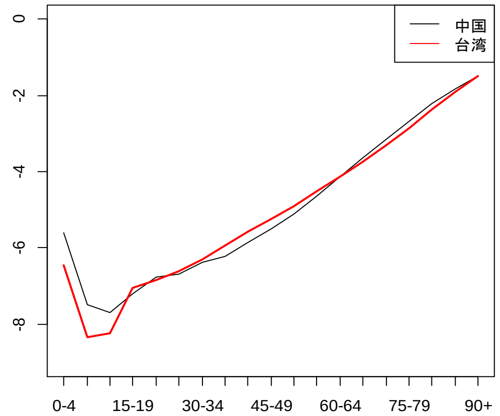

Figure 1 Estimate of male mortality rate α_x in Taiwan and China under the Lee-Carter model

The parameter α_x represents the average level of the x-year logarithmic mortality rate. It can be seen from Figure 1 that α_x shows a trend of decreasing first and then increasing with age, reflecting the tendency of the average of mortality rate to change with age, and The age-related death rates of the Chinese male population are roughly the same, that is, the average mortality rate of 0-4 years is higher, then it is lower, it reaches the lowest level when it is around 10 years old, and then it increases with age.

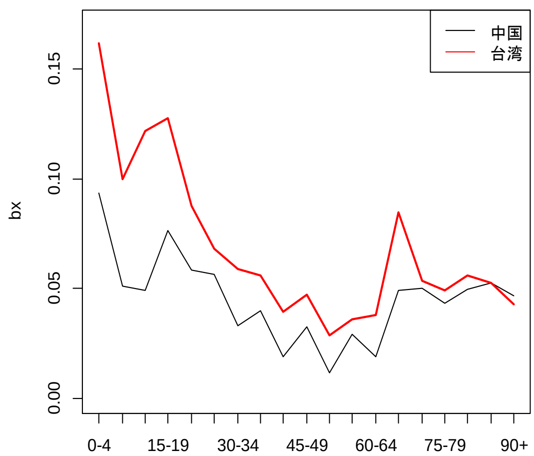

Figure 2 Estimation of male mortality rate β_x in Taiwan and China under the Lee-Carter model

The parameter β_x indicates the sensitivity of each age group to the change in mortality rate. As can be seen from Figure 2, with the increase of age, β_x basically shows a downward trend, and the value of β_x in the low age group is high, indicating that the change in mortality rate is even more significant. Sensitive; the β_x value of the high age group tends to 0, indicating that as the age increases, the population in each age group is no longer sensitive to changes in mortality. It is noteworthy that the β_x values in China and Taiwan have roughly the same trend. This is consistent with the sensitivity of changes in mortality levels across all age groups.

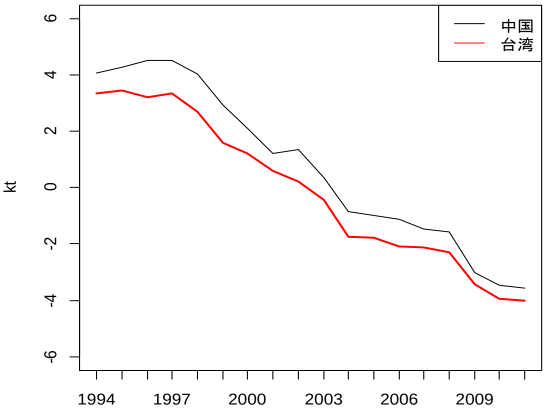

Figure 3 Estimated k_t for male mortality in Taiwan and China under the Lee-Carter model

The parameter k_t the overall level of mortality as a function of time. It can be seen from Figure 3 that k_t decreases year by year with the increase of natural years, which is consistent with the fact that the overall population mortality rate tends to decline, and that the Taiwan region and the Chinese mainland The k_t sequence in the region has a long-term stable trend, indicating that there may be a co-integration relationship between the two.

C. Prediction of China's Population Mortality Based on Cointegration Theory

In view of the lack of data for some years in China and the serious lack of demographic mortality data, this article draws on the method of coordinating k_t with the time series of Chinese population mortality in Taiwan proposed by Chen Ning (2015)[15], using the Lee-Carter model combined with Bell ((1997) proposed another method for predicting $\hat{m}_x(t)$, ie

$$\ln\dot{m}_x(t) = \ln\hat{m}_x(t_n) + \hat{\beta}_x\left(k_t - k_{t_n}\right), t > t_n \quad (8)$$

forecast China's population mortality.

Based on the characteristics of the k_t time series and the estimated time-to-factor k_{t_2} of the male population in Taiwan, a co-integration test was performed using the EG two-step test. The residual sequence was found to be a stationary time series, indicating that the mainland China and Taiwan The regional death time factors k_{t_1}, k_{t_2} have a cointegration relationship, so that the predicted value of the Chinese man's death time factor k_{t_1} is calculated, and the formula (8) is used to obtain the predicted value of the Chinese male mortality rate. Then compare the actual data from 2013 to 2014 with the forecast data, as shown in Figure 4.

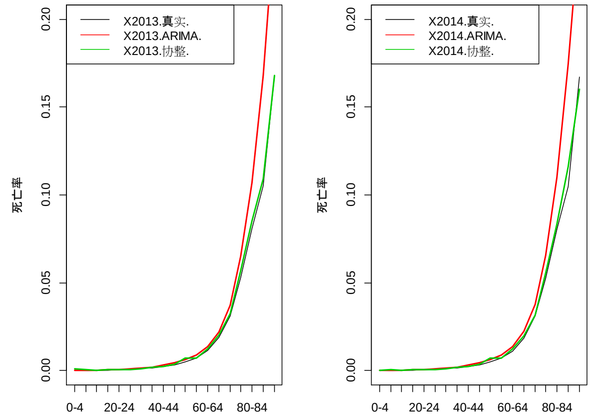

Figure 4 Comparison of actual and predicted mortality of Chinese male population from 2013 to 2014

Figure 4 shows that compared with the cointegration model, the ARIMA model predicts a high mortality rate, especially for population data over 65 years old, and the use of cointegration theory model to make the prediction of mortality more accurate. Except individual age groups There is some difference between the predicted value of mortality and the actual value, and other data fit well with the actual value. This shows that it is reliable to use the data of Taiwan's population mortality data to cope with the Chinese population mortality data. The mortality rate can be further extrapolated. The male mortality rate data for the next 15 years in China is shown in Figure 5.

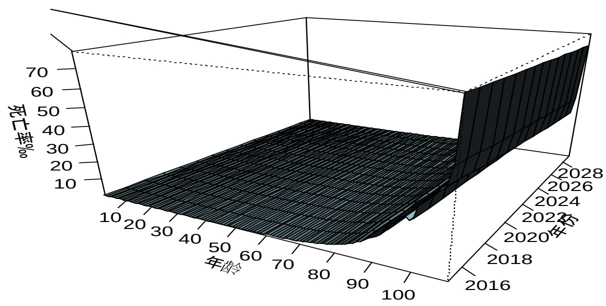

Figure 5 Mortality Prediction of Chinese Male Population from 2015 to 2029

From Figure 5, it can be seen that in the same year, the male population of China has an increasing mortality rate with age. In the same age group, with the increase of the year, the mortality rate shows a declining trend, which is similar to the mortality rate. The fact of diminishing is consistent. In summary, the average life expectancy of the future population is showing an increasing trend, and the longevity risk is increasingly serious.

D. Empirical Analysis of GSA Model

With the mortality forecast data, we can empirically analyze the payment models in China's old-age pension and social insurance. Assuming that the initial time is 2018, a group of male retirees, aged 65, will join the GSA plan with initial funding of 10,000 yuan. Using the mortality rate predicted in Section 3.3 and the recurrence formula of Section 2.3, the Monte Carlo simulation was used to observe the distribution of pensions received by participants in different situations, and then select the 95% quantile of the amount of pension received each year. Number of digits and 5% quantiles for analysis.

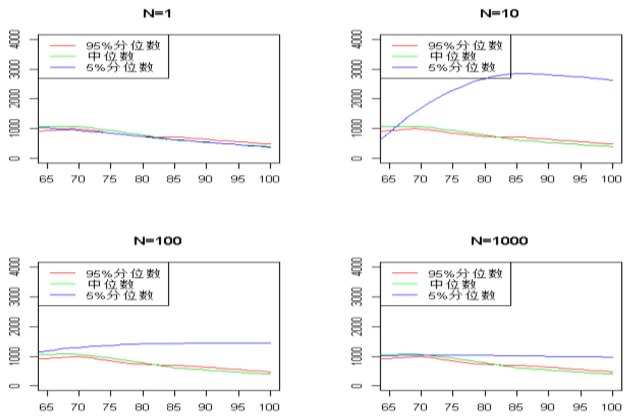

Fig. 6 Distribution of the pension amount of different members

Figure 6 analyzes the impact of an annuity pool on the future pension, but does not consider the improvement of future expected mortality. When N=1, it is an individual's annuity. With the increase of age, the amount received will decrease year by year. This is Since N = 1, there is no improvement in the mortality risk and mortality from other members of the annuity pool, resulting in a decline in the payment level of personal life, and accordingly, the annual amount of withdrawal will be reduced. When N = 10, the amount of money received The 95% quantile rises around the age of 80. This is due to the fact that the number of people in the annuity pool is small and the proportion of deaths over a period of time is relatively large. According to the principle of fund allocation of the GSA plan, the survival after this period of time Will receive a larger mortality credit allocation. When N is 100 and 1000 respectively, the distribution of pensions received becomes smoother, and the larger the N, the 95% quantile, the median and 5% points. The smaller the gap between the figures, the more members in the annuity pool, the more stable the distribution of mutual aid pensions. With the gradual reduction in the number of members of the annuity pool, the mortality credit is also reduced, called " Ontine effect". When an annuity pool is less in number and at When there is a large proportion of members who die within a period of time, there will be "tontine benefits".

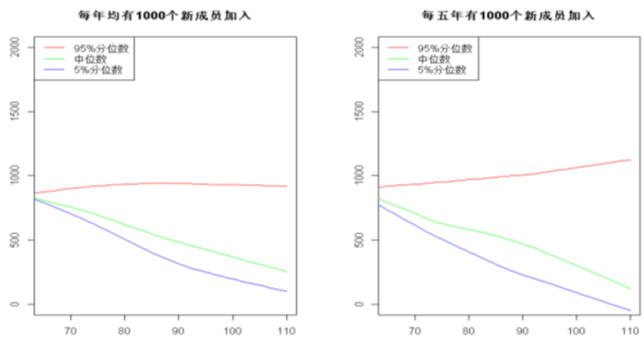

Figure 7 Distribution of pension payment amount received by the Open GSA Program

Changing the assumptions, the GSA plan is open, comparing the distribution of 1,000 new 65-year-old members each year and the distribution of 1,000 new 65-year-old members every five years. The figure can be seen from Figure 7. There is no significant difference in the annuity distribution in this case, except that the annual participation rate of 1,000 members is slightly smaller than the rate of increase of 1,000 members participating in the pension every five years. The number of members in the annuity pool changes over time. Become more stable. The dynamic annuity pool can reduce the volatility of the amount of old-age pension received by the elderly, because when the younger members join the queue, the specific fluctuations of death in the annuity pool will be reduced.

Compared with the closed annuity pool, the pension funds received from the open annuity pool tend to decline more slowly with age. When new members are added, the funds in the annuity pool are also increasing, which can be considered as an increase in annuities. The size of the members in the pool, so that the distribution of pensions is more stable.

IV. Conclusion

In view of the problems of low mortality data and lack of data for some years in China, this paper uses the cointegration theory to consider the long-term equilibrium relationship between male population mortality in Taiwan and mainland China, overcoming the limitations of the ARIMA model prediction and combining the GSA model. The results of the distribution of pensions received by pensions show that the prediction effect based on the cointegration theory is better than that predicted by the ARIMA model, which provides a new idea for the prediction of population mortality in China. At the same time, the GSA model will be compared with the average annuity. Individuals with longevity risk and system longevity risk are concentrated, do not involve payment guarantees, and do not need to use expensive funds to support reinsurance. They are retirement income similar to life annuity, and the consumer sharing model of the GSA model has a higher payoff. In the amount, participants can choose annuity structure and investment mode more flexibly according to their own preferences.

In view of China's increasingly severe population aging and the existing problems in the pension insurance system, we should establish an old-age insurance system with Chinese socialism characteristics. We must use family security as the foundation, basic national pension insurance as the focal point, and corporate annuity and business annuities. As an important supplementary and minimum living security system, the old-age pension security multiple safety nets fully realize the integration of old-age insurance.

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