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Availability Analysis of a Markovian System with Preventive Maintenance

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Abstract— **This paper deals with a Markovian queueing system, where the system can fail partially, fail completely, or be shutdown for preventive maintenance. The partially failed system can fail completely, or be shutdown for preventive maintenance. When completely failed, the system is repaired. The system works as new after preventive maintenance or repair. The steady-state availability is analized analytically and some numerical results are given.**

Index Terms—**Availability, preventive maintenance, Markovian system.**

I. Introduction

Several researchers have considered systems with server subject to breakdown and repairs [1-7]. Kadyan [1] considered the reliability and profit analysis of a single-unit system with preventive maintenance subject to maximum operation time, where the system fails completely either directly from normal state or via partial failure. As numerical results, they obtained the various performance measures with Markovian assumptions. The model in [1] is described as follows. There is a single-unit system, where the system can fail either partially or completely. The partially failed system can fail completely, or still be operating during pre-specified time, called maximum operating time. The partially failed operating system is shutdown after the maximum operating time for preventive maintenance. When completely failed, the system is repaired. The system works as new after preventive maintenance or repair. For numerical results, they assumed that the failure time from normal state to complete failure, the failure time from normal state to partial failure, the failure time from partial failure to complete failure, the repair time of the failed system, the maximum operating time after partial failure, and the preventive maintenance time of the system are all exponentially distributed with rate λ, λ_1, λ_2, θ, β, and α, respectively. They showed that the steady-state availability is given by

$$A_0 = \frac{\theta\beta(\alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1)}{\begin{array}{r}[\theta\beta(\alpha\lambda + \lambda\lambda_2 + \alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1) \\ +(\beta\lambda_1^2\lambda_2 + \alpha\theta\lambda_1^2 + \beta\lambda\lambda_1\lambda_2 + \alpha\theta\lambda\lambda_1)]\end{array}} \qquad (1)$$

However, there must be some errors in the above results [8]. When $\lambda_1 = 0$, the system becomes a simple on-off system with failure rate λ and repair rate θ, where the steady state availability A_0 should be

$$A_0 = \theta/(\theta + \lambda). \qquad (2)$$

However, A_0 given by (1) is 0.

In this paper, the modified analysis are presented under a more generalized model and some numerical results are given.

II. Analysis

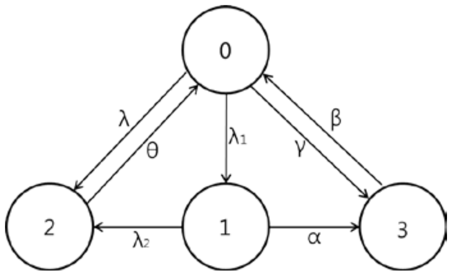

Figure 1. State transition diagram

In this paper, we generalize the model in [8]. We assume that the system in normal state can be shutdown for preventive maintenance with rate γ. Now we consider the state transition diagram of the system described in the previous section (Figure 1), where state 0 is the normal state, state 1 the partially failed state, state 2 the completely failed state, and state 3 the preventive maintenance state. Then, we obtain the following balance equations:

$$(\lambda + \lambda_1 + \gamma)\pi_0 = \theta\pi_2 + \beta\pi_3,$$
$$(\lambda_2 + \alpha)\pi_1 = \lambda_1\pi_0,$$
$$\theta\pi_2 = \lambda\pi_0 + \lambda_2\pi_1,$$
$$\beta\pi_3 = \gamma\pi_0 + \alpha\pi_1.$$

Solving the balance equations, we get the following expressions for steady-state probabilities:

$$\pi_0 = \frac{1}{1 + \frac{\lambda}{\theta} + \frac{\gamma}{\beta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \qquad (3)$$

$$\pi_1 = \frac{\frac{\lambda_1}{\lambda_2 + \alpha}}{1 + \frac{\lambda}{\theta} + \frac{\gamma}{\beta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \qquad (4)$$

$$\pi_2 = \frac{\frac{\lambda}{\theta} + \frac{\lambda_2}{\theta}\frac{\lambda_1}{\lambda_2 + \alpha}}{1 + \frac{\lambda}{\theta} + \frac{\gamma}{\beta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \qquad (5)$$

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$$\pi_3 = \frac{\frac{\gamma}{\beta} + \frac{\alpha}{\beta}\frac{\lambda_1}{\lambda_2+\alpha}}{1 + \frac{\lambda}{\theta} + \frac{\gamma}{\beta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2+\alpha}}, \quad (6)$$

Therefore, the steady-state availability A is obtained by

$$A = \frac{1 + \frac{\lambda_1}{\lambda_2+\alpha}}{1 + \frac{\lambda}{\theta} + \frac{\gamma}{\beta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2+\alpha}}. \quad (7)$$

III. Numerical Examples

Table 1. Parameter values

Cases	Parameters				
	λ	λ_1	λ_2	θ	β
Case 1	0.13	0.17	0.21	2.1	2.7
Case 2	0.16	0.17	0.21	2.1	2.7
Case 3	0.13	0.20	0.21	2.1	2.7
Case 4	0.13	0.17	0.21	2.6	2.7
Case 5	0.13	0.17	0.21	2.1	3.7

Let us provide some numerical examples. The parameter values are summarized in Table 1.

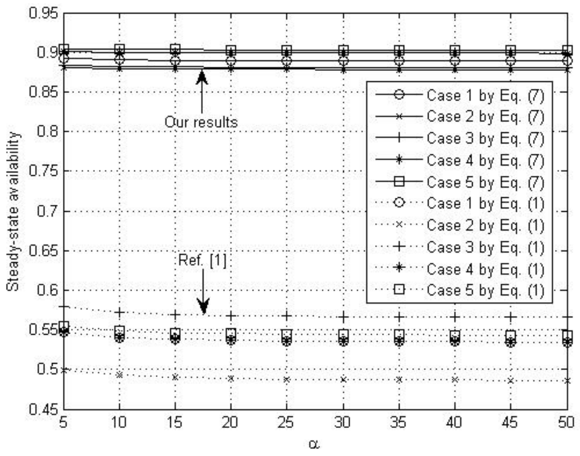

Figure 2. Compare our results with Ref. [1]

First, we compare our results with the results obtained by (1). The steady-state availability obtained in this paper is coincide with the availability in Table 3 of [1]. However, the results in the table of [1] do not coincide with those derived from their corresponding equations. Figure 2 shows the numerical results obtained in this paper as well as those derived from (1), where the prameter α is varied from 5 to 50 and the parameter γ is set to 0. Figure 2 shows that our results are significantly different from those derived from the equations in [1].

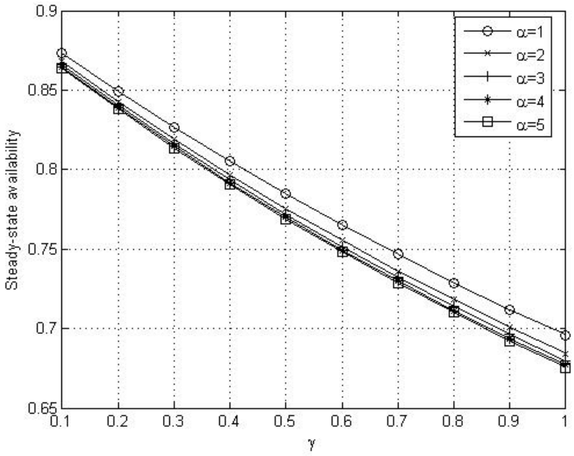

Figure 3. Steady state availability as a function of $\boldsymbol{\gamma}$

Figure 3 represents the steady-state availability as a function of the parameter γ. The figure shows that the steady-state availability decreases as the value γ increases, as expected.

IV. Conclusion

This paper dealt with a Markovian queueing system, in which the system can fail partially, fail completely, or be shutdown for preventive maintenance. The partially failed system is shutdown after the maximum operating time for preventive maintenance. The steady-state availability were analyzed analytically. Numerical results showed that our results are significantly different from those derived from the equations in [1] and the steady-state availability decreases as the value γ increases.

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