An Analytical Solution for Hoop Tension in Liquid Storage Cylindrical Tanks

Anand Daftardar, Shirish Vichare, Jigisha Vashi

Abstract— **Design codes for liquid storage cylindrical tanks give values of design forces (hoop tension, bending moment and shear force) through a dimensionless parameter H2/Dt. The values of this parameter range from 0.4 to 16. The parameter H2/Dt encompasses key dimensions of the tank and gives corresponding coefficients for hoop tension, bending moment and shear force. However, for values of H2/Dt range below 0.4 and above 16, the coefficients are not easily available. Availability of these coefficients are crucial for design engineers who deal with variety of tank dimensions. This paper calculates the values of hoop tension coefficients for H2/Dt range from 0.1 to 100 by providing analytical solution. Further, the paper also gives maximum value of hoop tension coefficients from 0.1 to 100. It is observed that for H2/Dt range from 0.4 to 0.1, the decrease in the maximum hoop tension coefficients value is about 4 % and for H2/Dt range from 16 to 100 the increase is about 22 %.**

Index Terms— **Liquid storage cylindrical tanks, hoop tension, design codes, tension coefficients, structural design, tension cracks, hydrostatic pressure**

I. INTRODUCTION

Storage reservoirs and overhead tanks are used to store water, liquid petroleum, petroleum products and similar liquids. In general, there are three kinds of storage tanks viz tanks resting on the ground, underground tanks and elevated tanks. These tanks are either rectangular or cylindrical in shape. In this paper, the scope is limited to cylindrical tanks resting on the ground. Some of the examples of tanks resting on the ground are clear water reservoirs, settling tanks and aeration tanks. The walls of these tanks are subjected to hydrostatic pressure and the base is subjected to the weight of liquid and upward soil pressure. The design of liquid storage cylindrical tanks involves calculations of the vertical bending moment, shear force, and hoop tension. This paper presents contribution on hoop tension calculations. Hoop tension causes tensile cracks in the walls which are undesirable as they cause of leakage. Design codes for liquid storage cylindrical tanks give values of design forces (hoop tension, bending moment and shear force) through a dimensionless parameter H^2/Dt. The values of this parameter range from 0.4 to 16. The parameter H^2/Dt encompasses key dimensions of the tank and gives corresponding coefficients for hoop tension, bending moment and shear force. However, for values of H^2/Dt below 0.4 and above 16, the coefficients are not easily available. Availability of these coefficients are crucial for design engineers who deal with variety of tank dimensions. It can be easily seen that many of the practically useful tanks do not fit in this range of H^2/Dt from 0.4 to 16. In this case, a design engineer is tempted to linearly extrapolate for unavailable coefficients, which, as this paper will show is not accurate way of calculating an important design force like hoop tension. Using fundamental equations for vertical bending moment and radial expansion of the tank wall, a systematic analytical solution is presented in section 3. The boundary conditions considered for the wall are hinged base and free top. Tension coefficients are calculated for the range of parameter H^2/Dt from 0.1 to 100. In addition, value of maximum tension coefficient for each H^2/Dt is also presented which may be useful for estimating wall thickness.

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II. NEED FOR THE STUDY

Considerable research effort has been carried out on the design, analysis, and assessment of liquid storage tanks. Timoshenko and Kreiger [1] gives an approximate analytical solution to differential equations governing the deformation of circular water tank whose axis is vertical. They considered a tank in which both inner and outer radii vary so that the mean radius of the tank remains constant. Thevendran and Thambiratnam [2] & Thevendran [3] purposed the Runge-Kutta numerical method of solution of ordinary differential equations and numerical optimization methods. Melerski [4],[5] presented a force-type approach in which the main elements, i.e. the base plate, wall and cover plate was first considered separately and then the global solution was obtained by making use of displacement compatibility conditions at the plate-wall junctions. Kukreti and Siddiqi [6] presented a differential quadrature solution for the flexural behavior of a cylindrical storage tank resting on isotropic elastic soil medium. Ziari and Kianoush [7] presented an element of tank wall which was subjected to a monotonic increasing direct tension load and its cracking behavior was closely monitored. The influence of direct tension cracks on water tightness of specimen was examined by exposing a major crack to pressurized water and evaluating the water leakage. Vichare and Inamdar [8] presented an analytical method based on fundamental equations of shells. They also presented an important observations on the variation of design forces across wall and raft with different soil conditions. Chen et al., [9] introduced a new weighted smeared wall method which was proposed to be a simple method to deal with stepped-wall cylinders of short or medium length with any thickness variation.

However, no significant research work has been done in the area of hoop tension coefficients of the tank wall, for designing liquid storage tank. From available design codes, hoop tension in the cylindrical tank can be calculated using the coefficient of H^2/Dt. However, no tension coefficients of cylindrical tank wall are available below 0.4 and beyond 16.

This causes limitation for the design of many practical tanks. For example a tank having dimensions H = 6m, D = 4m, and t = 0.25m [$H^2/Dt=36$] and/or a tank having dimensions H = 7m, D = 2m, and t = 0.25m [$H^2/Dt=98$], do not have readily available hoop tension coefficients. Even, there are no clear guidelines given in the design codes for such tanks. In these like situation, the design engineer may be tempted to linearly extrapolate for hoop tension coefficients which may give appreciably inaccurate results.

III. Problem Definition and Analytical Modeling

In this section, the systematic outline of the analysis is provided.

A. *Modeling*

The tank is modeled as an elastic cylindrical tank of diameter D, height H, and thickness t. The Young's modulus and Poisson's ratio of the tank material are E and μ, respectively. The density of liquid contained in the tank is γ. The problem is axisymmetric and the scope is limited to tanks resting on ground level only.

B. *Loads*

As the tank is considered to be resting on ground level, there is no uplift force on bottom raft due to ground water table. Walls are subjected to hydrostatic pressure from the inner side of the tank. The vertical load of the water body and self-weight of the base slab of the tank are balanced by the reaction from the soil.

C. *Analytical modeling*

The cross section of a cylindrical tank of finite diameter and height is as shown in figure 1(a). The wall–raft junction is assumed to be hinged. The tank wall is subjected to a hydrostatic loading as shown in figure 1(b).

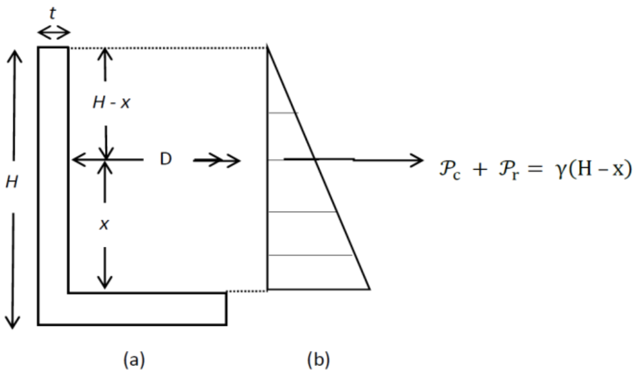

Fig. 1: (a) Cross section of circular tank. (b) Hydrostatic load

The hydrostatic pressure causes radial expansion and vertical bending of the tank wall.

At any height 'x' from base

$$\mathcal{P}_c + \mathcal{P}_r = \gamma(H - x) \tag{1}$$

Hoop tension at height 'x' from base

$$T = \mathcal{P}_r \frac{D}{2} \tag{2}$$

$$\text{Hoop stress} = \mathcal{P}_r \frac{D}{2t} \tag{3}$$

$$\text{Hoop strain} = \mathcal{P}_r \frac{D}{2tE} \tag{4}$$

Radial expansion at height 'x' from base

$$y_r = \mathcal{P}_r \frac{D^2}{4tE} \tag{5}$$

The 4th order of deflection due to the vertical bending of the tank wall is given by

$$\frac{d^4y}{dx^4} = \frac{\mathcal{P}_c}{EI} \tag{6}$$

For compatibility

$$y_c = y_r = y \tag{7}$$

From equations (5)

$$\mathcal{P}_r = y_r \frac{4tE}{D^2} \tag{8}$$

and from equation (1)

$$\mathcal{P}_c = \gamma(H - x) - \mathcal{P}_r \tag{9}$$

Substituting equation (3) & (5) in equation (9),

$$EI\frac{d^4y}{dx^4} = \gamma(H - x) - y\frac{4tE}{D^2} \tag{10}$$

$$\text{Where, } I = \frac{t^3}{12(1-\mu^2)} \tag{11}$$

$$\frac{Et^3}{12(1-\mu^2)}\frac{d^4y}{dx^4} = \gamma(H - x) - y\frac{4tE}{D^2} \tag{12}$$

The solution to differential equation (12) is

$$y = e^{kx}(ACos[kx] + BSin[(kx)]) + e^{-kx}(CCos[(kx)] + DSin[(kx)]) + \gamma(H - x)\frac{D^2}{4Et} \tag{13}$$

Where; *A*, *B*, *C*, and *D* are the four constants to be determined from the boundary conditions.

The boundary conditions for tank wall are hinged bottom and free top. Thus, shear force and bending moment are zero at the top, while deflection and bending moment are zero at the bottom. These conditions give the following equations.

At *x* = H, shear force is zero i.e. $EI.\frac{d^3y}{dx^3} = 0$

At *x* = H, bending moment is zero i.e. $EI.\frac{d^2y}{dx^2} = 0$

At *x* = 0, bending moment is zero i.e. $EI.\frac{d^2y}{dx^2} = 0$

At *x* = 0, deflection is zero i.e. $EI.y = 0$

After applying boundary conditions to the equation (13), the constants A, B, C, and D are solved and shown in appendix A.

D. Calculation of hoop tension

Hoop tension for the tank wall due to hydrostatic pressure is given by

$$T = \mathcal{P}_r \frac{D}{2} \tag{14}$$

From equation (5), the equation (14) becomes

$$T = \frac{2tEy}{D} \tag{15}$$

Hoop tension equation from design codes

$$T = coefficient\ \gamma\ H\ r \tag{16}$$

The hoop tension coefficients can be calculated from the equation (16) by substituting hoop tension value obtained from the equation (15).

IV. Validation and Extension of Hoop Tension Coefficients

Hoop tension coefficients for the cylindrical wall, hinge base, free top and subject to triangular hydrostatic load are given by design codes, which are limited to the H^2/Dt range from 0.4 to 16. Hoop tension coefficients for the cylindrical wall for H^2/Dt range from 0.4 to 16 was taken for verification problem by solving the differential equation (12). Input parameters taken to solve the differential equation (12) are as follows, the density of the liquid, γ is 10 kN/m3; Poisson's ratio, μ is 0.3 and modulus of elasticity of concrete, E is 2000000 kPa. The values of hoop tension coefficients for H^2/Dt range from 0.4 to 16, shown in Appendix B - Table 1 are matching with Reference [10] (Table No 12, Page No 38). Hence, the differential equation (12) used in this paper, to find out hoop tension from the hoop tension coefficients is validated.

The analysis was carried out to observe the behavior of hoop tension coefficients for cylindrical wall for higher range of H^2/Dt i.e beyond 16 up to 100. Also, the analysis was also carried out to observe the behavior for lower range of H^2/Dt i.e 0.1 to 0.4. Appendix B - Table B.1 show the table of hoop tension coefficients for cylindrical wall hinge base, free top and subjected to hydrostatic load for H^2/Dt range from 0.1 to 100.

V. Observations and Conclusions

An analytical solution for finding hoop tension in a liquid storage cylindrical tank is presented in section 3. The hoop tension coefficients which enable evaluation of hoop tension are provided in (Appendix B) for H^2/Dt range from 0.1 to 100. This section provides graphs which will help in highlighting the observations and conclusions derived from the analysis presented in this paper.

Figure 2 shows a plot of hoop tension coefficients along the height of the wall for H^2/Dt range from 0.1 to 100. Most of the design codes (Reference no.[10], Table No 12, Page No 38) give the values of hoop tension coefficients in the limited H^2/Dt range from 0.4 to 16. But it is observed from figure 2 that this limited range is not adequate. Though, it is observed from figure 3 that for the upper portion of the wall (i.e. from 0.0H to 0.6H), the hoop tension coefficients are not varying significantly for H^2/Dt range from 30 to 100 and figure 4 shows that for the bottom portion of the wall (i.e from 0.7H to 0.9H), the hoop tension coefficients are increasing significantly for H^2/Dt range from 0.1 to 100.

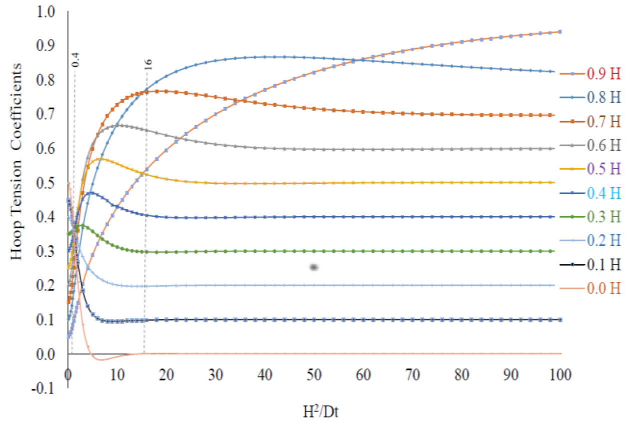

Fig. 2: Hoop tension coefficients for H^2/Dt range from 0.1 to 100

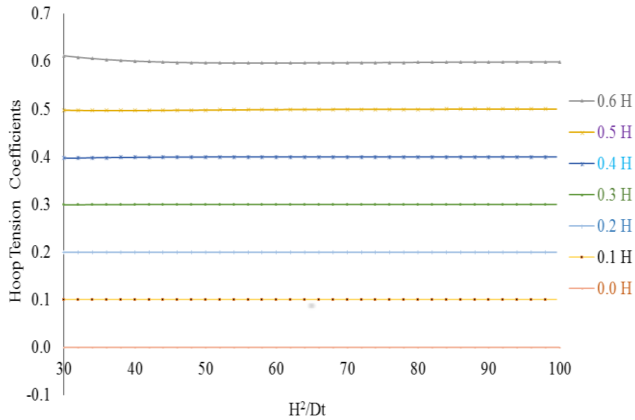

Fig. 3: Hoop tension coefficients v/s H^2/Dt range from 30 to 100 for the height of the tank (0.0H to 0.6H)

Figure 5 shows the plot of maximum and minimum hoop tension coefficients for H^2/Dt range from 0.1 to 100. It is observed that for H^2/Dt range from 0.4 to 0.1, the decrease in the maximum hoop tension coefficients value is about 4% and for H^2/Dt range from 16 to 100 the increase is about 22%.

Figure 6 shows the plot of location of maximum and minimum hoop tension along the height of the tank wall. It is observed that for the H^2/Dt range from 3 to 100, the minimum hoop tension is located at the top of the wall, whereas the maximum hoop tension is located at 0.7H for H^2/Dt range from 7 to 15, at 0.8H for H^2/Dt range from 16 to 58 and at 0.9H for H^2/Dt range from 59 to 100.

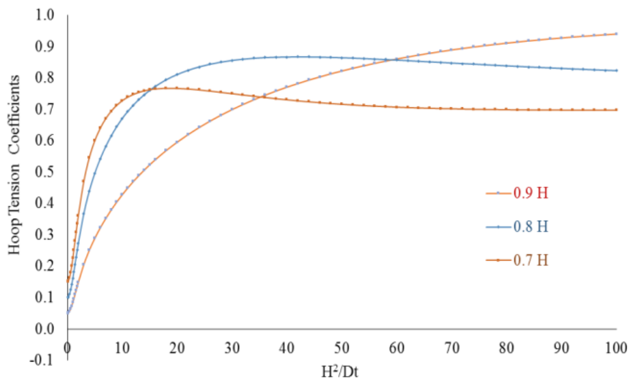

Fig. 4: Hoop tension coefficients v/s H^2/Dt range from 0.1 to 100 for the height of the tank (0.7H to 0.9H)

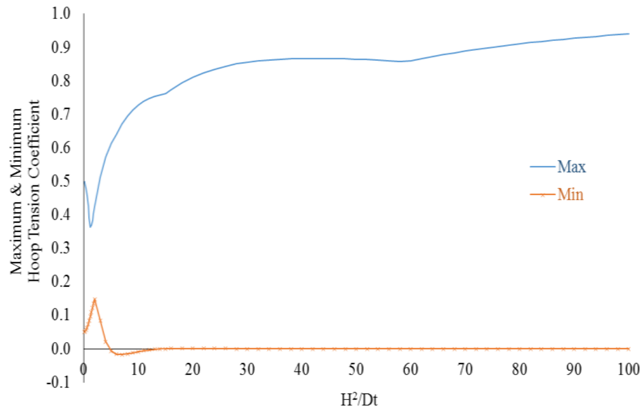

Fig. 5: Maximum & minimum hoop tension coefficients for H^2/Dt range from 0.1 to 100

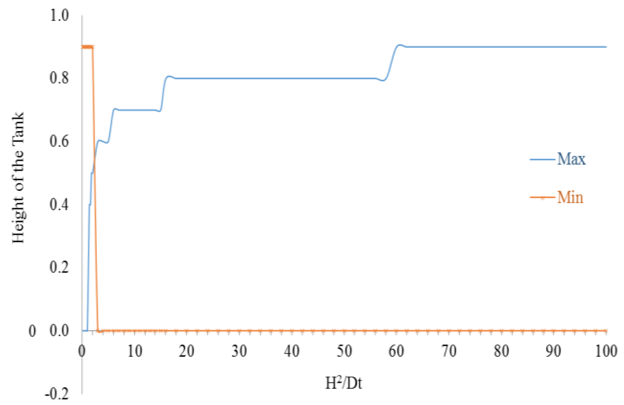

Fig. 6: Location of maximum and minimum hoop tension along the height of the tank wall v/s H^2/Dt

From these observations, we conclude that the availability of hoop tension coefficients (Appendix B) and analytical solution (Section 3) given in this paper will be of value to researchers and practicing engineers.

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APPENDIX A

$$A = -\frac{d^2\gamma(-2e^{2Hk}\mathrm{Cos}[Hk]^2 + Hk\mathrm{Cos}[Hk]^2 + 3e^{2Hk}Hk\mathrm{Cos}[Hk]^2 - 2e^{2Hk}Hk\mathrm{Cos}[Hk]\mathrm{Sin}[Hk] + Hk\mathrm{Sin}[Hk]^2 + e^{2Hk}Hk\mathrm{Sin}[Hk]^2)}{4ekt(\mathrm{Cos}[Hk]^2 + 6e^{2Hk}\mathrm{Cos}[Hk]^2 + e^{4Hk}\mathrm{Cos}[Hk]^2 + \mathrm{Sin}[Hk]^2 + 2e^{2Hk}\mathrm{Sin}[Hk]^2 + e^{4Hk}\mathrm{Sin}[Hk]^2)} \quad (A.1)$$

$$B = \frac{d^2\gamma(1 + e^{2Hk} + Hk - e^{2Hk}Hk + 2e^{2Hk}\mathrm{Tan}[Hk] - 2e^{2Hk}Hk\mathrm{Tan}[Hk] + \mathrm{Tan}[Hk]^2 + e^{2Hk}\mathrm{Tan}[Hk]^2 + Hk\mathrm{Tan}[Hk]^2 + e^{2Hk}Hk\mathrm{Tan}[Hk]^2)}{4ekt(1 + 6e^{2Hk} + e^{4Hk} + \mathrm{Tan}[Hk]^2 + 2e^{2Hk}\mathrm{Tan}[Hk]^2 + e^{4Hk}\mathrm{Tan}[Hk]^2)} \quad (A.2)$$

$$C = -\frac{d^2H\gamma}{4et} + \frac{d^2\gamma(-2e^{2Hk}\mathrm{Cos}[Hk]^2 + Hk\mathrm{Cos}[Hk]^2 + 3e^{2Hk}Hk\mathrm{Cos}[Hk]^2 - 2e^{2Hk}Hk\mathrm{Cos}[Hk]\mathrm{Sin}[Hk] + Hk\mathrm{Sin}[Hk]^2 + e^{2Hk}Hk\mathrm{Sin}[Hk]^2)}{4ekt(\mathrm{Cos}[Hk]^2 + 6e^{2Hk}\mathrm{Cos}[Hk]^2 + e^{4Hk}\mathrm{Cos}[Hk]^2 + \mathrm{Sin}[Hk]^2 + 2e^{2Hk}\mathrm{Sin}[Hk]^2 + e^{4Hk}\mathrm{Sin}[Hk]^2)} \quad (A.3)$$

$$D = -\frac{d^2e^{-1+2Hk}\gamma(-1 - e^{2Hk} - Hk + e^{2Hk}Hk + 2\mathrm{Tan}[Hk] + 2Hk\mathrm{Tan}[Hk] - \mathrm{Tan}[Hk]^2 - e^{2Hk}\mathrm{Tan}[Hk]^2 + Hk\mathrm{Tan}[Hk]^2 + e^{2Hk}Hk\mathrm{Tan}[Hk]^2)}{4kt(1 + 6e^{2Hk} + e^{4Hk} + \mathrm{Tan}[Hk]^2 + 2e^{2Hk}\mathrm{Tan}[Hk]^2 + e^{4Hk}\mathrm{Tan}[Hk]^2)} \quad (A.4)$$

Table B.1: Tension in circular ring wall hinge base, free top and subjected to hydrostatic load for H^2/Dt range from 0.1 to 100 by authors

	Coefficients at point									
$\frac{H^2}{Dt}$	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.1	0.4987	0.4491	0.3996	0.3501	0.3004	0.2507	0.2009	0.1508	0.1007	0.0503
0.2	0.4948	0.4467	0.3987	0.3504	0.3019	0.2530	0.2036	0.1535	0.1028	0.0515
0.4	0.4798	0.4374	0.3949	0.3518	0.3077	0.2619	0.2141	0.1638	0.1111	0.0562
0.6	0.4562	0.4227	0.3890	0.3540	0.3167	0.2760	0.2307	0.1801	0.1242	0.0635
0.8	0.4259	0.4039	0.3813	0.3567	0.3283	0.2940	0.2521	0.2012	0.1412	0.0731
1	0.3910	0.3821	0.3723	0.3597	0.3415	0.3148	0.2769	0.2258	0.1610	0.0843
1.2	0.3535	0.3586	0.3626	0.3528	0.3557	0.3372	0.3037	0.2525	0.1827	0.0966
1.4	0.3153	0.3346	0.3525	0.3558	0.3700	0.3601	0.3313	0.2802	0.2054	0.1095
1.6	0.2779	0.3109	0.3424	0.3584	0.3839	0.3826	0.3588	0.3079	0.2283	0.1226
1.8	0.2421	0.2882	0.3325	0.3707	0.3970	0.4042	0.3854	0.3351	0.2508	0.1356
2	0.2088	0.2669	0.3230	0.3726	0.4090	0.4244	0.4106	0.3611	0.2727	0.1483
3	0.0850	0.1853	0.2835	0.3749	0.4509	0.5007	0.5112	0.4697	0.3674	0.2048
4	0.0213	0.1395	0.2564	0.3683	0.4675	0.5917	0.5741	0.5456	0.4393	0.2504
5	-0.0066	0.1155	0.2376	0.3576	0.4694	0.5609	0.6128	0.5996	0.4958	0.2885
6	-0.0165	0.1035	0.2244	0.3459	0.4641	0.5680	0.6367	0.6397	0.5423	0.3220
7	-0.0178	0.0979	0.2150	0.3350	0.4559	0.5684	0.6515	0.6704	0.5816	0.3520
8	-0.0155	0.0955	0.2084	0.3255	0.4470	0.5654	0.6603	0.6942	0.6154	0.3793
9	-0.0120	0.0949	0.2038	0.3178	0.4348	0.5605	0.6650	0.7128	0.6448	0.4044
10	-0.0087	0.0951	0.2008	0.3118	0.4308	0.5548	0.6669	0.7273	0.6707	0.4276
11	-0.0058	0.0957	0.1989	0.3071	0.4241	0.5488	0.6668	0.7386	0.6931	0.4492
12	-0.0036	0.0963	0.1977	0.3037	0.4184	0.5429	0.6654	0.7473	0.7130	0.4694
13	-0.0019	0.0970	0.1971	0.3012	0.4138	0.5374	0.6630	0.7539	0.7307	0.4883
14	-0.0008	0.0977	0.1969	0.2995	0.4099	0.5322	0.6600	0.7588	0.7464	0.5061
16	0.0003	0.0987	0.1972	0.2976	0.4043	0.5232	0.6530	0.7647	0.7728	0.5389
18	0.0007	0.0993	0.1978	0.2970	0.4007	0.5159	0.6456	0.7668	0.7939	0.5684
20	0.0007	0.0997	0.1984	0.2971	0.3987	0.5103	0.6384	0.7665	0.8108	0.5951
22	0.0005	0.0999	0.1990	0.2975	0.3976	0.5061	0.6318	0.7646	0.8243	0.6195
24	0.0003	0.1000	0.1994	0.2980	0.3971	0.5029	0.6259	0.7616	0.8351	0.6418
26	0.0002	0.1000	0.1997	0.2985	0.3971	0.5007	0.6207	0.7579	0.8437	0.6624
28	0.0001	0.1000	0.1999	0.2989	0.3972	0.4992	0.6162	0.7539	0.8504	0.6814
30	0.0000	0.1000	0.2000	0.2992	0.3975	0.4981	0.6124	0.7498	0.8557	0.6990
32	0.0000	0.1000	0.2001	0.2995	0.3975	0.4975	0.6092	0.7456	0.8597	0.7153
34	0.0000	0.1000	0.2001	0.2997	0.3982	0.4972	0.6065	0.7415	0.8626	0.7306
36	0.0000	0.1000	0.2001	0.2998	0.3986	0.4971	0.6043	0.7376	0.8647	0.7448

$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
38	0.0000	0.1000	0.2001	0.2999	0.3989	0.4971	0.6025	0.7338	0.8660	0.7581
40	0.0000	0.1000	0.2001	0.3000	0.3991	0.4972	0.6011	0.7302	0.8667	0.7705
42	0.0000	0.1000	0.2000	0.3000	0.3993	0.4974	0.6000	0.7269	0.8670	0.7821
44	0.0000	0.1000	0.2000	0.3000	0.3995	0.4976	0.5991	0.7238	0.8668	0.7931
46	0.0000	0.0999	0.2000	0.3000	0.3997	0.4979	0.5984	0.7210	0.8662	0.8033
48	0.0000	0.0999	0.2000	0.3000	0.3998	0.4981	0.5979	0.7183	0.8654	0.8130
50	0.0000	0.0999	0.2000	0.3000	0.3999	0.4983	0.5975	0.7159	0.8643	0.8221
52	0.0000	0.0999	0.2000	0.3000	0.4000	0.4986	0.5973	0.7137	0.8630	0.8307
54	0.0000	0.0999	0.2000	0.3000	0.4000	0.4988	0.5971	0.7117	0.8616	0.8387
56	0.0000	0.0999	0.2000	0.3000	0.4000	0.4990	0.5971	0.7099	0.8600	0.8464
58	0.0000	0.0999	0.1999	0.3000	0.4001	0.4992	0.5971	0.7083	0.8584	0.8535
60	0.0000	0.0999	0.1999	0.3000	0.4001	0.4993	0.5971	0.7068	0.8566	0.8603
62	0.0000	0.0999	0.1999	0.3000	0.4001	0.4995	0.5972	0.7055	0.8549	0.8667
64	0.0000	0.0999	0.1999	0.3000	0.4001	0.4996	0.5973	0.7043	0.8530	0.8728
66	0.0000	0.0999	0.1999	0.3000	0.4000	0.4997	0.5974	0.7031	0.8512	0.8785
68	0.0000	0.0999	0.1999	0.3000	0.4000	0.4998	0.5976	0.7023	0.8493	0.8839
70	0.0000	0.0999	0.1999	0.3000	0.4000	0.4998	0.5977	0.7015	0.8475	0.8891
72	0.0000	0.0999	0.1999	0.3000	0.4000	0.4999	0.5979	0.7008	0.8456	0.8939
74	0.0000	0.0999	0.1999	0.3000	0.4000	0.4999	0.5980	0.7001	0.8438	0.8985
76	0.0000	0.0999	0.1999	0.3000	0.4000	0.5000	0.5982	0.6996	0.8420	0.9028
78	0.0000	0.0999	0.1999	0.3000	0.4000	0.5000	0.5983	0.6991	0.8402	0.9069
80	0.0000	0.0999	0.1999	0.3000	0.4000	0.5000	0.5985	0.6987	0.8384	0.9108
82	0.0000	0.0999	0.1999	0.3000	0.4000	0.5000	0.5986	0.6983	0.8367	0.9145
84	0.0000	0.0999	0.1999	0.3000	0.4000	0.5001	0.5988	0.6980	0.8350	0.9179
86	0.0000	0.0999	0.1999	0.3000	0.4000	0.5001	0.5989	0.6978	0.8334	0.9212
88	0.0000	0.1000	0.1999	0.3000	0.4000	0.5001	0.5990	0.6976	0.8318	0.9243
90	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5991	0.6974	0.8302	0.9273
92	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5992	0.6973	0.8287	0.9300
94	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5993	0.6972	0.8273	0.9326
96	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5994	0.6971	0.8259	0.9351
98	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5995	0.6971	0.8245	0.9375
100	0.0000	0.1000	0.2000	0.3000	0.4000	0.5001	0.5996	0.6971	0.8232	0.9397