Robust Least Squares Dummy Variable Estimation Of Dynamic Panel Models In The Presence Of Outliers

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***Abstract*— This research is focused on the consistent, robust least squares dummy variable ($LSDV_R$) estimator which is predicated on the correction of the bias of the inconsistency of the least squares dummy variable estimator of the parameters of the dynamic panel data model, as an extension of earlier results. We compared the results of the bias corrected least squares dummy variable estimator of the dynamic panel data models in the presence of outliers, at stated specifications of the model with the consistent instrumental variable (IV) and the generalized method of moments (GMM) estimators of Anderson and Hsiao (AH), Arellano and Bond (AB) and Blundell and Bond (BB) to validate the claims or otherwise of the estimators. We observe at $\rho = \gamma$ =0.8 and B=0.2 that the robust least squares dummy variable estimator ($LSDV_R$) performs better than the IV- GMM in finite and large samples in terms of predictive powers and in the estimation of the autoregressive coefficient in large samples followed by the LSDV, though, with maximum RMSE property while the Blundell and Bond (BB) performs better than the other contending models in estimation of the autoregressive coefficient in finite samples showing that the presence of an outlier does not affect the predictive power of the robust least squares dummy variable ($LSDV_R$) estimator.**

***Index Terms*—** Consistent estimator, dynamic, outliers, panel data model

I. INTRODUCTION

Leveraging on the exposition by [1] that the Least Squares dummy variable estimator is inconsistent in determining the estimates of the parameters of a first order autoregressive panel data model at finite time period, T, as the cross sectional units, N, becomes infinitely large, certain instrumental variable (IV) and generalized method of moments (GMM) estimators have been proposed in the econometric literature in the accounts of [2], [3] and [4].

However, the IV-GMM estimators which includes the Anderson Hsiao (AH) instrumental variable estimators, Arellano and Bond (AB) generalized method of moments estimators and Blundell and Bond (BB) system generalized method of moments estimator Could not provide all the cure for all the problems inherent in the model as a result of the violation of assumption of absence of correlation between the explanatory variable and the error term: a condition upon which the ordinary least squares and hence the least squares dummy variable estimator could be both consistent and efficient. In the accounts of [5], [6], [7], [8] and [9], the system GMM was proposed as a result of the weakness of the first-differenced instrumental variable IV-GMM estimators which suffered small sample bias due to weak instruments. In clear terms, all the IV-GMM estimators maintained their consistency property, with highly persistent data, when the cross section units, N, is large but could be severely biased in small samples as in [5]. In large samples, the LSDV estimator, though inconsistent, has a small variance, relatively compared to the IV-GMM methods as observed by [1], [10] and [11]. The fact that the LSDV may be consistent in large samples in the direction of T is buttressed by [5] and [7], but has higher variance relative to the IV-GMM estimators in small samples with highly persistent data says [12].

Also , in highly persistent data, the Bias corrected least squares dummy variable ($LSDV_R$) estimation of a first order autoregressive Panel data model which involves the approximation of the bias of the least squares dummy variable estimator and taking care of the bias to produce an estimator that could be consistent both in large and small samples emerged in the accounts of[1],[13] and [7].

[1] and [13] used higher order asymptotic expansion approximation techniques of order $N^{-1}T^{-1}$ and $N^{-1}T^{-2}$ respectively to obtain the small sample bias of the LSDV under the assumption of homoscedasticity. [14] obtained the bias corrected LSDV estimator for a case of cross section units heteroscadasticity. [7] obtained the bias corrected least squares dummy variable estimator for samples under the assumption of homoscedasticity and also worked on the bias correction model of the LSDV in the case of time series and cross section heteroscedasticity, an extension of the work by [14].

This paper is a further extension of the work of [14] and [7] and deals strictly with the comparison of the performances of the LSDV,$LSDV_R$, AH, AB and BB estimators in the presence of an outlier. An effort, still, in search of supportive evidence of their performances in the first order autoregressive panel data model that is still evolving.

A. Weak Instrument

An instrumental variable is a proxy which is highly correlated with the included endogenous variable in the dynamic panel data model but uncorrelated with the error term. The strength of the correlation can be determined using the F-statistics since the instrument and instrumented are observable.

In the first order autoregressive panel data model given by

$$y_{it} = \gamma y_{it-1} + X_{it}^{'}\beta + V_{it}$$

where $V_{it} = \alpha_i + \varepsilon_{it}$

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test to determine the absence of correlation between the instrument and the error term is conducted using the Sagan-Hansen test statistic calculated as TR^2 . R^2 is the coefficient of multiple determination obtained from the OLS residuals onto the exogenous variable and T is period as in [4]. In a system of linear models, the test is not feasible in exactly identified model but rather in over identified models where it is expected that the instruments are truly exogenous and uncorrelated with the error term says [15].The presence of instruments that are correlated with the error term or that are poorly correlated to the endogenous explanatory variable can make the estimates obtained to be inconsistent and are thus regarded as weak instruments. A weak instrument produced wrong estimates of parameters and standard errors while the good instrument is expected to be highly correlated with the instrumented and uncorrelated with the error to produce correct estimates of the parameters and their standard errors.

B. Generalized Method of Moments (GMM) Estimator

Generalized method of moments (GMM) estimation is the application of Instrumental variable to an over-identified model, i.e. when the number of instruments is greater than or equal to the number of covariates in the equation of interest. It should be recalled that if instruments are greater than the number of covariates, this is over identification. In other words, the GMM is a generalization of the just-identified instrumental variable estimator.

The danger of instrumental variable method is that it may be difficult to find a good instrument but may introduce multicollinearity. Hence [4] and [5] suggested the use of maximum likelihood estimation method with it limitations: methodologically and practical wise, especially in data involving large cross sections.

C. Heteroscedasticity

Heteroscedasticity is the presence of unequal variance of the error term in a model. Unequal error variance is a violation of the assumption of equal error variance (homoscedasticity) upon which the OLS is BLUE and efficient which is worth correcting only when it is severe as in [4]. However, in the presence of heteroscedasticity OLS is BLUE but not efficient. Under heteroscedasticity the estimates of the coefficients using OLS is unbiased but their standard errors may be biased in the accounts of [4] and [16].

D. Outlier

An outlier is an observation, which is much different in magnitude, i.e. either very large or very small compared to other observation within the same sample. In other words, they are perceived to be from a population other than that from which the other sample observations are generated as in [4] and [15]. Outliers could be a source of heteroscedasticity. Outliers could be a result of the unobservable individual effects in a panel data study, such as effects of government policies, available resources and their uses, political will of the leader, level of patriotism of the citizenry, and generally, the individual attributes of the units of a cross-section.

II. Approximating the Inconsistency of the LSDV

$$y_{it} = \gamma y_{it-1} + X_{it}^{'}\beta + \alpha_i + \varepsilon_{it}, \quad \text{I=1,2,…,N; t=1,2,…,T} \qquad (1)$$

where y_{it} is the value of y for the ith individual or group at period, t, a TX1 vector of dependent variable. $y_{i,t-1}$ is the immediate value of y at the immediate past one period t-1 for the ith cross section unit or group. X_{it} is the value of the exogenous explanatory variable at period, t for the ith cross section unit or group and ((N-1) X1) vector . α_i is the unobserved ith unit or group effect term and ε_{it} is an error term that has mean zero and variance σ_{ε}^2.

When we consider the LSDV by within estimation obtained by the application of the ordinary least squares on the transformed model:

$$\tilde{y} = \gamma\tilde{y}_{-1} + \tilde{X}\beta + \tilde{\varepsilon} \qquad (2)$$

such than: $$\hat{\mathrm{H}} = (\tilde{w}^{'}\tilde{w})^{-1}\tilde{w}^{'}y \qquad (3)$$

and: $$\tilde{w} = [\tilde{y}_{-1}, X] \qquad (4)$$

$\tilde{y}_{-1}$ and $\tilde{X}$ are observations that have been centered and stacked over time such that $\tilde{y}_{-1}$ is an (NTX1) vector of lagged endogenous explanatory variable and $\tilde{X}$ is an (NTX1) vector of strictly exogenous explanatory variables in the accounts of [7].

$$\hat{\mathrm{H}} = (\gamma, \beta^{'}) \qquad (5)$$

$\hat{\mathrm{H}}$ is an ((N+1)X1) vector of coefficients as in [5].
The inconsistency of the LSDV at finite period, T and large cross section units,N, is evidenced by

$$\mathrm{cov}[(\tilde{y}_{it-1} - \bar{y}_{i,-1})(\tilde{\varepsilon}_{it} - \bar{\varepsilon})] \neq o \qquad (6)$$

and can be estimated using partition regression technique in line with [7],for the errors of γ and β as

$$\hat{\gamma} - \gamma = (\tilde{y}_{-1}^{'}D\tilde{y}_{-1})^{-1}y_{-1}^{'}\tilde{\varepsilon} \qquad (7)$$

$$\hat{\beta} - \beta = -(\tilde{X}^{'}\tilde{X})^{-1}\tilde{X}^{'}\tilde{y}_{-1}(\hat{\gamma} - \gamma) + (\tilde{X}^{'}\hat{\beta})^{-1}\tilde{X}^{'}\tilde{\varepsilon} \qquad (8)$$

where: $$\tilde{\varepsilon} = \tilde{y} - \tilde{w}\hat{\mathrm{H}} \qquad (9)$$

and $D = 1 - \tilde{X}(\tilde{X}^{'}\tilde{X})^{-1}\tilde{X}^{'}$.

Then taking probability limits, we have:

$$P\lim_{N\to\infty}(\hat{\gamma} - \gamma) = (P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}D\tilde{y})^{-1}(P\lim_{N\to\infty}\frac{1}{N}\tilde{y}^{'}D\tilde{\varepsilon}) \qquad (10)$$

$$P\lim_{N\to\infty}(\hat{\beta} - \beta) = -P\lim_{N\to\infty}(\tilde{X}^{'}\tilde{X})^{-1}\tilde{X}^{'}\tilde{y}_{-1}P\lim_{N\to\infty}(\hat{\gamma} - \gamma) \qquad (11)$$

From (10): $$P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}D\tilde{\varepsilon} = P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}\tilde{\varepsilon} \qquad (12)$$

because X is assumed to be strictly exogenous.

Then from (12)

$$P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}\tilde{\varepsilon} = \underset{N\to\infty}{Lim}\frac{1}{N}(\sum E[\tilde{y}_{-1}^{'}\tilde{\varepsilon}]) \qquad (13)$$

By further decomposition, we obtain

$$E(\tilde{y}_{-1}^{'}\tilde{\varepsilon}) = tr(\Pi_T \Sigma_T) \quad (14)$$

Substituting (14) into (13), we obtain (15) in accordance with [7] and [14].

$$P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}\tilde{\varepsilon} = \underset{N\to\infty}{Lim}\frac{1}{N}\sum tr(\Pi_T\Sigma_T) = tr(\Pi_T \underset{N\to\infty}{Lim}\frac{1}{N}\sum_{i=1}^{N}\Sigma_i) = tr(\Pi_T\Sigma_T) \quad (15)$$

Under the assumption of homoscedasticity for which $\Sigma_T = \sigma^2 I_T$, we have that

$$P\lim_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}^{'}\tilde{\varepsilon} = tr(\Pi_T\sigma^2 I_T) = \sigma^2 tr(\Pi_T) = -\sigma^2(\frac{1}{1-\gamma} - \frac{1-\gamma^T}{T(1-\gamma)^2}) \quad (16)$$

which would result to the bias approximations below with reasonable level of accuracy:

$$B_1 = c_1(\overline{T}^{-1}); B_2 = B_1 + c_2(N^{-1}\overline{T}^{-1}); B_3 = B_2 + c_3(N^{-1}\overline{T}^{-2}) \quad (17)$$

where c_1, c_2 and c_3 depended on the unknown parameters σ_ε^2 and γ.

Substituting (15) into (10) we obtain:

$$P\lim_{N\to\infty}(\hat{\gamma}-\gamma) = tr(\Pi_T\Sigma_T)/\sigma_{y_{-1}/X}^2 \quad (18)$$

Also, substituting (14) into (9), we obtain

$$P\lim_{N\to\infty}(\hat{\beta}-\beta) = -\psi P\lim_{N\to\infty}(\hat{\gamma}-\gamma) \quad (19)$$

(18) and (19) are the bias approximation of the LSDV estimator derived by [7] directly from the data without initial resort to any consistent estimator

$$\Sigma_{xy_{-1}} = P\lim_{N\to\infty}\frac{1}{N}\tilde{X}^{'}\tilde{y}_{-1},$$

$\Sigma_{xx} = P\lim_{N\to\infty}\frac{1}{N}\tilde{X}^{'}\tilde{X}$ and $\sigma_{y_{-1}/X}^2 = (1-\rho_{xy_{-1}}^2)\sigma_{y_{-1}}^2$.

$\rho_{xy_{-1}}^2 = \Sigma_{xy_{-1}}^{'}\Sigma_{xy_{-1}}^{-1}\Sigma_{xy_{-1}}$ is the squared multilple correlation coefficient of y_{-1} regressed on X and $\psi = \Sigma_{xx}^{-1}\Sigma_{xy_{-1}}$ is the corresponding vector of regression coefficients for γ and ρ unknown.

III. ROBUST LEAST SQUARES DUMMY VARIABLE ($LSDV_R$) ESTIMATOR

The robust least squares dummy variable estimation is predicated on the derivation of an approximate expression for the inconsistency of the LSDV which could be used to correct for the bias of the LSDV. In [5], the robust LSDV estimator is implemented by finding consistent estimates for σ_ε^2 and γ, subtracting each or any of the bias approximation B_c in (17) from the LSDV obtained by within estimation, we obtain the robust LSDV, $LSDV_R$, estimator below:

$$LSDV_R = LSDV - \hat{B}_c,$$ c= 1, 2 and 3: AH=1, AB=2 and BB=3 (20)

The consistent estimator of σ_ε^2 is obtained by

$$\hat{\sigma}_{\varepsilon c}^2 = \frac{e_c^{'}De_c}{N-K-T}, \quad N > T+K \quad (21)$$

where $e_c = y - w\hat{H}_c$ and C= AH, AB and BB, are the consistent estimators of γ.

It is pertinent to point out, at this juncture, that the bias approximation derived by [1], [2] and [7] assumed homoscedasticity of the error variance in their studies. The additive bias corrected LSDV estimator by [7] , just like that of [2], relied upon the consistent IV-GMM estimators of γ to determine the bias such that the bias corrected LSDV estimators are:

$$\hat{\gamma} = \hat{\gamma}_{DV} - \frac{tr(\hat{\Pi}_{T.Gmm}\hat{\Sigma}_{T.gmm})}{\sigma_{y_{-1}/X}^2} \quad (22)$$

$$\hat{\beta} = \hat{\beta}_{DV} + \zeta\frac{tr(\hat{\Pi}_{T.Gmm}\hat{\Sigma}_{T.Gmm})}{\sigma_{y_{-1}/X}^2} \quad (23)$$

where $\Pi_{T.GMM}$ and $\Sigma_{T.GMM}$ are the variance structures which depended on γ and β respectively, and can be obtained by their sample equivalence as explained earlier in a section above while Σ_T can be estimated from:

$$\hat{\Sigma}_{T.GMM} = diag(\hat{\sigma}_{t.gmm}^2) \quad (24)$$

For which

$$\hat{\sigma}_{t.gmm}^2 = \frac{(\tilde{y}_t - \hat{\gamma}_{gmm}\tilde{y}_{t-1} - \tilde{X}_t\hat{\beta}_{gmm})^{'}(\tilde{y} - \hat{\gamma}_{gmm}y_{t-1} - \tilde{X}_t\hat{\beta}_{gmm})}{N(T-1)/T}$$

(25)

and $\tilde{y}_t, \tilde{y}_{t-1}$ and $\tilde{X}_t$ are stacked across individuals such that $\tilde{y}_t = (\tilde{y}_{1t}, \tilde{y}_{2t}, ..., \tilde{y}_{Nt})$

According to [7] , to take care of a non linear bias correction, the bias corrected estimator is not feasible for γ is obtained by solving

$\hat{\gamma} = \gamma - \frac{tr(\Pi_T\Sigma_T)}{\sigma_{y-1/X}^2}$ for γ while assuming, first, that the variance structure Σ_T is given by $\sigma_{y_{-1}}^2$ and ζ. Σ_T, $\sigma_{y_{-1}}^2$ and ζ are all unknown and the consistent estimator of Σ_T [7].

By a system of K equations, the bias corrected γ and β are obtained from

$$\hat{\gamma}_{DV} = \gamma - tr(\hat{\Pi}_T\hat{\Sigma}_T)\hat{\sigma}_{y_{-1}/X}^2 \quad (26)$$

$$\hat{\beta}_{DV} = \beta + \zeta tr(\hat{\Pi}_T\hat{\Sigma}_T)\hat{\sigma}_{y_{-1}/X}^2 \quad (27)$$

where

$$\hat{\Sigma}_T = diag(\hat{\sigma}_t^2);$$

$$\hat{\sigma}_t^2 = \frac{(\tilde{y}_t - \gamma\tilde{y}_{t-1} - \tilde{X}_t\beta)'(\tilde{y}_t - \gamma\tilde{y}_{t-1} - \tilde{X}\beta')}{N(T-1)/T}$$

IV. DESIGN OF THE EXPERIMENT

To provide for effective comparison of the performance of the robust LSDV estimator, ($LSDV_R$), against the AH, AB, and BB, we generated the y_t value as it is defined in (1), i.e. $Y_{it} = X^1_{it}\beta + \gamma Y_{it-1} + \alpha_i + \varepsilon_{it}$, a simple dynamic panel data model with fixed effects i.e. having a time invariant individual or group specific effects, α_i , and generated X_t using the generating equation $X_{it} = \rho X_{it-1} + e_t$, $e \sim N(O,1)$, $/\rho/<1$, making provision for an outlier.

We specified ρ =0.8, γ =0.8 and β=0.2 in the general models and specified two models: (1) for finite samples (i.e. n=11 and t=50) and (2) for large samples (i.e. n=51 and t=10). The start up values Y_{i0} and X_{i0} are obtained using the procedures by [17]. for $\alpha_i = Q_1\bar{X}_{it} + e_{it}$, $e_{it} \sim N(0,1)$, we fix Q_1 at 0.8 as in [18]. We then used the within estimation of (2) to obtain the LSDV parameter estimators which are $\hat{\gamma}_{DV}$ and $\hat{\beta}_{DV}$. The experiment conducted is replicated five hundred (500) times. The root mean square errors (RMSE) of the estimated model, the root mean square errors (RMSE γ) of the estimated autoregressive coefficients as well as the Akaike Information Criterion (AIC) are employed for model comparisons. We employed Stata 10.0, Excel 2007 and Minitab statistical packages in the analysis to cushion the cumbersome nature of some estimators,

V. RESULTS AND DISCUSSIONS

The results of the simulation analysis for the various estimators considered is shown in the table 1 below at the two specifications of time and cross-sections for $\rho = \gamma$ =0.8 and β=0.2.

Table 1. Simulation analysis of the various estimators of the dynamic panel data model in the presence of outliers

Estimator	1. n=11, t=50				2. n=51, t=10			
	RMSE	AIC	BIAS γ	RMSE γ	RMSE	AIC	BIAS γ	RMSE γ
$LSDV_R$	0.01109	1.308	0.00102	0.00121	0.00101	0.1601	0.0124	0.0106
BB	0.01201	1.4529	-0.0061	0.00695	0.01038	0.198	0.00224	0.0057
AB	0.01245	1.5616	-0.0076	0.0089	0.0113	0.1952	0.0023	0.0068
AH	0.01114	1.351	0.01645	0.01764	0.01113	0.1937	0.0038	0.0098
LSDV	0.0133	1.5212	0.01147	0.00143	0.01374	0.1765	0.0206	0.0401

From the results of the study, the Blundell and Bond (BB) generalized method of moments (gmm) consistent estimator recorded minimum error in the autoregressive term with RMSEY=0.0057 in finite samples with large number of cross-sections (n=51) and finite period of time (t=10) or specification 1: this buttressed the results of [2], [7], [12] and [5], while the robust least squares dummy variable estimator ($LSDV_R$) showed high predictive power of the model by returning the least values in the RMSE=0.0010 and AIC=0.1601 at the same specification 2 as well as in specification 1. The robust least squares dummy variable estimator ($LSDV_R$) recorded the minimum error values of the autoregressive term with RMSEY=0.00121 in specification 1. The error values are generally lower in specification 2, relatively, compared to those of the specification 1. The unsteady nature of the Arellano and Bond (AB) consistent estimator that led to the introduction of the BB system GMM estimator is seen as it recorded higher values of RMSE's of 0.01245 and 0.0113 in specifications 1 and 2 respectively, relative to the other consistent estimators. However, the $LSDV_R$ recorded minimum RMSEY of 0.00101 in finite samples with large cross-section of n=51 and t=10 which is in agreement with the report of [7]. The bias of the least squares dummy variable estimator is approximated by the Blundell and Bond (BB) and its effects on the results of the robust least squares dummy variable estimator is quite glaring as the robust least squares dummy variable estimator ($LSDV_R$) produced the minimum error in large samples of small number of cross sections (n=11) and long time period (t=50). It is observed that even in the presence of outliers the predictive power of the robust least squares dummy variable estimator ($LSDV_R$) is more powerful than the other competing models and it is the most efficient except in the finite sample where the BB is the most efficient model in estimating the autoregressive coefficient.

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