Analysis Simulation of Interaction Information in Chaotic Systems of Fractional Order

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Abstract— **In article, in terms of Tsallis entropy, a model of generalized information losses for chaotic systems of fractional order in proposed. It is shown that the generalized model is the interaction of memory losses and process losses (transition from a state of relaxation to a gate of equilibrium) caused by the duration of the relaxation processes. Interaction demonstrates a variable mode of work.**

Index Terms—**Tsallis entropy, memory losses, relaxation, variable mode of work.**

I. INTRODUCTION

In the direction of the physics of open systems, in the study of transient and recurrent processes, an important place is occupied by the paradigm of non-additive systems.

For example, they include systems with anomalous diffusion of levy, quantum systems, fractional-order systems, and others.

In such systems, exponentially rapid mixing becomes power low character, as a result of which there is a weak phase space chaotization.

It is known that these systems have long-range effects, non-Markovian behavior, multifractal initial conditions, some dissipation mechanisms, and so on.

Formally, the theory of non-additive systems is based on the deformation of the logarithmic and exponential functions, which modifies the Bolthmann-Gibbs entropy in such a way that the distribution function acquires either long-range asymptotic or is cut off at finite values energy.

In addition, it should be noted that a characteristic of non-additive systems is the self-similarity of their phase space, the volume of which remains unchanged under deformation.

When studying transient and recurrent processes in network structures with mixing, it turns out that they belong to complex (anomalous) systems in which there is a strong interaction between its individual parts.

That is, non-Markovian of the dynamic process, some motivation for the movement of its individual elements, which leads to a violation of the hypothesis of complete chaos and thus to the non-additive of its thermodynamic characteristics – entropy.

For the reasons listed above, its description on the basis of classical statistics is not adequate, since the exponential distribution of the probability of a state is true for simple systems, and for complex ones the probability decreases according to the power law Pareto [1].

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As a result, the process of self-organization can proceed spontaneously.

The theory, within which adequate modeling of the above features of transport systems with mixing is possible, can become intensively developed non-extensive statistical mechanics of Tsallis, which is intended for describing the collective properties of complex systems (with a long memory, with strong correlation between elements etc.) [1].

The account of these states makes it possible to analyze various consequences (bifurcated regime, branching process, information loss etc., in the functioning of complex systems).

In this connection, the paper sets the tasks of developing a mathematical model for information loss in transient processes in chaotic system of fractional order.

II. EQUILIBRIUM AND NON-EQUILIBRIUM STATES

An important aspect in the study of transient chaotic systems is keeping their equilibrium and non-equilibrium states.

The account of these states makes it possible to analyze various consequences (bifurcated regime, branching process, information loss, etc. in the functioning of complex systems).

In this connection, the paper sets the tasks of developing a mathematical model for information loss in transient processes in chaotic system of fractional order.

2.1. Basic concepts

A. Topological Order Space

Definition 1. The number is called as a metric order of a compact A

$$k = lim\left(-\ln N_A(\varepsilon)/\ln\varepsilon\right), \tag{1}$$

where ε - the sphere of radius ε; $N(\varepsilon)$ - number of spheres in a final sub covering of a set.

The lower bound of metric orders for all metrics of a compact A (called by metric dimension) is equal the Lebesgue dimension.

However it appeared that the metric order entered in [2], coincides with the lower side the fractal dimension of Hausdorff-Bezikovich defined in the terms “box-counting”.

Takes place

Theorem 1 [2]. For any compact metric space X .

$$dim\, X = inf\left\{\underline{\lim_{\varepsilon\to 0}}\frac{\log N_{\varepsilon,d}(X)}{-\log\varepsilon} : d \text{ is a metric on } X\right\}, \tag{2}$$

where

$N_{\varepsilon,d}(X) = min\{|U| : U \text{ is a finite open covering of } X \text{ with mesh} \le \varepsilon\}$.

From here (X, d_f) - compact fractal metric space with dimension d_f .

Here it is important to note that at the description of properties of systems with fractional structure it is impossible to use representation of Euclidean geometry.

There is a need of the analysis of these processes for terms of geometry of fractional dimension.

Remark. In [3] presented results of communication of a fractional integro-differentiation (in Riman-Liouville or Gryunvalda-Letnikov's terms) with Koch's curves.

It is noted that *biunique communication between fractals and fractional operators does not exist*: fractals can be generated and described without use of fractional operations, and defined the fractional operator not necessarily generates defined (unambiguously with it

connected) fractal process or fractal variety.

However use of fractional operations allows generating other fractal process (variety) which fractal dimension is connected with an indicator of a fractional integro-differentiation a linear ratio on the basis of the set fractal process (variety).

In [3] fractional integrals of Riman-Liouville are understood as integrals on space of fractional dimension. Thus the indicator of integration is connected with dimension of space an unambiguous ratio.

In this regard consideration of dimension of chaotic systems of a fractional order causes interest. So, in [4] was noted that dimension of such systems can be defined by the sum of fractional exponents Σ, and $\Sigma < 3$ is the most effective.

Let the chaotic fractional system of Lorentz take place [4]:

$$\frac{d^{\alpha}}{dt^{\alpha}}x = \sigma(y-x) \quad \frac{d^{\beta}}{dt^{\beta}}y = \rho x - y - xz' \quad \frac{d^{\gamma}}{dt^{\gamma}}z = xy - bz \cdot \tag{3}$$

Here $\sigma = 10,\ \rho = 28,\ b = 8/3;\ 0 < \alpha,\beta,\gamma \le 1,\ r \ge 1$.

Then fractional dimension of system of the equations (3) will have an appearance [4]:

$$\alpha + \beta + \gamma = \Sigma \,. \tag{4}$$

So, for example, for Lorentz's system with fractional exponents $\alpha = \beta = \gamma = 0.99$, effective dimension $\Sigma = 2.97$.

This, in the context of fractional dynamics let $\widetilde{X}$ - any set of nonlinear physical systems, A^{α} - a subset of a set $\widetilde{X}$ of systems of a fractional order with memory $A^{\alpha} \subset \widetilde{X}$. Then a triad $\left(\widetilde{X}, A^{\alpha}, \Sigma\right)$ – compact fractional metric space with dimension Σ.

Let's designate $W \in \left(X, d_f\right)$. On the basis [5] and Remarks $\left(\widetilde{X}, A^{\alpha}, \Sigma\right) \subset W$.

Let's consider transformation W at an angle of communications of average time of return of Poincare $\langle\tau\rangle$ with d_f and "residual" memory $J(t)$.

Here $g : \langle\tau\rangle \Rightarrow d_f l : d_f \Rightarrow J(t)\chi : \langle\tau\rangle \Rightarrow (g,l)$.

From here $U \in \left(X, \langle\tau\rangle\right)$ - the generalized compact metric space of Poincare with dimension $\langle\tau\rangle$.

B. Generalized Memory

As said, the analysis and synthesis of multidimensional chaotic system of fractional-order there was a problem with memory estimation.

Axiomatic

Let the trajectory of the generalized memory system is of the form [6-8]:

$$Q_{GM} = Q_{\ge 0} \bigcup Q_{\le 0} \,. \tag{5}$$

Definition 2. $Q_{\ge 0} \subset Q_{GM}$ - it called semi-trajectory to the trajectory Q_{GM}, if each $t > 0$ it the inclusion

$$Q_{\ge 0} \subset O_{\varepsilon}^{mem} \bigcup_{j=0}^{j-1} \left(\left[t_j, t_{j+1}\right], j\right) \cdot \tag{6}$$

Here $Q_{\ge 0}\left[t_j, t_{j+1}\right]$ - segment of the semi-trajectory memory of responsible values $t \subset \left[t_j, t_{j+1}\right]$ and O_{ε}^{mem} - ε - neighborhood of the corresponding set.

Definition 3. $Q_{\le 0} \subset Q_{GM}$ it called semi-trajectory to the trajectory Q_{GM}, if each $t < 0$ it the inclusion

$$Q_{\le 0} \subset O_{\varepsilon}^{lm} \bigcup_{k=1}^{k} \left(\left[s_k, s_{k-1}\right], -k+1\right) \cdot \tag{7}$$

Here $Q_{\le 0}\left[s_k, s_{k-1}\right]$ - segment of the semi-trajectory "loss memory" of responsible values $t \in \left[s_k, s_{k-1}\right]$, and O_{ε}^{lm} - ε - neighborhood of the corresponding set.

Definition 4. Semi-trajectory $Q_{\ge 0} \subset Q$ recurrent if for every $t > 0$ inclusion

$$Q_{\ge 0} \subset \mathrm{O}_{\varepsilon} \bigcup_{j=0}^{j-1} \left(\left[t_j, t_{j+1}\right], j\right) \cdot \tag{8}$$

Here $Q_{\ge 0} \subset \left[t_j, t_{j+1}\right]$ - segment of the semi-trajectory recurrent of responsible values $t \subset \left[t_j, t_{j+1}\right]$.

Definition 5. Semi-trajectory $Q_{\le 0} \subset Q$ not recurrent if for every $t < 0$ it the inclusion

$$Q_{\le 0} \subset \mathrm{O}_{\varepsilon} \bigcup_{k=1}^{k} \left(\left[s_k, s_{k-1}\right],\ k+1\right) \cdot \tag{9}$$

Remark. It is known the average return time of Poincare is determined by the fractal dimensions the trajectories of generalized memory, GM. Hence the memory loss will be determined by the difference between global and local fractal dimensions, which means that the recurrence and no-recurrence semi-trajectories respectively.

C. Formation of Loss Memory

It is a known that during the Poincare recurrence characterizes as a "residual", and the real memory of the fractional-order system. Hence the equivalence between the spectrums of the Poincare returns time and distribution of generalized memory.

Los memory is determined by the difference between the global and the local fractal dimensions, which means, respectively, reversible and irreversible processes. Loss of information numerically defines the entropy.

D. Spectrum dimensions of the Poincare Return Time

From the perspective of mathematics synergetic aspect of the process of evolution is a change of a topological structure of the phase space of an open system.

Tracking change this structure as transient process requires the formation of a generalized criterion for recognizing the system in a certain state of chaos-quasi-periodic-hyper chaos and so on.

In the work, as a generalized criterion proposed spectrum of dimensions Poincare return time, describing how geometric, information, and dynamics characteristics of the transient process. That is generalized criterion is formed on the basis of synergetic principles. Spectrum of dimensions for the Poincare return time $\tilde{\alpha}$ is a functional dependence $\tilde{\alpha}_q$ dimension type characteristics from the scale q expressed in terms of the Hausdorf dimension $\dim_H$ [9]:

$$\tilde{\alpha}_q = \dim_H(x)(1-q/h), \qquad (10)$$

where q - scale, x - invariant set, h - topological entropy.

Shown [10] that the transport exponent μ defined by the relationship $\lim h(\mu) = h(d_f)$ and $\lim h(\mu) = h(d_f + 1)$ respectively.

Noted that μ^{opt} defined as [10]:

$$\mu^{opt} \approx d_f + 2 - \frac{\log(2)}{2(\log(2)+3+d_f)}, \qquad (11)$$

where d_f - fractal dimension.

However, in practical, is used

$$\mu^{opt} \approx d_f + 2. \qquad (12)$$

In addition, knowing that $h \le \sum_{\lambda_i > 0} \lambda_i$ [11], where λ_i - Lyapunov exponents, spectrum dimensions of the Poincare return time will look:

$$\tilde{\alpha}_{\tilde{q}} = d_f(X)\{(1-q/h)(\mu-2)(Q_{\ge 0})(h(\lambda))\}. \qquad (13)$$

Thus, there is functional dependence of the memory of the transport exponent mixing heterogeneous chaotic maps, as well as the Lyapunov exponent.

III. Formulation of the Problem

Definition. 6 If the system is not only all parameters are constant over time, but there is no steady flow through the action of any external source, such a state is called the *equilibrium*.

Period of time during which the system returns to the equilibrium state, called the *relaxation time*.

Definition 7. A system in a non-equilibrium state means that unbalanced potential exist within the system.

Let the functioning of fractional order chaotic system is carried out with the efficiency control system. That is, we can assume that in addition to a complex system of tasks by the main industrial activity goes into the efficiency control regime (a transition from a non-equilibrium state to an equilibrium, the equilibrium state of test mode, etc).

Thus, it is possible to present the interaction of a complex system with a control system as the efficiency of *functioning of the structure with a variable operating mode*. Known [12,13] that after nating operation successfully describes semi-Markov processes (SMP).

3.1. Model I.

Let the system works in there successive modes:

e_1 - normal operation;

e_2 - transition system in the equilibrium state of test mode;

e_3 - transition system to a non-equilibrium state.

Graph of the system shown in figure 1.

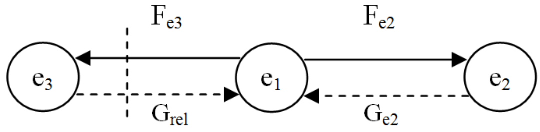

Fig 1.

In the first mode, the system is time

$$\theta_1 = \min\{\eta_{e_1}, \eta_{e_3}\}, \qquad (14)$$

that is, there is a mode e_1 until it will go off e_2 or e_3 mode.

Random variable θ randomly allocated:

$$P\{\theta_1 < t\} = 1 - P\{\eta_{e_1} \ge t\}P\{\eta_{e_3} \ge t\} = 1 - [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)], \qquad (15)$$

where $P\{\eta_{e_1} < t\} = \Phi_{e_1}(t)$ - arbitrary distribution function η_{TE} (testing equilibrium);

$P\{\eta_{e_3} < t\} = \Phi_{e_3}(t)$ - arbitrary distribution function η_{TE} (testing equilibrium).

Then expected value for θ_1

$$M\theta_1 = \int_0^\infty t dP\{\theta_1 < t\} = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt. \qquad (16)$$

Let θ_2 - the time of the test works, e_2.

The distribution function of time

$$P\{\theta_2 < t\} = G_{e_1}(t) \qquad (17)$$

θ_3 - the time in the system e_3.

The distribution function for θ_3 is:

$$P\{\theta_3 < t\} = G_{rel}(t). \qquad (18)$$

Hence the expectation of relaxation time

$$M\theta_3 = \int_0^\infty t dG_{rel}(t) \qquad (19)$$

Thus, the *transition of the states at the system: e_2 in e_1 and e_3 to e_1 completely recovery work.*

The task is to determine the mathematical expectation of the time in a state e_1, e_1 state probabilities and the mathematical expectation of the number of states e_3.

In accordance with work V.S.Korolyuk semi-Markov process (SMP) with a finite number of states $E = (e_1, e_2, e_3)$ is completely determined by residence times θ_i $(i = 1,\ldots,3)$ with distribution functions $P\{\theta_i < t\}$ of the transition probabilities of the states e_i is in the state e_j, provided that the SMP was in e_i state for a time t [14].

Let τ_1 the time before reaching into e_3, beginning e_1;

τ_2 - the time before reaching into e_3, beginning with e_2.

In [14] shows that the expectation of $\tau_1 = M\tau_1$ satieties the system of algebraic equations of the form:

$$M\tau_1 = M\theta_1 + \sum_{j\in E_+} P_{ij}\cdot M\tau_j \tag{20}$$

Then the system of stochastic equations for time reaching in state e_3 will have the form [14]:

$$\begin{aligned}\tau_1 &\overset{\bullet}{=} \theta_1 + \sigma_{12}\tau_2\\ \tau_2 &\overset{\bullet}{=} \theta_2 + \tau,\end{aligned} \tag{21}$$

where $\overset{\bullet}{=}$ - means equality on distribution; σ_{12} - transition indicator.

According to the formula of total probability

$$\begin{aligned}&M\tau_1 = M\theta_1 + M\sigma_{12}M\tau_2 =\\ &= \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + P\{\eta_{e_1} < \eta_{e_3}\}M\tau_2 =\\ &= \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty P\{\eta_{e_1} < t\}d\Phi_{e_3}(t)M\tau_2 =\\ &= \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)M\tau_2,\end{aligned} \tag{22}$$

$$M\tau_2 = M\theta_1 + M\tau_1 = \int_0^\infty [1-G_{e_1}(t)]dt + \mu\tau_1 \cdot \tag{23}$$

Then the system of algebraic equations for determining the mathematical expectation of the output time e_3, beginning with i -th state has the form:

$$\begin{cases} M\tau_1 = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)M\tau_2\\ M\tau_2 = \int_0^\infty [1-\sigma_{e_1}(t)]dt + M\tau. \end{cases} \tag{24}$$

Where the average time of the system in the state e_1 has the form [14]:

$$\begin{aligned}&M\tau_1 = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \left\{\int_0^\infty [1-G_{e_1}(t)dt + M\tau_1]\right\}\times\\ &\times \int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t) = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt +\\ &+ \left\{\int_0^\infty [1-G_{e_1}(t)]dt\right\}\left\{\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)\right\} + M\tau_1\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t),\\ &M\tau_1 = \left[1-\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)\right] = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty [1-G_{e_1}(t)dt]\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t).\end{aligned} \tag{25}$$

Here

$$M\tau_1 = \frac{\int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty [1-G_{e_1}(t)dt]\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)}{1-\int_0^\infty \Phi_{e_1}(t)d\Phi_{e_3}(t)}. \tag{26}$$

In the case of the gamma-distribution with parameter $m = 2$ we have:

$$\Phi_{e_1}(t) = \int_0^t \frac{\lambda_{e_1}^2}{\Gamma(2)} x e^{-\lambda_{e_1} x}dx = 1-\lambda_{e_1} t e^{-\lambda_{e_1} t} - e^{-\lambda_{e_1} t}. \tag{27}$$

Similarly

$$\Phi_{e_3}(t) = 1-\lambda_{e_3} t e^{-\lambda_{e_3} t} - e^{-\lambda_{e_3} t}. \tag{28}$$

$$G_{e_1}(t) = 1-\mu_{e_1} t e^{-\mu_{e_1} t} - e^{-\mu_{e_3} t}. \tag{29}$$

$$d\Phi_{e_3}(t) = \lambda_{e_3}^2 t e^{-\lambda_{e_3} t}dt. \tag{30}$$

Then the equation for $M\tau_1$ will be form [15]:

$$[M\tau_1] = \frac{4\mu_{e_1}(\lambda_{e_3}+1.38\lambda_{e_1})(\lambda_{e_3}+2.62\lambda_{e_1}) + \lambda_{e_1}^2(3\lambda_{e_3}+\lambda_{e_1})}{2\mu_{e_1}\lambda_{e_3}^2(3\lambda_{e_1}+\lambda_{e_3})}. \tag{31}$$

In the case of the exponential distribution equation for $M\tau_1$ has the form [15]:

$$[M\tau_1]_{\Im} = \frac{2\lambda_{e_1}}{\lambda_{e_3}(\mu_{e_1}+\lambda_{e_3})} + \frac{1}{\mu_{e_1}}\frac{\mu_{e_1}+\lambda_{e_1}}{\mu_{e_1}+\lambda_{e_1}}. \tag{32}$$

Laplace-Stieltjes transform (L-S) for distribution values [16]:

$$\begin{aligned}&\Phi_{e_1}(s) = \int_{e_3}^\infty e^{-st}d\Phi_{e_1}(t) = P_{12}(s); \quad \Phi_{e_3}(s) = \int_{e_3}^\infty e^{-st}d\Phi_{e_3}(t) = P_{13}(s);\\ &G_{rel}(s) = \int_{e_3}^\infty e^{-st}dG_{rel}(t) = P_{31}(s); \quad G_{e_1}(s) = \int_{e_3}^\infty e^{-st}dG_{e_1}(t) = P_{21}(s).\end{aligned} \tag{33}$$

L-S transform distribution time to SMP output $\{e_1, e_2\}$ beginning with e_1:

$$T_i(s) = \int_0^\infty e^{-st}dP\{\tilde{\tau}_1 < t\}. \tag{34}$$

Then on the basis of [17] we have the system of equation:

$$\sum_{j\in\{1,2\}}[P_{ij}(s) - \delta_{ij}]T_j(s) = \sum_{j\in\{3\}} P_{ij}(s), \quad i \in \{1,2\} \tag{35}$$

and

$$\begin{cases} -T_1(s) + P_{12}(s)T_2(s) = -P_{13}(s)\\ 12_{21}(s)T_1(s) - T_2(s) = 0 \end{cases} \tag{36}$$

or

$$\begin{cases} T_1(s) + \Phi_{e_1}(s)T_2(s) = -\Phi_{e_3}(s)\\ G_{e_1}(s)T_1(s) - T_2(s) = 0 \end{cases} \tag{37}$$

From whence

$$\begin{aligned}&T_2(s) = G_{e_1}(s)T_1(s)\\ &-T_1(s) + \Phi_{e_1}(s)G_{e_1}(s)T_1(s) = -\Phi_{e_3}(s)\\ &T_1(s)[\Phi_{e_1}(s)G_{e_1}(s) - 1] = -\Phi_{e_3}(s)\end{aligned} \tag{38}$$

Here

$$\begin{aligned}T_1(s) &= \frac{-\Phi_{e_3}(s)}{\Phi_{e_1}(s)G_{e_1}(s)-1} = \frac{\Phi_{e_3}(s)}{1-\Phi_{e_1}(s)G_{e_1}(s)}\\ T_2(s) &= \frac{G_{e_1}(s)\Phi_{e_3}(s)}{1-\Phi_{e_1}(s)G_{e_1}(s)}.\end{aligned} \tag{39}$$

With the transformation of L-S probability of normal works of the system, starting with the state i $P_i(s)$ expressed by the formula [18]:

$$P_i(s) = \int_0^\infty e^{-st}P_i(t)dt = \frac{1-T_i(s)}{s}. \tag{40}$$

Transformation of L-S probability of normal works, starting with e_1 it is expressed as [15]:

$$P_1(s) = \frac{1-\Phi_{e_1}(s)G_{e_1}(s) - \Phi_{e_3}(s)}{s[1-\Phi_{e_1}(s)G_{e_1}(s)]} \tag{41}$$

$$P_2(s) = \frac{1-\Phi_{e_1}(s)G_{e_1}(s) - G_{e_1}(s)\Phi_{e_3}(s)}{s[1-\Phi_{e_1}(s)G_{e_1}(s)]} \tag{42}$$

Let

$$\Phi_{e_3}(s) = \int_0^\infty e^{-st}\frac{\lambda_{e_3}^m}{\Gamma(m)}t^{m-1}e^{\lambda_{e_3}t}dt = \frac{\lambda_{e_3}^m}{(s+\lambda)^m}. \tag{43}$$

Similarly

$$G_{e_1}(s)=\frac{\mu_{e_1}^m}{(s+\mu_{e_1})^m};\qquad \Phi_{e_1}(s)=\frac{\lambda_{e_1}^m}{(s+\lambda_{e_1})^m}.$$

The equation (39) will have the following:

$$T_1(s)=\frac{\lambda_{e_3}^m(s+\mu_{e_1})^m(s+\lambda)^m}{(s+\lambda_{e_3})^m\left[(s+\mu_{e_1})^m+(s+\lambda_{e_1})^m-\mu_{e_1}^m\lambda_{e_1}^m\right]} \qquad (44)$$

and equation for

$$P_1(s)=\frac{(\lambda_{e_1}+s)^m(s+\mu_{e_1})^m(s+\lambda_{e_3})^m-\mu_{e_1}^m\lambda_{e_1}^m(s+\lambda_{e_3})^m-\lambda_{e_3}^m(s+\lambda_{e_3})^m(s+\mu_{e_1})^m}{s(s+\lambda_{e_3})^m\left[(\mu_{e_1}+s)^m+(s+\lambda_{e_1})^m-\mu_{e_1}^m\lambda_{e_1}^m\right]}. \qquad (45)$$

$$P_2(s)=\frac{(\lambda_{e_1}+s)^m(s+\lambda_{e_3})^m(\mu_{e_1}+s)^m-\lambda_{e_1}^m\mu_{e_1}^m(\lambda_{e_3}+s)^m-\lambda_{e_3}^m\mu_{e_1}^m(s+\lambda_{e_1})^m}{s(s+\lambda_{e_3})^m\left[(\mu_{e_1}+s)^m+(s+\lambda_{e_1})^m-\mu_{e_1}^m\lambda_{e_1}^m\right]}. \qquad (46)$$

Original finding in the general form for the equations (45) and (46) is difficult.

For the case of $m=2$, we have [15]:

$$T_1(t)=\frac{\mu_{e_1}\lambda_{e_1}}{2(\mu_{e_1}+\lambda_{e_1})}+\left[A-\frac{\lambda_{e_3}(\mu_{e_1}-\lambda_{e_3})^2(\lambda_{e_1}-\lambda_{e_3})^2 t}{(a-\lambda_{e_3})(b-\lambda_{e_3})(\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3})}\right]e^{-\lambda_{e_3}t}- \qquad (47)$$
$$-\frac{\lambda_{e_3}\mu_{e_1}\lambda_{e_1}e^{-(\mu_{e_1}+\lambda_{e_1})t}}{2(\mu_{e_1}+\lambda_{e_1})(\lambda_{e_3}-\mu_{e_1}-\lambda_{e_1})}+\frac{\lambda_{e_3}^2abe^{-at}}{8A_1(\lambda_{e_3}-a)^2}-\frac{\lambda_{e_3}^2abe^{-bt}}{8A_1(\lambda_{e_3}-b)^2},$$

$$P_1(t)=f_{e_1 1}-\frac{\mu_{e_1}\lambda_{e_1}t}{2(\mu_{e_1}+\lambda_{e_1})}+\left[B-\frac{(\mu_{e_1}-\lambda_{e_3})^2(\lambda_{e_1}-\lambda_{e_3})^2 t}{(\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3})(a-\lambda_{e_3})(b-\lambda_{e_3})}\right]e^{-\lambda_{e_3}t}- \qquad (48)$$
$$-\frac{\lambda_{e_3}^2\mu_{e_1}\lambda_{e_1}e^{-(\mu_{e_1}+\lambda_{e_1})t}}{2(\mu_{e_1}+\lambda_{e_1})^2(\lambda_{e_3}-\mu_{e_1}-\lambda_{e_1})}+\frac{\lambda_{e_3}^2\mu_{e_1}\lambda_{e_1}e^{-at}}{4aA_1(\lambda_{e_3}-a)^2}-\frac{\lambda_{e_3}^2\mu_{e_1}\lambda_{e_1}e^{-bt}}{4bA_1(\lambda_{e_3}-b)^2}.$$

If the parameter is limited to positive integers, the gamma-distribution is Erlang distribution for $m=2$, we have Erlang distribution of the Ist order:

$$P_2(t)=f_{e_1 2}-\frac{\mu_{e_1}\lambda_{e_1}t}{2(\mu_{e_1}+\lambda_{e_1})}+\left[C-\frac{\mu_{e_1}^2(\lambda_{e_1}-\lambda_{e_3})^2 t}{(\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3})(a-\lambda_{e_3})(b-\lambda_{e_3})}\right]e^{-\lambda_{e_3}t}- \qquad (49)$$
$$-\frac{\lambda_{e_3}^2\mu_{e_1}^4e^{-(\mu_{e_1}+\lambda_{e_1})t}}{ab(\mu_{e_1}+\lambda_{e_1})^2(\lambda_{e_3}-\mu_{e_1}-\lambda_{e_1})}+\frac{\lambda_{e_3}^2\mu_{e_1}^2(\lambda_{e_1}-a)^2e^{-at}}{2A_1a^2b(\lambda_{e_3}-a)^2}-\frac{\lambda_{e_3}^2\mu_{e_1}^2(\lambda_{e_1}-b)e^{-bt}}{2A_1ab^2(\lambda_{e_3}-b)^2},$$

where

$$f_{e_1 1}=\frac{(\mu_{e_1}+\lambda_{e_1})(a\lambda_{e_3}+b\lambda_{e_3}+2ab)+\lambda_{e_3}ab}{4\lambda e(\mu_{e_1}+\lambda_{e_1})^2}$$

$$f_{e_1 2}=\frac{2\lambda_{e_1}^2\lambda_{e_3}ab(\mu_{e_1}+\lambda_{e_1})+\mu_{e_1}^2\lambda_{e_1}^2\left[2ab(\mu_{e_1}+\lambda_{e_1})ab\lambda_{e_3}+\lambda ab(\mu_{e_1}+\lambda_{e_1})\right]}{2\lambda_{e_3}a^2b^2(\mu_{e_1}+\lambda_{e_1})^2}+$$
$$+\lambda_{e_3}a(\mu_{e_1}+\lambda_{e_1})\left[\lambda_{e_3}a^2b^2(\mu_{e_1}+\lambda_{e_1})^2\right]$$

If e_2 test frequency exceeds the number of states e_3, then equations for $P_1(t)$ and $P_2(t)$ will have the approximate values [15]:

$$P_1(t)=f_{e_1 1}-\frac{\mu_{e_1}\lambda_{e_1}t}{2(\mu_{e_1}+\lambda_{e_1})}-\frac{\left[\mu_{e_1}^2\lambda_{e_1}^2-2\mu_{e_1}\lambda_{e_1}\lambda_{e_3}(\mu_{e_1}+\lambda_{e_1})\right]te^{-\lambda_{e_3}t}}{2\mu_{e_1}\lambda_{e_1}(\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3})-\lambda_{e_3}\mu_{e_1}(\mu_{e_1}+\lambda_{e_3})} \qquad (50)$$

$$P_2(t)=f_{e_1 2}-\frac{\mu_{e_1}\lambda_{e_1}t}{2(\mu_{e_1}+\lambda_{e_1})}-\frac{\lambda_{e_1}\mu_{e_1}^2(\lambda_{e_1}-2\lambda_{e_3})te^{-\lambda_{e_3}t}}{2\mu_{e_1}\lambda_{e_1}(\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3})-\lambda(\mu_{e_1}+\lambda_{e_3})^2} \qquad (51)$$

For the exponential distribution law equations $P_1(t)$ and $P_2(t)$ have the form [15]:

$$P_1(t)=\frac{\lambda_{e_1}\mu_{e_1}(\lambda_{e_3}+\mu_{e_1}+\lambda_{e_1})}{\lambda_{e_3}(\mu_{e_1}+\lambda_{e_1})^2}-\frac{\lambda_{e_1}\mu_{e_1}}{\mu_{e_1}+\lambda_{e_1}}+e^{\lambda_{e_3}t}+\frac{\mu_{e_1}\lambda_{e_1}}{\lambda_{e_1}+\mu_{e_1}-\lambda_{e_3}}\times \qquad (52)$$
$$\times\left[\frac{\lambda_{e_3}e^{-(\mu_{e_1}+\lambda_{e_1})t}}{(\mu_{e_1}+\lambda_{e_1})^2}-\frac{1}{\lambda_{e_3}}e^{-\lambda_{e_3}t}\right]$$

$$P_2(t)=\frac{\lambda_{e_3}\mu_{e_1}(\mu_{e_1}+\lambda_{e_1})+\lambda_{e_1}\mu_{e_1}(\lambda_{e_3}+\mu_{e_1}+\lambda_{e_3})}{\lambda_{e_3}(\mu_{e_1}+\lambda_{e_1})^2}+\frac{\lambda_{e_1}\mu_{e_3}t}{\mu_{e_1}+\lambda_{e_1}}+ \qquad (53)$$
$$+\frac{\lambda_{e_1}}{\mu_{e_1}+\lambda_{e_1}-\lambda_{e_3}}\left[\frac{(\lambda_{e_3}-\mu_{e_1})e^{-\lambda_{e_3}t}}{\lambda_{e_3}}-\frac{\lambda_{e_1}\lambda_{e_3}e^{-(\mu_{e_1}+\lambda_{e_1})t}}{(\mu_{e_1}+\lambda_{e_1})(\lambda_{e_3}-\lambda_{e_1}-\mu_{e_1})}\right]$$

The effect of different distributions on the parameter λ is 15%, figure 2.

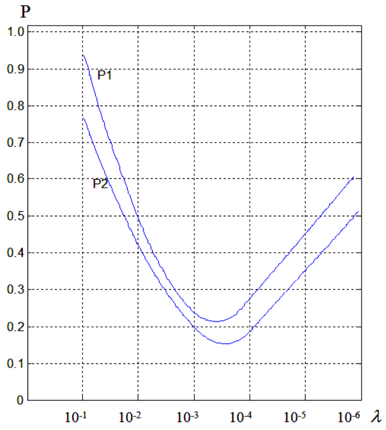

Fig. 2.

3.2. Definition of the mathematical expected value of relaxation

The average time between two test have $\theta_3+\tau$.

Distribution function of relaxation time is defined as:

$$\mu_I=MG_3+\mu\tau_1=\int_0^\infty\left[1-G_{rel}(t)\right]dt+M\tau \qquad (54)$$

where $M\tau$ was determined from the formula (20).

On the basis of (31) μ_I for the gamma-distribution has the form [15]:

$$\mu_I=\frac{2}{\mu_{rel}}+\frac{4\mu_{e_1}(\lambda_{e_3}+1.38\lambda_{e_1})(\lambda_{e_3}+2.62\lambda_{e_1})+\lambda_{e_1}^2(\lambda_{e_1}+3\lambda_{e_3})}{2\lambda_{e_3}^2\mu_{e_1}(3\lambda_{e_1}+\lambda_{e_3})} \qquad (55)$$

Based on theory of recovery V.Smite [19] mathematical expected value of relaxation has the form:

$$H(T_f)=\frac{T_f}{\mu_I}, \qquad (56)$$

where T_f - time operation of the system.

3.3. Model II.

Here we consider the case when the system is in e_1 and e_2, that is the system is not fully restored, that is, the system enters e_1. Graph of the system in figure 3.

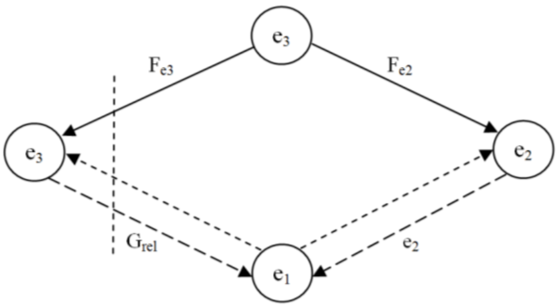

Fig. 3.

Model II is similar algorithm to the system of stochastic equation Model I.

The system of equation for the time of intering the e_3 has the form [15]:

$$\begin{cases} M\tau_0 = M\theta_{e_3} + e_1\{\eta_{e_1} < \eta_{e_3}\}M\tau_2 \\ M\tau_1 = M\theta_1 + e_1\{\eta_{e_1} < \eta'_{e_3}\}M\tau_2 \\ M\tau_2 = M\theta_2 + M\tau_1 \end{cases} \quad (57)$$

Then the system of algebraic equations for determining the mathematical expectation of the output time e_3, beginning with i th state has the form [15]:

$$\begin{cases} M\tau_0 = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi_{e_3}(t)]dt + \int_0^\infty \Phi_{e_3}(t)d\Phi_{e_3}(t)M\tau_2, \\ M\tau_1 = \int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi'_{e_3}(t)]dt + \int_0^\infty \Phi_{e_1}(t)d\Phi'_{e_3}(t)M\tau_2, \\ M\tau_2 = \int_0^\infty [1-G_{e_1}(t)]dt + M\tau_1. \end{cases} \quad (58)$$

Here

$$M\tau_1 = \frac{\int_0^\infty [1-\Phi_{e_1}(t)][1-\Phi'_{e_3}(t)]dt + \int_0^\infty [1-G_{e_1}(t)]dt \int_0^\infty \Phi_{e_1}(t)d\Phi'_{e_3}(t)}{1-\int_0^\infty \Phi_{e_1}(t)d\Phi'_{e_3}(t)}. \quad (59)$$

After transformation expression (59) will be of the form [15]:

$$M\tau_1 = \frac{4\mu_{e_1}(\lambda'_{e_3} + 0.38\lambda_{e_1})(\lambda'_{e_3} + 2.62\lambda_{e_1}) + \lambda_{e_1}) + \lambda^2_{e_1}(\lambda'_{e_3}3 + \lambda_{e_1})}{1-\int_0^\infty \Phi_{e_1}(t)d\Phi'_{e_3}(t)} \quad (60)$$

Probability of normal works, beginning with e_0 state in the L-S:

$$P_{e_3}(s) = \frac{1-\Phi_{e_1}(s)G_{e_1}(s) - \Phi_{e_3}(s) + \Phi_{e_1}(s)G_{e_1}(s)[\Phi_{e_3}(s) - \Phi'_{e_3}(s)]}{s[1-\Phi_{e_1}(s)G_{e_1}(s)]} \quad (61)$$

The original expression for (61) λ_{e_3}, $\lambda'_{e_3} >> \lambda_{e_1}$ has the form [15]:

$$P_{e_3}(t) = -\lambda_{e_3}e^{-\lambda_{e_3}t} + F\lambda'_{e_3}e^{-\lambda'_{e_3}t} + \frac{fbe^{-bt}}{4ab^4\bar{A}(0{,}5-\lambda_{e_3}-\lambda'_{e_3})} - \frac{fae^{-at}}{4a^5b\bar{A}(0{,}5-\lambda_{e_3}-\lambda'_{e_3})} - \\ - \frac{-\mu_{e_1}\lambda_{e_1}\lambda_{e_3}\lambda'_{e_3}e^{-(\mu_{e_1}+\lambda_{e_1})t}}{(\mu_{e_1}+\lambda_{e_3})^2(\mu_{e_1}+\lambda_{e_1}-2\lambda_{e_3})(\mu_{e_1}+\lambda_{e_1}-2\lambda'_{e_3})}. \quad (62)$$

Here the coefficients $f^o_{e_1}, a, b, f_a, f_b, F'_{\lambda_0}, A, B, C$ are not.

3.4. Definition of mathematical expectation of the number of relaxations

Is similarly to the Model I expression for μ_{II} has the form [15]:

$$\mu_{II} = M(\tau_1 + \theta_3) = M\tau_1 + M\theta_3 \quad (63)$$

or

$$\mu_{II} = \frac{2}{\mu_{rel}} + \frac{4\mu_{e_1}(\lambda'_{e_3} + 0.38\lambda_{e_1})(\lambda'_{e_3} + 2.62\lambda_{e_1}) + \lambda^2_{e_1}(3\lambda'_{e_3} + \lambda_{e_1})}{2\mu_{e_3}(\lambda'_{e_3})^2(3\lambda_{e_1} + \lambda'_{e_3})}, \quad (64)$$

here

$$H(T_f)_{II} = \frac{T_f}{\mu_{II}}.$$

Lack e_2 graphs models I and II – trivial. Here

$$P(t) = 1 - \Phi_{e_3}(t) \quad (65)$$

$$M\tau'_1 = \int_0^\infty [1-\Phi_{e_3}(t)]dt \quad (66)$$

$$M\theta_3 = \int_0^\infty [1-G_{rel}(t)]dt \quad (67)$$

Values μ in this case:

$$\mu' = M\theta_3 + M\tau'_1 = \int_0^\infty [1-G_{rel}(t)]dt + \int_0^\infty [1-\Phi_{e_3}(t)]dt \quad (68)$$

Producing transformation expression (68) will have the form

$$\mu' = 2\frac{\lambda_{e_3} + \mu_{rel}}{\lambda_{e_3}\mu_{rel}}. \quad (69)$$

Function of the number of relaxation is:

$$H'(T_f) = \frac{T_f}{\mu'} = 0{,}5\frac{\lambda_{e_3}\mu_{rel}}{\lambda_{e_3} + \mu_{rel}}T_f \quad (70)$$

3.5. The construction of a mathematical model of transients processes. Entropy of Tsallis

Strong interaction thermodynamically anomalous systems significantly changes the picture of research, which leads to new degrees of freedom, to other statistical physics of non-Boltzmann type. This means that the expression for entropy will be different.

So, Tsallis used the standard expression and instead of the logarithm introduced a new function-power [20,21]:

$$ln(x) \to ln_q(x) = (x^{1-q} - 1)/(1-q), \quad (71)$$

where q - to same numerical parameter.

Thus, the new formula for q -entropy takes the form [20,21]:

$$S_q = -\sum_i (P_i^q\, ln_q(P_i)) = \left(1 - \sum_i P_i^q / (q-1)\right). \quad (72)$$

For $q \longrightarrow 1$, the q -entropy goes over into the standard Boltzmann entropy and, thus the entropy is no longer an extensive function.

In the case of independent of freedom [20,21]:

$$S_q(A,B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B), \quad (73)$$

where q - measure of non-flexibility of the system; A and B - independent subsystems.

To construct a mathematical model of transient processes in the context of accounting: memory loss, in the implementation of fractional dynamics problems; loss of information depending of the scenarios of the functioning (beginning the system into states e_i), we use the entropy of Tsallis (73). Let the generalized model of memory GM in terms of Tsallis entropy called as $S_q(GM) = A$; loss of information from scenarios e_i in terms of Tsallis entropy as $S_q(M_{\tau_i})B$.

Then the total loss of information in the realization of fractional dynamics problem, in terms of the entropy of the Tsallis will have the form (73).

IV. CONCLUSION

The model of generalized information losses in term of Tsallis entropy for chaotic systems of fractional order is proposed.

It is shown that the model is a generalized memory interacting with the situational system (transitions from the nonequilibrium to the equilibrium state). The relaxation time corresponds to the loss of information.

The interaction of structures that determine the loss of information is a system that operates in an alternating mode (semi – Markov process).

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