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Analysis of Optimal Connected Designs Using Minimal Replicates

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***Abstract*— Minimizing type I and type II errors with appropriate sample sizes in order to have convincing conclusions often pose a great challenge to experimenter. REML and ML criteria are popular for estimating variance-covariance matrix. Which one to use might pose another challenge to an experimenter. Huge error has effect on experiment. How to get the best information on Incomplete Block Design experiment is the focus of this paper.**

***Index Terms*— Optimal design, Power, type I error, type II error.**

I. INTRODUCTION

Incomplete block designs play a significant role most especially in agricultural sciences when a researcher has a great number of treatment with a constraint that these treatments cannot be accommodated in any block of the experiment Divecha and Ghosh (1994), Sharma (1996) and Sharma and Fanta (2010), leading to "between block" variability larger than "within block variability" improving precision in an experiment. These designs eliminate heterogeneity to a greater extent compared to Randomized complete block design (RCBD).

Apart from eliminating all possible controllable and measurable uncontrollable effects (i.e., covariate(s)) from the experimental error to have a precise estimate of the parameter of interest, the concept of replication is very important as well Sharma and Fanta (2010).

A class of incomplete block designs in which all treatment effects are estimated with the same variance are called Balanced Incomplete Block Design (BIBD) otherwise, they are called Partially Balanced Incomplete Block Designs (PBIBD) Das (1998). Balanced Incomplete Block Design could also be called Partially Incomplete Block Design with one associate class denoted as PBIBD/1 while PBIBD/k is defined as partially balanced incomplete block design with "k" associate classes.

The objective of an experimenter is important in planning and must be established fully before the experiment is carried out. At this phase, all possible things which may affect research work negatively must be accounted for both in the design and in the statistical model.

A design which could account for assignable causes of variation must be selected from a catalogue of designs; such designs are often called **optimal designs**. An optimal design is known to possess three important properties which are Unbiasedness, Minimum variance and small number of unit utilization. These properties make an experiment cheap to run and providing reliable results. In this paper, **A-**, **D-**, **E-** and **SS-** optimality criteria were used to select optimal designs and their performances were also checked. If a researcher fails to find an optimal design, larger sample size would be required to detect true effect. Minimum variance is the idea of precision. Accuracy is broader than precision due to the effect of biasedness but precision affects accuracy. Mean Square Error (***MSE***) is used as a measure of accuracy here. Accuracy is affected by type I error, type II error and effect size. The smaller the ***MSE*** the better.

There are two types of error associated with statistical test; type I and type II errors Hun Myoung Park (2010). These errors cannot be avoided due to a number of reasons. The major concern of an experimenter is to minimize these errors as much as possible by finding a suitable sample size which optimal designs provide. By this, the power of the test would be high, and the conclusion from such test would be convincing Karin Meyer (1987). As mentioned above, effect size affect accuracy, large effect size means high probability of detecting effect when it is actually there (i.e., power) holding other components constant Hun Myoung Park (2010). High power of a statistical test with small test size is an indication of a good test otherwise, it is bad.

Error comes into an experiment through uncountable sources, and no matter how hard an experimenter tries to avoid this error, it exists. In fact, that is why, this error is sometimes called unexplained variation. It is difficult for an experimenter to explain how this happens but must be minimized or else it may distort the conclusion of an experiment. Such errors are called experimental error.

II. MATERIALS AND METHOD

Once experimental design capable of minimizing variability transmitted from a noise factor has been detected through appropriate optimality criteria, a statistical model that complement such design must be chosen in the planning phase and if possible, by guiding against extra sources of variation, the statistical model must not be changed after the experiment Dieter Rasch et al (2011).

Simulation

Without data, no analysis. Incomplete block design data were simulated using mean model defined as

$$y_{ij} = \mu_{ij} + e_{ij};\, i = 1,2, \dots t,\, j = 1,2, \dots b \quad (1)$$

Where

y_{ij} Is the observation from ith treatment and jth block

μ_{ij} Is the sum of treatment effect, block effect and the overall mean. e_{ij} Is the error from ith treatment and jth block; $e_{ij} \sim N(0, \sigma^2)$

The variability in μ_{ij} depend heavily on model parameters and e_{ij} is added to it in order to have the response variable as

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a random variable. In this experiment, the addition of the e_{ij} to the μ_{ij} varies with changes in error term variance such that $0 \leq var \leq 2$. The effects of these error terms were also observed on both optimal and less optimal designs. Here, treatments and blocks are no longer orthogonal Bayrak and Buluf (2006), only treatment contrasts are estimable because block effects are eliminated Das (1998).

P-value approach was used in the decision making regarding whether or not to reject H_0. This approach gives information about how deep the test statistic enters the rejection region, a plus to a researcher Hun Myoung Park (2010). Actually, P-value show case the smallest level of significance possible to reject H_0, this will help an experimenter to take a decision at any recognized level of significance.

Heteroscedasticity between treatments tends to inflate the random (or experimental) error Bronislaw Ceranka and Malgorzata Graczyk (2008) and Sharma and Fanta (2010), making the test statistic less sensitive to detect true effects. When interaction occurs between treatment and block and this interaction is not accounted for in the model, it invalidates the outcome of the experiment. All these and others were considered in the simulation.

The crossdes package in R was used to construct designs. Simulation and analysis were done using agricolae package in R.

Statistical Methods

A block design without interactions and covariates has a model as defined below

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}; i = 1,2, \dots t, j = 1,2, \dots b \qquad (2)$$

Where

y_{ij} Is the observation from ith treatment and jth block

μ Is the overall mean.

α_i Is the ith treatment effect

β_j Is the jth block effect

e_{ij} Is the error from ith treatment and jth block; $e_{ij} \sim N(0, \sigma^2)$

The model above is defined as incomplete block design model if the number of treatment is larger than each block size i.e., $t > k$

III. RESULTS AND DISCUSSION

In this paper, moderate P-values suggest little signal against the null hypothesis whereas small P-values suggest that the null hypothesis is false. R package was used for the simulation and the analysis.

Table 1. PERFORMANCE OF REML AND ML CRITERIA IN INCOMPLETE BLOCK ANALYSIS D(9,9,3,3) PBIB(2)

A=0.727272	**D=0.7377879**	**E=0.66667**	**SS=0.166667**

Random variation RML	**Random variation ML**	**Bahavior of randomness**	
Df SSq MSq F Pr(>F)	**Df SSq MSq F Pr(>F)**	**Min**	**-0.13425**
trt 8 927 115.9 68681 < 2.2e-16	**trt 8 618.0 77.25 82418 <2.2e-16**	**Max**	**0.072988**
Re 10 0.02 0.002	**Re 10 0.01 0.001**	**Var**	**0.001957**
Efficiency = 0.7272727	**Efficiency =0.7272727**	**SS**	**0.06757**
Mean =31.30847	**Mean = 31.30847**	**MD**	**0.034514**
CV=0.1311968	**CV = 0.09778834**		
AIC 72.62688	**AIC 34.737609**		
BIC 83.31134	**BIC 50.287651**		
-2 Res Log Likelihood =-24.31344	**-2 Res Log Likelihood =-5.368804**		

Table 2. PERFORMANCE OF REML AND ML CRITERIA IN INCOMPLETE BLOCK ANALYSIS D (15,10,3,2) PBIB(3)

A=0.5645161	**D=0.6411746**	**E=0.3333333**	**SS=1.190476**

D(15,10,3,2) PBIB(3)	**(5, 7,10),(5,13,15),(6, 7,11),(3,12,14), (4, 8,15),(6, 8,12),(2, 9,11),(2,13,14), (1, 4, 9),(1, 3,10)**

Random variation RML	**Random variation ML**	**Bahavior of randomness**	
Df SSq MSq F Pr(>F)	**Df SSq MSq F Pr(>F)**	**Min**	**-0.0822**
trt 14 1922.6 137.3 91430 8.7e-15	**trt 14 961 68.7 152384 1.9e-15**	**Max**	**0.0792**
Re 6 0.01 0.002	**Re 6 0.0 0.000**	**Var**	**0.0022**
Efficiency = 0.6363636	**Efficiency =0.6363636**	**SS**	**0.0664**
Mean =34.57319	**Mean = 34.57319**	**MD**	**0.03944**
CV = 0.1120972	**CV = 0.06139811**		
AIC = 104.59691	**AIC = 33.713686**		
BIC = 117.34181	**BIC = 58.935239**		
-2 Res Log Likelihood= -34.29845	**-2 Res Log Likelihood =1.143157**		

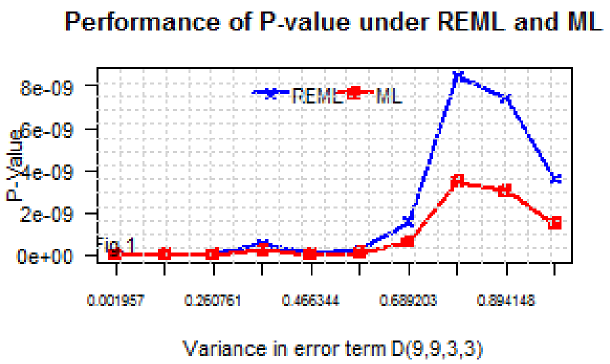
Table 3. PERFORMANCE OF REML AND ML CRITERIA IN INCOMPLETE BLOCK ANALYSIS D(30,20,3,2) PBIB(15)

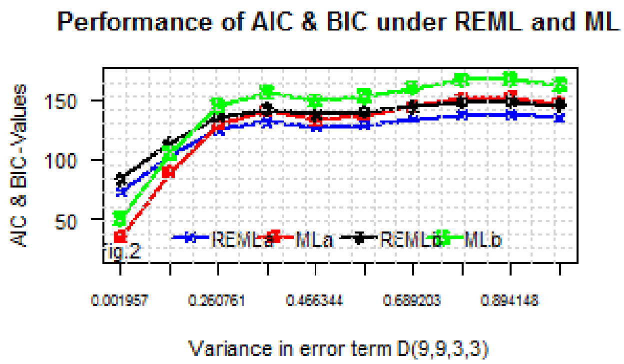
A=0.4706192	**D=0.5885551**	**E=0.1774279**	**SS=2.873563**

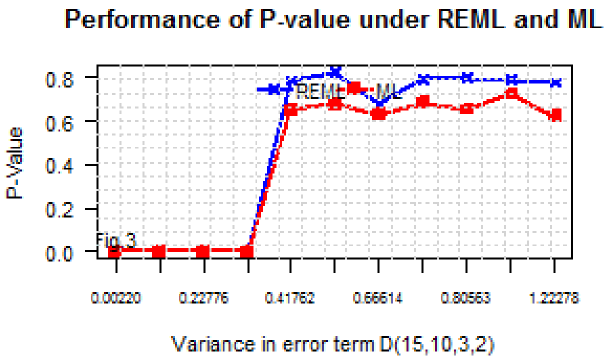
D(30,20,3,2) PBIB(15)	**(6, 11,30),(9,14,21),(12,16,30),(3,22,27), (11,20,24),(14,17,29),(2,10,19),(13,16,26), (7,25,28),(18,22,23),(5, 6,18),(3, 26,29), (2,15,20),(8, 9,12),(4, 5, 28),(19,21,23), (1,15,27),(4,10,13),(1, 8, 25),(7, 17,24)**

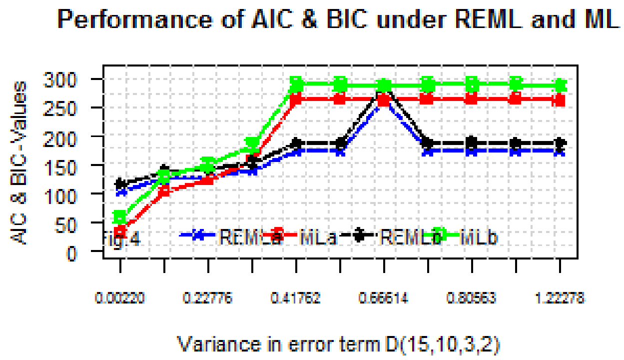
Random variation RML	Random variation ML	Bahavior of randomness
Df SSq MSq F Pr(>F) trt 29 2101.6 72.470 0.8653 0.642 Re 11 921.3 83.754 Efficiency 0.6170213 Mean = 42.09938 CV = 21.7384 AIC 338.0973 BIC 384.3368 -2 Res Log Likelihood -136.0486	Df SSq MSq F Pr(>F) trt 29 1036.91 35.755 1.4158 0.2774 Re 11 277.81 25.255 Efficiency 0.6170213 Mean = 42.09938 CV = 11.93717 AIC 501.7789 BIC 570.8923 -2 Res Log Likelihood -217.8894	Min -0.11023 Max 0.108798 Var 0.001935 SS 0.114188 MD 0.034467

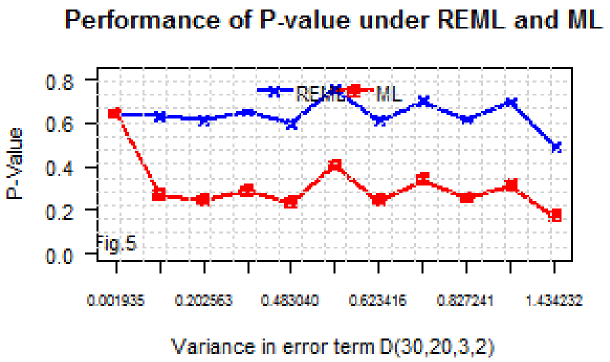
In this paper, three major designs and their performances were checked. The first design is considered to be an optimal design due to A-, D- and E- optimality criteria values greater than 0.5 with low SS value. The second design is considered to be a less optimal design due to one of the A-, D- and E- optimality criteria value less than 0.5 having others higher with medium SS value. The third design is considered to be a non optimal design due to two or all of the A-, D- and E- optimality criteria values less than 0.5 with high SS value.

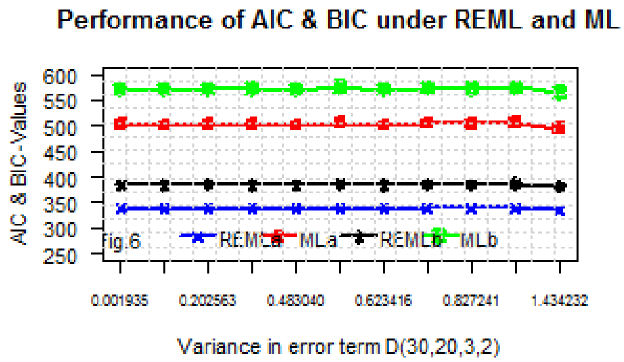












In the analysis, it is discovered that ***ML*** criterion detect true effect faster than ***REML***, indicated by the P-value on Table 1, Table 2 and Table3 even as the variance in error term increases. This can be graphically seen in Fig. 1, Fig.3 and Fig. 5. The error term variance has different effect on optimal, less optimal, and non optimal designs. In the presence of true effect for the optimal design, even as the variance in the error term increases, it is more likely to detect it, observed in Fig. 1.

In the presence of true effect for a less optimal design, if increase in the error term variance gets to a point, true effect would no longer be detected, noticed in Fig.3. it is unlikely to detect true effect under non optimal designs, Fig. 5 shows this. Akaike Information Criterion (***AIC***) and Bayesian Information Criterion (***BIC***) are useful for model selection. The performances of these two criteria were also observed for the three designs mentioned above. ***REML*** criterion produces lower ***AIC*** and ***BIC*** values for the three designs observed in Fig. 2, Fig. 4 and Fig. 6, but only consistent on average with optimal design even as the error term variance increases. As error term variance increases, the ***AIC*** and ***BIC*** increase on a reasonable average. ***ML*** criterion produces higher ***AIC*** and ***BIC*** values for the three designs, but only consistent on average with optimal design and less optimal design even as the error term variance increases. As error term variance increases the ***AIC*** and ***BIC*** increase on a reasonable average seen in Fig. 2 and Fig. 4. When the variance in error term increases to a point for less optimal design, the ***AIC*** and ***BIC*** reach their peak noticed in Fig. 4.

Both ***REML*** and ***ML*** criteria produce constant values for ***AIC*** and ***BIC*** even as the variance in error term increases but each having unique value.

CONCLUSION

In this paper, it is noticed that optimal design helps to control both type I and type II errors by minimizing ***MSE*** appropriately, determining an appropriate cost effective sample sizes with high precision, are the benefits of the experimenter. Through these, quality conclusions would be made on well planned experiments. Even at increase of error in an experiment, optimal design has the tendency to detect true effect.

This paper also testify to the effectiveness of using A-, D-, E- and SS- optimality criteria in selecting optimal designs.

In this work, we are able to solidify that there is high tendency to find significant difference when it is actually there for an optimal design than for non- optimal design according to A-, D- E- and SS- optimality criteria. If a researcher fails to find an optimal design, larger sample size would be required to detect true effect by increasing replications. Less optimal design may lack capability to minimize type I and type II errors thus causing confusion in conclusion, is another problem a researcher may face.

Huge error in experimental procedures affects the outcome of the experiment negatively Gauch and Zobel (1997), most especially the less optimal and non optimal designs. Experimenter should know that as the inflow of error increases, it is more likely to commit type II error.

REML and ***ML*** criteria were considered in the analyses above, ***ML*** often get significant difference faster than ***REML***. Often ***REML*** are considered better than ***ML*** using ***AIC*** and ***BIC***. Due to inconsistency showed in Fig.4, it would be better to use ***ML*** when fixed effects only are for consideration, since ***REML*** was developed for Mixed Models Karin Meyer (1987)

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