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Torsional oscillation model in circular shaft

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***Abstract*— In this paper is analyzed the phenomenon vibration of shaft rotation which in fact creates dynamic problems, has low work efficiency and creates deformation. It should be taken into consideration the equilibrium as well because might be caused large dynamic forces in the shaft and in the rotation element and must be equilibrated the rotating masses which are placed in the shaft and the machine itself should be equilibrated.**

***Index Terms*— Torsion, torque moment, shaft, rotating angle, button, calculated scheme**.

I. INTRODUCTION

Initial general realized model determining torsional oscillation and torque moments it is shown to be as very efficient, in such a simple fashion could be derived other characteristics special model which for practical application would be important. Models which consider amortization features response more to the real state. Torque moments are constant and are dependent from the time, also do not change the character of torsional oscillation and momentum as well. In the case where one of the moments is function of time and in fact it is a frequent case in practice, the torsional oscillation problem could be solved by applying mathematical method of variations. Changeable forces in transitional working period of the machine, due to the unequal rotation (speeding up and slowing down), being followed in working shafts in particular on the main shaft, and because of the elasticity and features of amortization, present dislocation of forces system that consequently announced additional charges in relation to the static ones.

II. EQUIVALENT AND REAL SCHEME OF SHAFT OSCILLATIONS

In working shafts of the machine, except the torsional oscillations, appear even the longitudinal and transverse oscillation. For torsional oscillation analysis can be started from a transmission to set a car into motion. Torsional oscillation model, in the general case for any shaft appear as below;

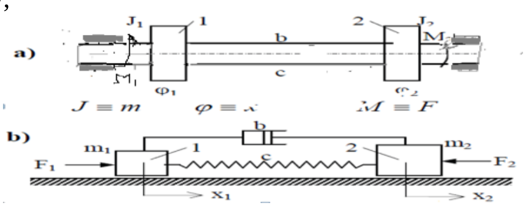

Fig 1. analogy between torsional and oscillation chain systems with two degree of freedom, a) the real scheme, b) equivalent scheme

Working shaft transmission is presented in Figure 1, where in the discs 1 and 2 operate external moments M1 and M2 Working transmitter 2. Moments M1 and M2 are with the opposite directions. In order to solve the problem easily can be initiated by the equivalent analog scheme of chain system with two masses m1 and m2, as in Figure 1b, in which masses operate external forces F1 and F2. Masses are connected with springs.

Torsional oscillation model with two masses by moments of inertia J1 and J2, operate in elastic shaft with stiffness c and amortization b. In the first case will be considered that masses 1 and 2 are much larger than the mass of the shaft, which is usually empty on the inside. Disc 1 represents the gear with the help of which, the movement is transmitted to the main shaft until the disc 2 may presents head austerity, in which the working piece is set .

It will also supposedly real case from the practice when the moments M1 and M2 during the working process are approximately constant. Such case often encountered in practice taking into account that we are dealing with rolling bearings, friction forces will be neglected. Here it should be noted that shaft sinking in the material represents the basic form of depreciation, as various materials, otherwise react to this phenomenon.

The momentum in the transmission element 1, in the case of uniform motion will be:

$$M_1 = M_d = M_h \times i_h \times \eta_h (1)$$

where;

M_h - Input momentum in transmission

M_d - Output momentum for examined kinematic chain (element no. 1)

i_h - transmission ratio

η_h - the degree of utilization.

The momentum in the transmission element 2, will be:

$$M_2 = \frac{F_1 \times D_r}{2} (2)$$

where;

F_1 - main resistance during cutting

D_r - working shaft diameter

In the case of work regime unchanged, the moment M2, will be approximately constant. Because here we are dealing with rotary motion, then the rotational movement of main shaft will be not continual at the time of issuance of work machine. Also can be noted even the case of stopping machine when the angular acceleration rather have slower angular.

III. EQUATIONS OF OSCILLATIONS SYSTEM

Torsional oscillation of mechanical system with two degrees of freedom, for the case of except conservative force and resistant forces operates any forced force, can efficiently be

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studied by applying the Lagrange equations of the second order, which apply in the case of small motions of the system

$Q_y = \frac{\partial}{\partial t}\frac{\partial E_k}{\partial \varphi_1} - \frac{\partial E_k}{\partial \varphi_1} + \frac{\partial E_p}{\partial \varphi} + \frac{\partial \Phi}{\partial \varphi}$ (3)

Kinetic energy of the system will be;

$E_k = \frac{(Jw_1^2 + Jw_2^2)}{2}$ (4)

Disk rotation angles which determine the position of system φ1 and φ2 and measured from the position of the center of aggravation. These represent the shaft torsion angles in places where drives are .

IV. MATHEMATIC MODEL OF OSCILLATION

Torsional oscillation model, in the general case for any working shaft of transmission is presented in Figure 1a, where in the discs 1 and 2 operate external moments M1 and M2. Moments M1 and M2 are with opposite directions.
In order to solve the problem easily can be initiated by the equivalent analog scheme chain system with two masses chain m1 and m2, as in Figure 1b, in which masses operate external forces F1 and F2. Measures are connected by spring and amortization (b), respectively.
Torsional oscillation model with two masses by moments of inertia J1 and J2, elastic shaft with stiffness c and amortization b .In the first case will be considered that measures 1 and 2 are much larger than the mass of the shaft, which is usually empty inside. It will also supposedly real case from the practice when the moments M1 and M2 during the working process are approximately constant. The case is very common in practice till the machines for processing of elements.
Here it should be noted that amortization in shaft material represents the basic form of amortization, as various materials, otherwise react to this phenomenon. On the basis of the relations shows that the momentum of the resisting forces is proportional to the first derivative of the angular velocity φ.
Virtual basic work of moments for rotational movement is;

$dA\varphi = M_{\varphi 1} d_{\varphi 1} - M_2 d_{\varphi 2}$(5)
where;

$d\varphi$ - rotation angel

Generalized forces are coefficients for virtual work

$Q_{\varphi 1} = M_{1\varphi 1}$ (6)
$Q_{\varphi 2} = -M_{2\varphi 2}$

By replacing the differential equations that describe the movement would be,
$\Phi_1 + cJ_{1(\varphi_1 - \varphi_2)} + bJ_{1(\varphi_1 - \varphi_2)} = M_{1\varphi 2} - cJ_{2(\varphi_1 - \varphi_2)} - bJ_{2(\varphi_1 - \varphi_2)} = -M_2 J_2$
(7)

If the second equation is derived by first equation, there is;

$\Phi_1 - \varphi_2 + (cJ_1 + cJ_2)(\varphi_1 - \varphi_2) + (bJ_1 + bJ_2)(\varphi_1 - \varphi_2) = M_1 J_1 + M_2 J_2$
(8)

By analyzing the system of equations shows that $c(\varphi_1 - \varphi_2) \times b(\varphi_1 - \varphi_2)$ present elastic moments of forces or resistant forces, respectively which resist external moments M1 and M2. Also, the magnitude Jφ represent moment of inertia of the system.
Elastic momentum is;

$M_e = c\psi = c(\varphi_1 - \varphi_2)$ (8)

Expression (8) present the momentum of elastic forces which appear in the shaft during rotation between discs (Figure 2):

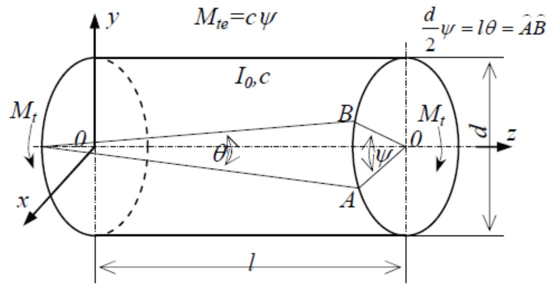


Fig 2. Torque elastic momentum and deformation during torquing of the main shaft.

The angle of shaft rotation can be expressed;

$\varphi_1 - \varphi_2 = \psi$ (9)

Deriving by time, angular velocity and angular acceleration will be:

$\varphi_1 - \varphi_2 = \psi$ (10)
$\varphi_1 - \varphi_2 = \psi$

Replacing the expressions above we have solutions based on characteristic equation of homogenous part,

$2 + 2\delta + \omega 2 = 0$ (11)

For real conditions δ<ω (amortization), solution of the equation will be;

$\lambda_{1/2} = -\delta \mp \delta 2 - \omega 2$ (12)

For the case of large amortizimeve δ≥ω, we have no solution of swinging character , so that this case does not match reality. According to the above equation, derived complex solution;

$\lambda_{1/2} = -\delta \mp i\omega 2 - \delta 2$ (13)

Homogeneous solution of the equation will be:

$\psi_h = e - \delta t(A \cos pt + B \sin pt)$(14)

We are seeking a particular solution in the form of constant;

$\psi p = D$ (15)

where, $\psi p = 0$

By substituted the expressions in differencial equation solution wii be,

$0 + 0 + \omega_2 D = M_1 J_1 + M_2 J_2$

the D constant has its own value;

$D = 1\omega_2 (M_1 J_1 + M_2 J_2)$ (17)

The particular solution of the equation will be;

$\psi p = 1\omega 2 (M_1 J_1 + M_2 J_2)$ (18)

The solution of the equation is the sum of the homogenous and particular solution;

$\psi = \psi h + \psi p$ (19)

respectively,

$\psi = e - \delta t(A \cos pt + B \sin pt) + 1\omega 2(M_1 J_1 + M_2 J_2)$
(20)

If we do take into consideration the initial conditions; $t = 0, \psi 0 = 0$, after differentiating the equation we do have;

$\psi = e - \delta t[\cos pt\ (B\ p - \delta A) - 0 \sin pt\ (\delta\ B + Ap)]$
(21)

according to the equation (21);

$\psi = 1[1(Bp - \delta A) - 0(\delta\ B + Ap)]$

And from the above expression we have do obtain the second constant;

$B = -\delta p \omega 2(M_1 J_1 + M_2 J_2)$ (22)

The final solution is;

$\psi = 1\omega 2(M_1 J_1 + M_2 J_2) - e - \delta t p \omega 1(M_1 J_1 + M_2 J_2)(p \cos pt + \delta \sin pt)$
(22)

where; $p = R \cos \alpha$
$\delta = R \cos \alpha$

So;

$R2 = p2 + \delta 2$
$R = p2 + \delta 2$ (23)

Then; $\tan \alpha = p\delta$
$\alpha = arctg\ p\delta$

The sum of the trigonometric functions equation;

$p \cos pt + \delta \sin pt = R \sin \alpha \cos pt + R \cos \alpha \sin pt = R \sin(pt + \alpha)$
(24)
The rotation angle is;

$\psi = cM_1 J_2 + M_2 J_1 + J_2(1 - p2 + \delta 2p\ \sin(pt + \alpha) e \delta t)$
(25)

The above equation presents the change of angle ψ in function of time. The circular frequency is;
$T_p = 2\pi p$

V. DYNAMIC COEFFICIENT

Using analysis of oscillation of dynamic moment, it is shown in Figure 3, the graph which shows the change of elastic torsional moment of shaft in time dependencies,

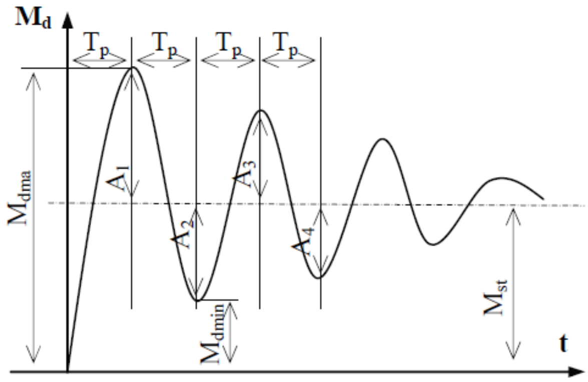


Fig 3. Replacement of elastic torsion moment depending on the time

The speed of growth momentum is determined by the circular frequency ***Md*** .If we take the relationship between dynamic maximum momentum and static during torquing, will be:

$k_d = \frac{M_d max}{M_{st}} = 1 - e^{\pi \partial / p}$ (28)

boundary values according to the coefficient; $k_{d\ max} = 1 + e^0 = 2$ this responde to the case of non amortization $(\delta = 0, b = 0)$. This case is valid when the report δp ⇒∞ or $\delta \gg p$.

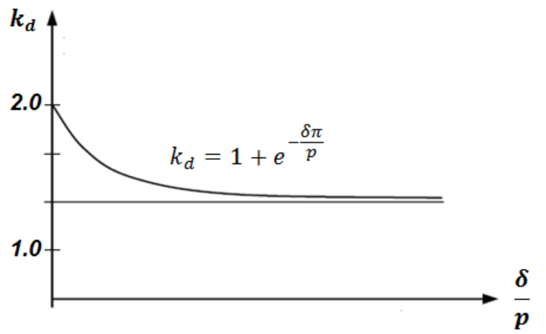


Fig 4. The dependence of the maximum value of the torsion elastic from the ratio δp

It is clear that in the case of the existence of amortization, ***kd*** coefficient ranges in the interval of 1 to 2. The diagram in Figure 3 shows that the coefficient ***kd*** falls with the increase of ***δp***.
By analyzing the expression ***kd*** decreases with increasing case of amortization ***b*** and reducing torsional stiffness ***c***, while other conditions are unchanged. Also for other unchanged

conditions , ***kd*** coefficient will be smaller for the smaller moments of inertia ***J1*** and ***J2***, while the moment of inertia ***J12*** falls with increasing moments of inertia ***J1*** and ***J2***.
If we suppose that;

For $\frac{\delta}{p} = 1.80$;

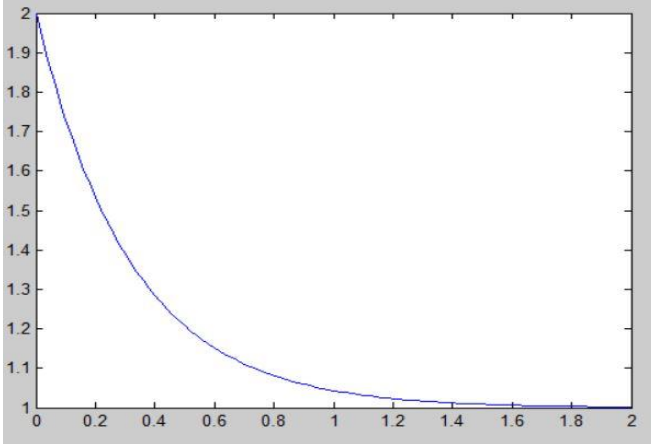
$$k_d = 1 + e^{-\frac{\delta\pi}{p}} = 1 + e^{-1.8\pi} = 1 + \frac{1}{e^{1.8\pi}} = 1 + \frac{1}{291.871} = 1.003$$

For $\frac{\delta}{p} = 2.00$;

$$k_d = 1 + e^{-\frac{\delta\pi}{p}} = 1 + e^{-2\pi} = 1 + \frac{1}{e^{2\pi}} = 1 + \frac{1}{584.405} = 1.001$$

Table 1. Dynamic coefficient

k_d	2	1.532	1.283	1.150	1.080	1.042	1.012
δ/p	0	0.2	0.4	0.6	0.8	1	1.2



VI. Conclusion

Examined elastic torsional moment of the main shaft, as disscused in the paper, changes over time, which means that there is dynamic character. For qualitative and quantitative analysis the most important is the maximum amplitude. The amplitude of the moment under the influence of the resistance is reduced by geometric progression. From the overall model results that dynamic effect of torque moment for other unchanged conditions, decrease along with increased amortization and reducing the moment of inertia of the rotating masses, which can be important for designing. Adopted coefficient kd can be considered as an indicator during the comparisson of the dynamic and static torque momentum. As shown, the theoretical value of the coefficient in the general case in interval ranges from 1 to 2, in which the high value applies if neglected suspension system. In reality it means that the coefficient values will be higher than theoretical.

Although in the paper was disscussed the elastic shaft of the machine, application dependencies realized it is possible to determine the relative proportions allocated to different variants of the design, considering that the main purpose does not mean to be profit the absolute values of certain characteristics. Approximation made only affects the kinetic energy of the system.

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