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Criterion to ensure uniqueness for minimum solution by algebraic method for inventory model

Chinshin Lau, Enju Chou, Julian Dementia

***Abstract*— This paper aims to solve inventory models without depending on calculus. This is an important issue such that many practitioners who are not familiar with calculus, then, can realize inventory models. There are more than one hundred papers that had worked on this research issue, however, most of them have overlooked the positivity of the item in a square root, and then some researchers can claim that they created a simplified solution procedure for algebraic approach. Their derivations are partially corrected under the restriction that the positivity of the sign of the item inside the square root is preserved. However, for other cases, when the positivity of the sign of the item inside the square root is violated, then their results will contain questionable derivations. Ignore the sign of the term inside a square root is a serious matter in algebraic methods such that other practitioners should provide revisions to constitute a well-defined algebraic approach. In this paper, we provided a detailed analysis to construct criterion to guarantee the existence and uniqueness of the optimal solution by algebraic method. Our findings will provide a prior work for researchers to further study inventory models under algebraic method environment.**

***Index Terms*— Algebraic method, Analytic approach, Economic production quantity**

I. INTRODUCTION

Grubbström and Erdem [5] is the first paper to derive optimal solution for inventory model by algebraic method without referring to calculus. Following this trend, Ronald et al. [8] revised Grubbström and Erdem [5] to present a well-constructed algebraic approach. Chang et al. [4] improved Ronald et al. [8] to simplify their complex solution procedure. Furthermore, Lan et al. [6] provided a detailed analysis of Grubbström and Erdem [5], Càrdenas-Barrón [1], Ronald et al. [8] and Chang [3] for algebraic methods for inventory models. Tuan and Himalaya [9] amended Xiao et al. [10] by algebraic method. Lin et al. [7] adjusted Càrdenas-Barrón [2] to show that the solution procedure of EPQ model can be easily obtained by that of the EOQ model. In this paper, we tried to solve the open question proposed by Chang et al. [4] to extend their open question with a more generalized setting. We focused on criterion to ensure the existence and uniqueness for the optimal solution.

Chinshin Lau is studying his Master Program in Traffic Science in Central Police University. He is an internet expert in crime prevention and investigation. His research interests are traffic model and intelligent wisdom.

Enju Chou is studying her Master Program in Traffic Science in Central Police University. She has worked as an expert for child rescuer and teenage protection police policy implementation.

Julian Dementia is presently working as a Professor in Department of Traffic Science, Central Police University. His research interests are analytic hierarchy process, inventory model, and traffic model.

II. DERIVATION OF CRITERION BY ALEGBRAIC METHOD

Chang et al. [4] proposed the following open question: To work out the minimum problem of

$$\sqrt{(1+\alpha)B^2+\beta}-B \qquad (1)$$

Where, $B>0$, $\alpha>0$, and $\beta>0$ without using calculus.

We will extend their problem to a more general setting for the following minimum problem:

$$f(x)=\sqrt{ax^2+bx+c}-x \qquad (2)$$

for $x>0$ to find the criterion among parameters a, b, and c.

We will find criterion to guarantee that there is a unique minimum solution for positive numbers such that $f(x)>0$ for $x\in(0,\infty)$.

Additionally, to ensure that $\sqrt{ax^2+bx+c}$ is meaningful, we know that

$$a\geq 0 \qquad (3)$$

and

$$b^2-4ac\leq 0. \qquad (4)$$

We derive that

$$f'(x)=\frac{2ax+b-2\sqrt{ax^2+bx+c}}{2\sqrt{ax^2+bx+c}} \qquad (5)$$

and

$$f''(x)=\frac{4ac-b^2}{4(ax^2+bx+c)^{1.5}}. \qquad (6)$$

From $\lim_{x\to 0^+} f(x)=\sqrt{c}$, we imply that

$$c\geq 0. \qquad (7)$$

If $c=0$, then the minimum point will be $x^*\to 0^+$. However, for our original problem, an inventory model, when the replenishment cycle during time goes to zero, then the backordered quantity will go to zero then there will be degenerated to a boundary. Hence, it is violated to the original purpose by Càrdenas-Barrón [1] to find an interior minimum solution. Consequently, we cannot accept that $x^*\to 0^+$. Therefore, we derive that

$$c>0. \qquad (8)$$

We rewrite $f(x)$ as

$$f(x)=\frac{(a-1)x^2+bx+c}{\sqrt{ax^2+bx+c}+x}. \quad (9)$$

If $a<1$, then $\lim_{x\to\infty} f(x)=-\infty$ that is unreasonable so we revise our restriction from $a\geq 0$ of Equation (3) to

$$a\geq 1. \quad (10)$$

For the numerator of $f'(x)$, we try to compare $g(x)$ with $h(x)$, where

$$g(x)=2ax+b \quad (11)$$

and

$$h(x)=2\sqrt{ax^2+bx+c}. \quad (12)$$

In the following, we divide into two cases: $b\geq 0$ and $b<0$.

When $b\geq 0$, we know that $2ax+b$ is always positive, then $g(x)\geq h(x)$ if and only if $g^2(x)\geq h^2(x)$.

We compute

$$g^2(x)-h^2(x)=4a(a-1)x^2+4b(a-1)x+b^2-4c. \quad (13)$$

When $b<0$, we know that $g(x)\leq 0$ for $x\in(0,-b/2a)$ and $g(x)>0$ for $x\in(-b/2a,\infty)$.

If $x\in(0,-b/2a)$, we have $f'(x)<0$ so $f(x)$ is a decreasing function for the interval $x\in(0,-b/2a)$.
For $x\in(-b/2a,\infty)$, we know that $g(x)>0$, then $g(x)\geq h(x)$ if and only if $g^2(x)\geq h^2(x)$.

We consider the case of $a=1$. Under the condition that $b^2-4ac\leq 0$ of Equation (4), we further divide our problem into two sub cases: $b^2-4ac<0$ and $b^2-4ac=0$.

If $a=1$, $b\geq 0$ and $b^2-4ac<0$, from Equation (13), we obtain that $g(x)<h(x)$, so $f'(x)<0$, then $f(x)$ is a decreasing function with minimum point as $x^*\to\infty$. It is unreasonable for the original inventory model proposed by Càrdenas-Barrón [1] to have an infinite during replenishment time cycle.

If $a=1, b<0$, and $b^2-4ac<0$, we can reduce our domain from $x\in(0,\infty)$ to $x\in(-b/2a,\infty)$ in order to find the minimum value. Hence, we still refer to Equation (13) to imply that $g(x)<h(x)$, and then the minimum occurs with $x^*\to\infty$ that is unreasonable for the original inventory model of Càrdenas-Barrón [1].

Next, we consider the problem with $a=1$ and $b^2-4ac=0$. We further divide our analysis into two cases: $b=2\sqrt{c}$ and $b=-2\sqrt{c}$.

When $a=1$, $b=2\sqrt{c}$, then $f(x)=\sqrt{c}$ is a constant function, so the minimum problem is solved.

When $a=1$, $b=-2\sqrt{c}$, then $f(x)=\sqrt{(x-\sqrt{c})^2}-x=|x-\sqrt{c}|-x$ such that $f(x)=\sqrt{c}-2x$ for $x\in(0,\sqrt{c})$ and $f(x)=-\sqrt{c}$ for $x\in(\sqrt{c},\infty)$ to imply the minimum value is $-\sqrt{c}$ to imply a negative value for our minimum problem. It is unreasonable for our original inventory model.

From above discussion, when $a=1$, our problem will imply two results: (a) unreasonable for our inventory model, or (b) a constant objective function, so we can claim that $a=1$ is not in the scope of our discussion. Therefore, we derive that the proper restriction should be expressed as

$$a>1. \quad (14)$$

For the restriction of $b^2-4ac\leq 0$, we consider the condition of $b^2=4ac$, then we divide into two cases: $b=2\sqrt{ac}$ and $b=-2\sqrt{ac}$

When $b=2\sqrt{ac}$, we rewrite

$$f'(x)=\frac{2(\sqrt{a}-1)(\sqrt{a}x+\sqrt{c})}{2(\sqrt{a}x+\sqrt{c})}=\sqrt{a}-1>0 \quad (15)$$

to imply that the minimum as $x^*\to 0^+$ that is unreasonable for our inventory model.

When $b=-2\sqrt{ac}$, we obtain that

$$f'(x)=\begin{cases}2(\sqrt{a}-1)(\sqrt{a}x-\sqrt{c}), & \text{for } x>\sqrt{c}/\sqrt{a}\\ 2(\sqrt{a}+1)(\sqrt{a}x-\sqrt{c}), & \text{for } x<\sqrt{c}/\sqrt{a}\end{cases} \quad (16)$$

to imply that $f'(x)<0$, for $x<\sqrt{c}/\sqrt{a}$ and $f'(x)>0$, for $x>\sqrt{c}/\sqrt{a}$ to derive that the minimum point is $x^*=\sqrt{c}/\sqrt{a}$. However, we find that equation (17) is unreasonable.

$$f(x^*)=-\sqrt{c}/\sqrt{a}<0 \quad (17)$$

From the above discussion, when $b^2=4ac$, we imply two unreasonable results such that in the following, we can simplify $b^2-4ac\leq 0$ of Equation (4) to assume that

$$b^2-4ac<0. \quad (18)$$

We summarize our results in the next theorem.

Theorem 1

Based on our derivations of Equations (8), (14), and (18), we obtain necessary conditions for the existence and uniqueness for the minimum solution for Equation (1) as

$$a>1,\ c>0 \text{ and } 4ac>b^2. \qquad (19)$$

III. EXAMINATION BY CALCULUS

From Equations (5) and (6), we know $f'(x)$ and $f''(x)$ to derive $f''(x)>0$ to yield that $f(x)$ is convex up such that $f'(x)=0$ will be the minimum point.

To solve $f'(x)=0$, after simplification, we find a quadratic polynomial and then

$$x=\frac{-4b(a-1)\pm\sqrt{16(a-1)(4ac-b^2)}}{8a(a-1)}. \qquad (20)$$

To derive a positive value for x, we find a necessary condition that

$$x=\frac{-b+\sqrt{(4ac-b^2)/(a-1)}}{2a}. \qquad (21)$$

Moreover, we further divide into two cases: $b\le 0$ and $b>0$.

When $b\le 0$, from Equation (21), we know that the expression of x is positive without adding other restriction.

When $b>0$, form Equation (21) again, to guarantee $x>0$ if and only if

$$(4ac-b^2)/(a-1)>b^2 \qquad (22)$$

to simplify that we find another restriction $4c>b^2$.

We summarize our findings in the next theorem 2.

Theorem 2

If $a>1$, $b\le 0$, $c>0$ and $4ac>b^2$, then the minimum solution x^* is expressed as Equation (21). If $a>1$, $b>0$, $c>0$ and $4c>b^2$, then the minimum solution x^* is expressed as Equation (21).

IV. DIRECTION FOR FUTURE RESEARCH

Our results of Theorem 2 was derived by calculus, the direction for future research is to obtain Theorem 2 by pure algebraic approach that will be a serious challenge for researchers.

V. CONCLUSION

We studied the open question proposed by Chang et al. (2005) under a general setting. We provided two theorems to demonstrate our derivations. Our findings will be useful for other researchers to solve inventory model by algebraic method.

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