

# Application of Fuzzy Optimal Path Algorithm for Bus Route Expansion in Thai Nguyen City

Thi Mai Thuong Duong, Thu May Duong, Phuong Huy Nguyen

**Abstract**—One frequently encountered problem in building and applying Geographical Information Systems (GIS) is to find the optimal path (shortest path). Bus route expansion is one demonstration example. The essence of this problem is the implementation of two options. The first is selecting potential locations for bus stop, and the other is choosing one optimal route which go through some potential locations with “lowest cost”. In the past, the problem of finding the optimal path in GIS will be implemented under hard computing. However, geographical data are inherently inaccurate and imprecise. Therefore, a fuzzy logic approach will make the solution to the problem of optimal path becomes more flexible. In this paper, the authors present four steps to expand bus routes in Thai Nguyen city of Vietnam in which using the fuzzy optimal path algorithm. It includes map database collection, map overlay (to identify potential locations), data fuzzification and applying the fuzzy optimal path algorithm.

**Index Terms**—Geographic Information System, Bus Route Expansion, Fuzzy Shortest Path, Fuzzy Optimal Path Algorithm

## I. INTRODUCTION

Development of public transport bus services is a compelling need of Thai Nguyen city to meet the travel needs of the people, to avoid traffic congestion, to reduce pollution and to ensure traffic safety. Therefore, the city should have suitable strategy and schedule in developing appropriate public transport bus service, attracting people to change from traveling by personal vehicles (especially motorcycles) to public transportation and completing appropriate bus transport network.

Currently, due to the need to travel by bus of the people is increasing, bus routes are overloaded. Some places have not yet deployed bus services. Therefore, it is necessary to expand some new bus routes.

The essence of bus route expansion problem is the implementation of two options. The first is selecting potential locations for bus stop between the starting point and the end point, and the other is choosing one optimal route which go through some potential locations with “lowest cost”. In the opinion of experts, the design should ensure the following requirements:

- (1) The new bus route selection should be based on the analysis of the current status of bus transport network to ensure better services.
- (2) Bus stops should be placed adjacent to public centers

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such as banks, offices, schools, hospitals, stations...

- (3) The bus stop should be placed in areas with high population density

- (4) Bus routes that have partial overlap must choose same bus stops

- (5) Distance between two adjacent bus stops on a bus route should be reasonable option (for example, for urban routes, the maximum distance is 1.5 km and a minimum of 500 m)

Currently, in the world, so many GISs have been completely built with the ability to store, manage, access, process, analysis and provide necessary information to implement decisions in many fields of public service [7]. Therefore, the use of GIS for the problem of expanding bus routes in the city of Thai Nguyen is a reasonable choice.

To perform this task, in addition to common data processing techniques in GIS, the application of algorithms to find the shortest path or optimal path is necessary [4]. We absolutely can apply these algorithms under hard computing. However, we will encounter some difficulties as follows:

- (1) GIS cannot accurately determine the absolute optimal location to place bus stops. On the other hand, during the construction of bus stops in real life, we may encounter some obstacles in people's houses, trees, land clearance and compensation ... so bus stops can be shifted in the range up to hundred meters. These bus stops shifting will lead to shifting of the other bus stops and affect the global results. There are even cases that we have to recalculate.

- (2) The handling of GIS data that accepts these incorrect factors is very difficult for traditional (hard computing) algorithms of optimal path.

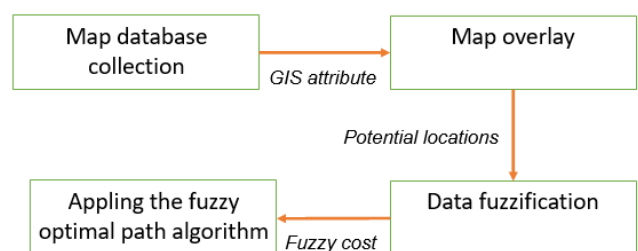


Fig. 1: Four steps for the problem of bus route expansion

Based on the above analysis, this article presents four steps to expand bus routes in Thai Nguyen city of Vietnam in which using fuzzy optimal path algorithm. It includes map database collection (to determine the necessary attributes of map data), map overlay (to identify potential locations), data fuzzification (to solve the problem according to the fuzzy logic approach) and applying the fuzzy optimal path algorithm (to find the optimal bus route). These four steps are illustrated on Fig. 1

## II. MAP DATABASE COLLECTION

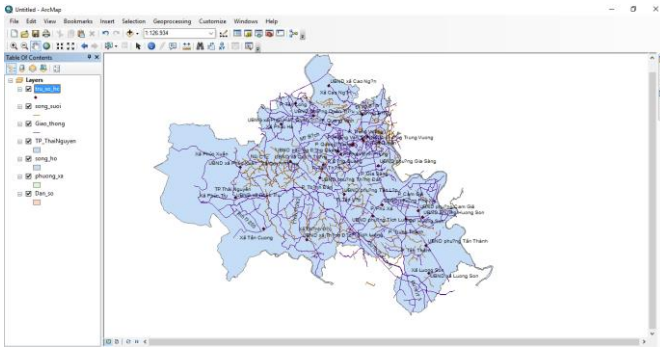


Fig. 2: Map of Thai Nguyen city with attributes in ArcMap

The purpose of this step is to collect the necessary attributes of map data that correspond to the first four criteria as described in the Introduction. So we need to collect map data containing the relevant attributes such as spatial, population, traffic, administrative offices, old bus stops ... This work has been done with the help of ArcMap module in ArcGIS software. The result of this step is to create the shape file of the data as *traffic.shp*, *administrative\_office.shp*, *population.shp* and *old\_bus\_stops.shp* ... (Fig. 2)

## III. MAP OVERLAY (SHAPEFILES)

Each shapefile that was obtained in the above step describes only one or a small number of attributes of the map according to spatial data. Thus, in order to detecting points so that contain full attributes required for placing bus stops (such as: near crowded residential areas, near public places or administrative offices, a priority bus stations available...), we need to overlay all shapefiles collected in the map database collection step. The results are shown in Fig. 3. Positions that have been marked in red x symbol are potential locations to place bus stops.

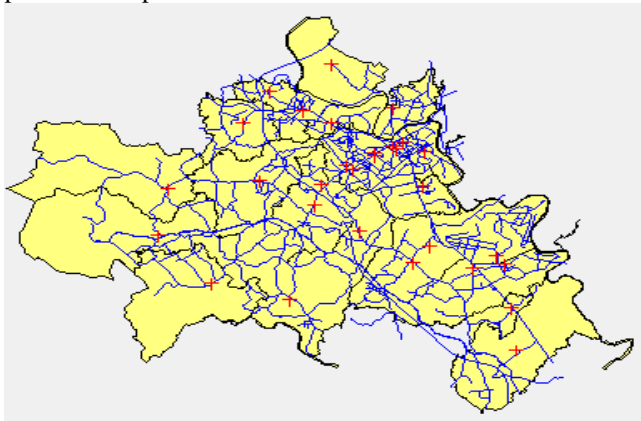


Fig. 3: Illustration of potential positions after data overlay

## IV. DATA FUZZIFICATION

To solve the problem of finding the optimal path according to the fuzzy logic approach, we must fuzzify crisp data (crisp locations). From the potential locations for the construction of bus stations which have received in map overlay step, we do the data fuzzification step. In fact, there are many complex criteria for fuzzification such as distance, population density, implementation costs. In this paper, we just illustrate the distance fuzzification.

As mentioned above, the placement of a bus stop on the

map can be shifted away due to many different causes. Therefore, the location of a bus stop will be fuzzified in two dimensions (x and y) by triangular fuzzy numbers. The tip of a triangle is a crisp potential location, two points on the left and right are based on allowed deviation of position in two dimensions (Fig. 4). After fuzzification, data are saved to the *point.mat* file.

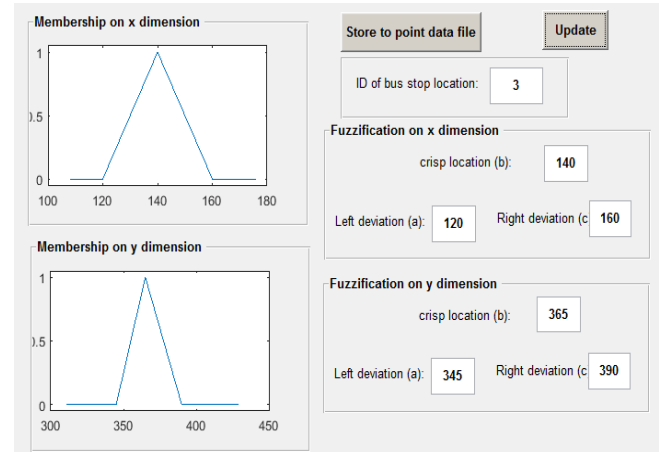


Fig. 4: Illustration of point data fuzzification

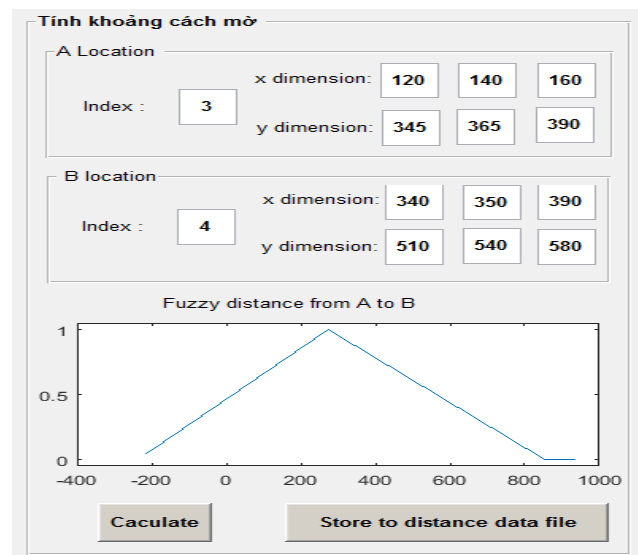


Fig. 5: Illustration of fuzzy distance calculation

The next step, we shall determine the cost between two points. In fact, this cost depends on many economic and social aspects. In terms of distance aspect, we can represent the fuzzy distance between two fuzzy points  $A\{(x_{a1}, x_{b1}, x_{c1}), (y_{a1}, y_{b1}, y_{c1})\}$ ,  $B\{(x_{a2}, x_{b2}, x_{c2}), (y_{a2}, y_{b2}, y_{c2})\}$  on a bus route in triangular fuzzy numbers  $AB(AB_a, AB_b, AB_c)$ . This step is shown in Fig. 5.

Where:  $AB_b = \sqrt{(x_{b1} - x_{b2})^2 + (y_{b1} - y_{b2})^2}$  (crisp distance between A and B)

$AB_a$  and  $AB_c$  are maximum deviation of position in both x,y dimensions.

If A and B are not on the same road, the distance AB is infinity. After calculating the entire distance between the points with the ability to set the bus stops, data are saved to the *distance.mat* file.

## V. APPLING THE FUZZY OPTIMAL ALGORITHM

### A. Figures and Tables

Up to now, in the world, a lot of fuzzy shortest path algorithms are proposed. These algorithms are based on fuzzy number of distance. For example, the FSA algorithm (proposed by Petrik [4]), the algorithm of finding the best path based on fuzzy numbers by Kiran Yadav [3], the fuzzy Dijkstra algorithm by Yong Deng [5].

In this paper, the key idea of the fuzzy optimal path algorithm is to represent the distance between two bus stop locations by a triangular fuzzy number and then applying the fuzzy optimal path algorithm.

A triangular fuzzy number  $\tilde{A}$  can be defined as  $A = (a, b, c)$  where the membership can be determined as follows and shown in Fig. 6

$$A(x) = \begin{cases} 0 & \text{if } x \leq b-a \\ \frac{x-(b-a)}{a} & \text{if } b-a < x < m \\ 1 & \text{if } x = b \\ \frac{(b+c)-x}{c} & \text{if } b < x < b+c \\ 0 & \text{if } x \geq b+c \end{cases} \quad (1)$$

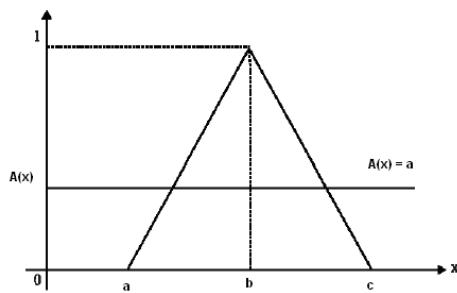


Fig. 6: A triangular fuzzy number  $A$

Given a triangular fuzzy number  $A$ , the graded mean integration representation of triangular fuzzy number  $P(A)$  is defined as:

$$P(A) = \frac{1}{6}(a_1 + 4b_1 + c_1) \quad (2)$$

The representation of the canonical representation of the addition operation  $\oplus$  on triangular fuzzy numbers  $A$  and  $B$  can be defined as:

$$P(S) = P(A \oplus B) = \frac{1}{6}(a_1 + 4b_1 + c_1) + \frac{1}{6}(a_2 + 4b_2 + c_2) \quad (3)$$

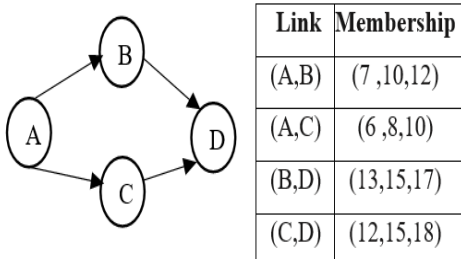


Fig. 7: An example on fuzzy distance calculating

For example, see Fig. 7. From node A, there are two routes to node B. One is  $A \rightarrow B \rightarrow D$  and the other is  $A \rightarrow C \rightarrow D$ . The canonical representation of the addition operation on fuzzy numbers in shortest path finding problem can be illustrated as follows. For the first route  $A \rightarrow B \rightarrow D$ , the length can be obtained as

$$\text{Cost}_1(A, D) = \text{Cost}(A, B) \oplus \text{Cost}(B, D) = (7, 10, 12) \oplus (13, 15, 17) \\ = 1/6(7 + 4 \times 10 + 12) + 1/6(13 + 4 \times 15 + 17) = 149/6$$

For the second route, the length can be obtained as

$$\text{Cost}_2(A, D) = \text{Cost}(A, C) \oplus \text{Cost}(C, D) = (6, 8, 10) \oplus (12, 15, 18) \\ = 1/6(6 + 4 \times 8 + 10) + 1/6(12 + 4 \times 15 + 18) = 138/6$$

The result shows that the first route is worse than the other since  $149/6 > 138/6$ . As can be seen from the example above, one merit of the canonical representation of the addition operation is that its result is a crisp number. The decision making can be easily obtained without the process of ranking fuzzy numbers, commonly used in many other fuzzy shortest path problems.

Thus, from two data file named *point.mat* and *distance.mat*, based on the requirement of the start point and the end point of the new bus route, we can apply the algorithm to find the fuzzy optimal path as follows:

**function** fuzzy\_optimal\_path (Graph, source):

**for each** vertex  $u$  in Graph;

dist[ $u$ ]:=infinity;

previous[ $u$ ]:=undefined;

**end for;**

dist[source]:=0;

Q:=the set of all nodes in Graph

**while** Q is not empty

$v$ :=vertex in Q with smallest dist[];

**if** dist[ $v$ ]=infinity;

**break;**

**end if**

remove  $v$  from Q

**for each** neighbor  $u$  of  $v$

cost:=dist[ $v$ ]+dist\_between( $v, u$ );

**if** cost<dist[ $u$ ];

dist[ $u$ ]:=cost;

previous[ $u$ ]:=v;

decrease-key  $v$  in Q

**end if;**

**end for;**

**end while;**

**Return** dist[];

**end** fuzzy\_optimal\_path

The results obtained after applying the algorithm is a new bus route with the best suited bus stop locations. Based on this optimal route, we carried out final adjustment stages in two steps:

- Dispose of the potential bus stops obtained from the step of map overlay but not on the optimal route
- Remove some bus stops on the route to meet the 5<sup>th</sup> criteria in requirements for bus stop design as mentioned in the introduction

Table. 1

Result after running the algorithm to find the optimal path

ID	Source	Destination	Distance (km)	No. bus stop (Before adjustment)	No. bus stop (After adjustment)
1	Tan Long subdistrict	NI town	39.8	40	25
2	Gang Thép town	Yen Lang district	33.6	34	23
3	Thai Market	Ky Phu	36.6	17	14
4	Dong Hy district	Song Cong town	39	33	27
5	Tan Long subdistrict	Phu Binh district	31.5	32	24
6	Thai Market	Dinh Hoa district	52.9	42	22
7	Quyet Thang subdistrict	Dinh Ca town	44.8	39	21
8	Song Cong town	Trai Cau subdistrict	42.8	31	17
9	Thinh Dan subdistrict	Quan Chu subdistrict	27.6	24	12
10	Thai Nguyen station	Coc lake	17	28	15

Table. 1 shows the results obtained when applying the fuzzy optimal algorithm to expand 10 bus routes in Thai Nguyen city of Vietnam. In which, results of the 10th row is illustrated in Figures 8 and Figures 9.

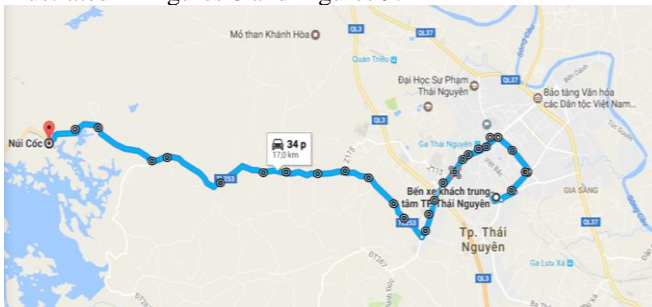


Fig. 8: The best new route with 28 potential bus stop

Fig. 8 shows the expansion bus route with the starting point is the center station of Thai Nguyen city and the end point is the Coc lake (a famous tourist destination of the city). This bus routes will provide very good conditions for tourists from other areas. After applying the fuzzy optimal algorithm, we obtain the optimal route with 17 km long passing through 28 potential bus stops. After adjustment, we get the new bus route with 15 bus stops.

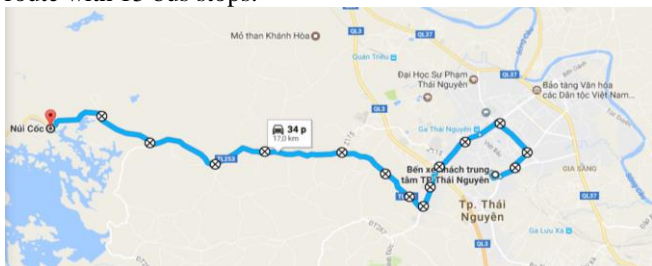


Fig. 9: Fifteen point of potential bus stops after adjustment

## VI. CONCLUSION

In this paper, the authors present main steps to apply the fuzzy optimal path algorithm to bus route expansion problem in Thai Nguyen city of Vietnam. At first, we collected GIS data with attributes that meet the requirements for a bus stop such as population, traffic, administrative offices, old bus stops. Then we overlaid obtained maps to identify potential

points that are suitable for bus stops. After identifying these crisp potential locations, we carried out determine fuzzy cost (distance) between two potential locations by a triangular fuzzy number. Finally, depending on the source and destination of the new bus route, we obtained the optimal route according fuzzy optimal path algorithm. Results show that the algorithm works well and can be applied in the real world.

## ACKNOWLEDGEMENT

This research was supported by a grant for the university research from the ICTU (University of Information and Communication Technology), under project # T2016-07-13. We thank our colleagues from the ICTU who provided insight and expertise that greatly assisted the research. We would also like to show our gratitude to the TNUT (Thai Nguyen University of Technology) which partially supported for the research.

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