An Optimal Replenishment Policy under Conditions of Permissible Delay in Payment and Shortages

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Abstract— **Most of the past research on economic replenishment quantity did not consider the trade credit period. In practice, however, companies in Taiwan have shown that taking the trade credit period into account when they are making inventory decisions decreases costs substantially. The aim of this paper was to develop an inventory model that incorporates a delay in payments for permissible shortages to minimize the total inventory-relevant cost. The optimal replenishment policy of the proposed model was identified by utilizing the global optimum conditions and non-constrained quadratic nonlinear programming. A numerical example was also generated.**

Index Terms— **Optimal Replenishment Policy, Delay in Payment, Permissible Shortages.**

I. INTRODUCTION

Inventory is one of the most important assessments when a firm attempts to improve the efficiency of the operations in its supply chain. Inventory control, which is linked to most aspects of production organization, can influence a company's business and has a critical role in the activities of operations management. Therefore, a company can take advantage of tracking stock by shortening the lead time for replenishment and estimating inventory cost as long as the company maintains a sturdy inventory management system.

Goyal(1985) argued that IBM's efforts in integrating their national spare part stock network, developing a new inventory system, and modifying their customer service quality have reduced their total inventory cost by more than $250,000,000 and continuously saves the company up to $20,000,000 in annual expenses. These findings demonstrate the importance of inventory management in a company's business. High-working capital on inventory could result in expenditures such as holding costs and insurance, and the costs of other personnel matters might dramatically rise. Most manufacturers believe that flawlessly managing their inventory can not only reduce inventory cost but also increase their freedom when they use the capital in their companies. In contrast, shortages would occur if companies fail to control their inventory. Shortages would also lower their service levels, harm their goodwill and damage their competitiveness.

Researchers like Wilson(1934) extended the original model and applied it to solve inventory problems in the real world. In addition, most Economic Production Quantity (EPQ) models incorporate the following assumptions:

- Production rate must be greater than demand
- There must be instantaneous replenishment

In fact, the order quantity submitted by retailers usually limits the production problems that suppliers need to deal with. Therefore, it is necessary to relax the EPQ assumptions to create an inventory model that will find a better solution for the issues that suppliers face. Furthermore, demand and supply do not have to be balanced because shortages occur when an insufficient quantity of product is produced by suppliers. When the order quantity cannot be satisfied, the retailer either waits for replenishment or switches the order to other suppliers who offer the same products.

Most of the traditional EPQ models assume that retailers pay for the goods immediately when the transaction is completed. In reality, few retailers pay for their goods immediately because most of them use trade credit. The use of trade credit violates the EPQ assumptions. Furthermore, to create a win-win situation, both the supplier and the retailer usually set up a reasonable credit transaction deadline by negotiating with each other. By setting a deadline, the supplier will accrue extra interest, and the retailer will not need to initiate loan financing. When delayed payment is provided by suppliers, the retailer also has the advantage of reducing interest yielded by the fund which uses in the payment of backlog.

Based on the reasons we mentioned above, we constructed a theoretical model that incorporates a delay in payments under conditions that allowed shortages. We then demonstrated some important properties associated with implementing an optimal replenishment policy. After obtaining the model properties, we generated an example. In the last section of the paper, we provided our conclusions as well as suggestions for further research.

II. LITERATURE REVIEW

A. *Credit Deadline for Payment Delays*

Goyal(1985) suggested an EOQ model with a fixed credit deadline for paying for goods as a promotion that could be provided by the supplier. If the retailer sells the product within the limit, he/she will be spared the interest generated by the funds that must be used to pay for the goods. That is a type of chance cost. If the pay date passes, he/she must pay for the products and the interest generated by the funds that have already been paid to the supplier for the rest of the products that are in stock. Many researchers have extended the above model in different ways. Chung(1998) investigated the interactions between credit transactions and the time value of currency, and he then employed a discounted cash flow (DCF) model to build a total variant cost function for inventory. In his research, an optimal replenishment policy was provided. Chung(2003) revised the model by Goyal(1985) and also found an optimal order quantity. Huang(2004, 2007) rewrote the assumptions made by Goyal(1985) and argued that the selling price and the purchase cost for the product might not be equal in real-world scenarios. Huang also considered the rate of replenishment to be finite and developed his research in two phases: one where the supplier allows the retailer to pay for the products before the credit deadline, and another phase where the retailer also provides his/her customers with permission to delay payments for the sake of increasing the amount of product that is sold. Huang(2007) also discussed a possibility in which the supplier offers partial permission for delays in payment when the actual order quantity is less than a certain amount. He minimized the total cost to ensure that the inventory cycle and the order quantity were optimal. Liao(2007) investigated a non-instantaneous inventory model of deterioration for items that incorporated delays in payment. Tsao and Sheen(2007) found an optimal selling price and maximized the profit for deterioration items while including permission for delays in payment. Chang, Teng and Goyal(2008) reviewed the literature for inventory models published in the last twenty years and classified these models as non-deterioration items, deterioration items, models that allow shortages, order quantity and inflation. They also suggested further research directions for every model they reviewed. Ouyang, Teng, Goyal and Yang(2009) extended the work by Goyal(1985) by making a rule that the promotion cannot be offered if the order quantity submitted by the retailer is below a certain threshold.

B. Permissible Shortages

Shortages occur when production cannot satisfy the quantity of orders. If customers allow for delays in delivery within a reasonable deadline, it is a permissible shortage. According to Cohen(1977), complete backlogging means that customers will not switch to other suppliers or cancel the order because of loyalty to the product during shortages. In a competitive market, if the supplier cannot deliver products to the retailer and solve shortages, the retailer will likely switch his/her order to other suppliers, which results in a loss to the suppliers. Wee(1995, 1999) considered conditions that included partial backlogging and set up a constant between 0 and 1 to be the backlogging rate. He also argued that waiting time was the most important factor that could affect whether the customers wait for replenishment or not. Philip(1974) argued that the deterioration rates of deterioration items should follow a three-parameter Weibull distribution. Papachristos and Skouri(2000, 2003) argued that the quantity that needs to be replenished is a decreasing exponential function of the time until product delivery. Jolai, Tavakkoli-Moghaddam, Rabbani and Sadoughian(2006) considered the factor of inflation and assumed that deterioration rates for deteriorated items should follow a two-parameter Weibull distribution to solve the EPQ model. Tsao and Sheen(2007) found an optimal selling price and a replenishment cycle with permissible delays in payment. They also sought to maximize the unit time profit. Dye(2007) determined a unitary selling price under an optimal replenishment policy. Dye(2007) also found that total profit was a concave function. Chung and Huang(2009) extended the work of Goyal(1985) by using dual variables and the year relevant cost to construct an inventory model. They also proved that the year relevant cost function was convex. A year later, Hu and Liu(2010) adopted the finite replenishment rate assumption and relaxed a condition in which purchase price and selling price must not be consistent as part of their effort to build upon the work of Chung and Huang(2009). Furthermore, they found an optimal inventory policy in the environment they had made.

III. The Model

A. Notation

D	Demand rate
P	Replenishment rate
H	Holding cost per unit of product in stock
I_{max}	Maximum inventory level
t_1	The time point when the quantity of products in stock reaches I_{max}
t_2	The time point when all of the products in stock are sold out
t_3	The time point when the quantity of backlog reaches N
T	Inventory cycle
$I_i(t)$	Total inventory in phase i and time point t, i=1,2,3,4; $t=t_1,t_2,t_3,T$
s	Sale price
a	Shortage cost per unit of product
N	Maximum shortage level
c	Purchase price
I_m	Inventory level at the time point of $t = M$
I_e	Interest income for the unit fund
I_k	Interest yielded by the unit fund, which is used to pay for the product in stock
M	Credit transaction deadline
$TVC(T)$	Total cost function

B. Assumptions

A3.1 Single product with non-deteriorating properties

A3.2 If both D and P are known and constant, it is reasonable to assume that $P > D$

A3.3 Purchase price and sale price of the product are both constant and $s > c$

A3.4 Instant replenishment

A3.5 $I_k \geq I_e$

A3.6 When $0 \leq T \leq M$, the retailer does not have to pay for the interest, I_e, for the products in stock. Otherwise, when $0 \leq M \leq T$, the retailer needs to pay the interest, I_k.

C. Model Formulation

Based on the above notation and assumptions, the inventory model was developed and is presented in Figure 1.

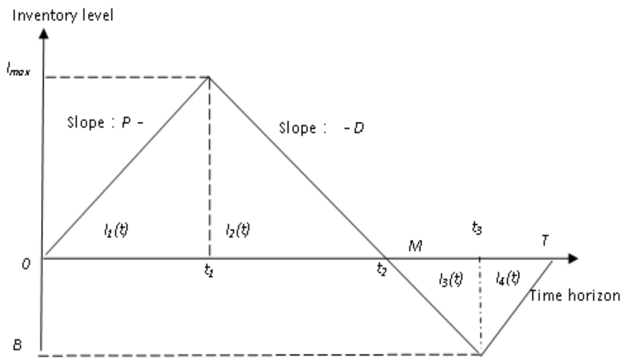

Figure 1 The inventory model

According to Figure 1, we partitioned the model into the following phases.

Phase 1: We start at $t=0$, which is the time point when the inventory level equals zero. As products are replenished at a rate P, the inventory level increases and reaches its maximum, I_{max}. Its increasing rate can be presented as follows:

$$\frac{dI_1(t)}{dt} = P - D,\ 0 \le t \le t_1 \quad (1)$$

Phase 2: Let $I_2(t)$ be the inventory level when $t \in [t_1, t_2]$. The retailer stops purchasing any product, but market demand results in a gradual decrease in inventory level. At $t = t_2$, the inventory level is reduced to zero. There is a variation in the inventory level during this period.

$$\frac{dI_2(t)}{dt} = -D,\ t_1 \le t \le t_2 \quad (2)$$

Then, the state of the inventory system enters Phase 3.

Phase 3: Shortages happens in this period. The quantity of backlog reaches it maximum level, N, at $t = t_3$. Then, Phase 4 begins. The rate of change of the inventory level during this period is:

$$\frac{dI_3(t)}{dt} = -D,\ t_2 \le t \le t_3 \quad (3)$$

Phase 4: The retailer decides to replenish products to meet market demand. Because of the assumption $P>D$, the inventory level reaches zero when all backlogs are satisfied.

$$.\ \frac{dI_4(t)}{dt} = P - D,\ t_3 \le t \le T \quad (4)$$

In the following content section, we investigate the optimal replenishment policy under the conditions $0 \le M \le t_2$ and $0 \le t_2 \le M$, respectively.

The objective of the model is to minimize the total cost. Thus, we first defined the total cost as equal to the sum of the holding cost, shortage cost, the interest yielded by the funds used to pay for the product in stock and the negative interest income. By using (1) to (4) as well as some triangular properties, we found the following formulas for computing other cost functions and determining the properties of an optimal policy.

$$t_1 = \frac{Dt_2}{P} \quad (5)$$

$$t_3 = \left(1 - \frac{D}{P}\right)T + \frac{Dt_2}{P} \quad (6)$$

$$I_{\max} = \left(1 - \frac{D}{P}\right)Dt_2 \quad (7)$$

$$N = D\left(1 - \frac{D}{P}\right)(T - t_2) \quad (8)$$

$$I_m = (t_2 - M)D \quad (9)$$

According to Figure 1 and (5) to (9), the following cost functions can be derived.

The holding cost during the period of $0 \le t \le t_2$ is:

$$h\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_2(t)dt\right] = \frac{hDt_2^2}{2}\left(1 - \frac{D}{P}\right) \quad (10)$$

The shortage cost, which occurs during the period of $t_2 \le t \le T$, can be formulated as follows:

$$a\left[\int_{t_2}^{t_3} I_3(t)dt + \int_{t_3}^{T} I_4(t)dt\right] = \frac{aD}{2}\left(1 - \frac{D}{P}\right)(T - t_2)^2 \quad (11)$$

Case 1: $0 \le M \le t_2$

The interest yielded by the unit funds used to pay for the product in stock, I_k, during the period $M \le t \le t_2$ is shown in Figure 2.

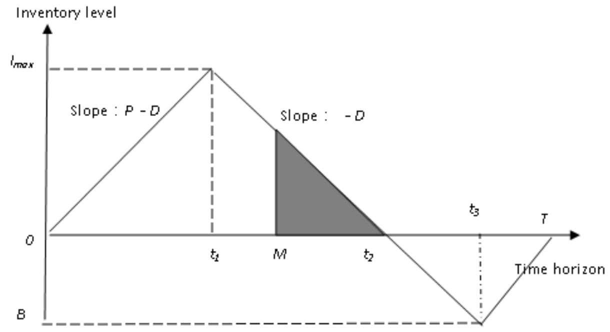

Figure 2 The interest yielded by unit funds used to pay for the product in stock during the period $0 \le M \le t_2$.

Based on Figure 2, we constructed the interest yielded by the unit funds used to pay for the product in stock as follows:

$$cI_k \int_M^{t_2} I_2(t)dt = \frac{cDI_k}{2}(t_2 - M)^2 \quad (12)$$

When $0 \le M \le t_2$, the interest income is constructed as written in formula (13).

$$cI_e\int_0^{t_1} Pdt + sI_e\int_0^{t_2} Ddt + I_e\int_{t_3}^{T}(sD - cP)dt = I_e((c+s)Dt_2 + \frac{D^2}{2P}(sD - cP)(1 - \frac{D}{P})(T - t_2)^2) \quad (13)$$

According to the aforementioned definition of total cost, we defined the total cost function as follows:

$$TVC_1(t_2, T) = \frac{hDt_2^2}{2}(1 - \frac{D}{P}) + \frac{aD}{2}(1 - \frac{D}{P})(T - t_2)^2 + \frac{cDI_k}{2}(t_2 - M)^2 - I_e((c+s)Dt_2 + \frac{D^2}{2P}(sD - cP)(1 - \frac{D}{P})(T - t_2)^2 \quad (14)$$

Case 2: $0 \le t_2 \le M$

Under this condition, retailers sell all of the products in stock before they have to pay for the products they purchased. Therefore, they do not have to pay interest. In other words, retailers are not affected by interest income during the periods $(0, t_2)$ and (t_2, M), as shown in Figure 3.

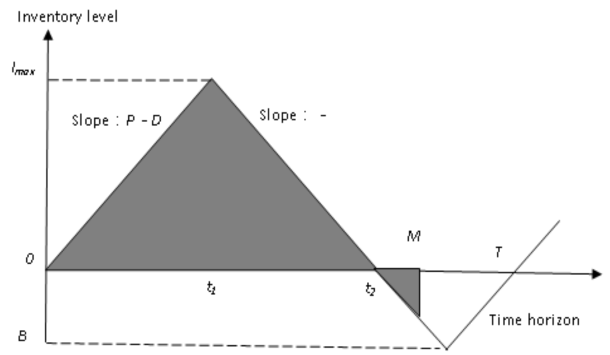

Figure 3: The chance cost of interest income during $0 \le t_2 \le M$

For this condition, the total cost can be constructed as follows:

$$TVC_2(t_2, T) = \frac{hDt_2^2}{2}(1 - \frac{D}{P}) +$$

$$\frac{aD}{2}(1-\frac{D}{P})(T-t_2)^2 -$$

$$I_e((c+s)Dt_2 + \frac{D^2}{2P}(sD-c)(1-\frac{D}{P})(T-t_2)^2) \quad (15)$$

D. Properties

At the point when $T = t_2 = M$, if $TVC_1(M) = TVC_2(M)$, the cost function, $TVC(T)$, is continuous.

Based on the above assumption, we employed the optimal conditions for finding the replenishment time to minimize the total cost.

$$\frac{\partial TVC_1(t_2,T)}{\partial t_2} = (hDt_2 - aD(T-t_2))(1-\frac{D}{P}) + cDI_k(t_2 - M)$$
$$-I_e(D(c+s) - \frac{1}{P}(D^2(sD-cP)(1-\frac{D}{P})(T-t_2))) \quad (16)$$

$$t_2^* = (P^2(aT + cI_kM + I_e(c+s+cDT)) - P(aDT + I_e(D^2T(c+s)) + sI_eD^3T)/$$
$$(P^2(a+h+cI_k+cDI_e) - P((h+a)D + D^2I_e(c+s)) + sD^3I_e) \quad (17)$$

Let

$$A = P^2(a+h+cI_k+cDI_e) - P((h+a)D + D^2I_e(c+s)) + sD^3I_e$$

By substituting (17) with (14), taking a derivative with respect to the inventory cycle, T, and setting the resulting formula equal to zero, we can solve the equation and find the optimal inventory cycle, T_1^*.

$$\frac{dTVC_1(T)}{dT} = \frac{1}{PA}(P^2(D^2I_e(hT(s+2c) + csI_k(T-M) - c^2I_kM - I_e(c+s)^2)$$
$$+ D(acI_k(T-M) - aI_e(c+s) + 2ahT))$$
$$+ P(D^3I_e(sI_e(c+s) - hT(c+2s) - csI_k(T-M)) - ahD^2T)$$
$$+ shI_eD^4T) \quad (18)$$

$$T_1^* = \frac{(cI_kM + I_e(c+s))P}{h(P-D) + cI_kP} \quad (19)$$

To assure that T_1^* is the global minimum, we took the derivative of (18) with respect to T again.

$$\frac{d^2TVC_1(T)}{d^2T} = \frac{1}{PA}(P^3(a(h+cI_k) + cDI_e(h+cI_k))$$
$$-P^2(D^2I_e(h(2c+s) + cI_k(c+s)) + aD(2h+cI_k))$$
$$+P(D^3I_e(h(c+2s) + csI_k) + ahD^2) - shI_eD^4) \quad (20)$$

Lemma 1: The sufficient conditions under which $TVC_1(t_2,T)$ is a convex function are

$$P > \frac{sD}{c} \text{ and } I_k > \frac{(s-c)DI_e}{s}$$

Proof: To show that $TVC_1(t_2,T)$ is convex, we needed to prove that (20) is greater than zero.

Because $P > D$, the condition of $A > 0$ is $cP^2I_k - sD^2I_e(P-D) > 0$

Let $P = \frac{sD}{c}$ and substitute this into the formula above. Then, we obtain $I_k > \frac{(s-c)DI_e}{s}$ by utilizing (20). We then employed $I_k > \frac{(s-c)DI_e}{s}$ again. It is clear that $A > 0$ and (20) > 0. □

By repeating the procedure for finding T_1^*, we attempted to find the derivatives of $TVC_2(t_2,T)$ and T_2^*.

By taking the first-order derivative of $TVC_2(t_2,T)$ with respect to T, we obtain

$$\frac{\partial TVC_2(t_2,T)}{\partial t_2} = hDt_2(1-\frac{D}{P}) - aD(1-\frac{D}{P})(T-t_2)$$
$$-I_e(D(c+s) - \frac{1}{P}(D^2(sD-cP)(1-\frac{D}{P})(T-t_2)) \quad (21)$$

When (21) equals zero, we obtain the optimal replenishment time after some rearranging.

$$t_2^{**} = (P^2((a+cDI_e)T + I_e(c+s)) - PT(aD + I_eD^2(c+s) + sI_eD^3T)/$$
$$((P-D)(P(a+h) - I_eD(sD-cP))) \quad (22)$$

Denote $B = (P-D)(P(a+h) - I_eD(sD-cP))$

By taking the derivative of $TVC_2(t_2,T)$ with respect to T and letting the resulting function be equal to zero, we can solve the equation to obtain the optimal inventory cycle.

$$\frac{dTVC_2(T)}{dT} = \frac{D}{PB}(-P^2(cDI_e^2(s+c) + I_e(a(c+s) - chDT) - ahT)$$
$$+P(sD^2I_e^2(c+s) - hD^2TI_e(c+s) - ahDT) + shD^3TI_e) \quad (23)$$

$$T_2^* = \frac{I_eP(c+s)}{h(P-D)} \quad (24)$$

Now, to examine the convexity of $TVC_2(t_2,T)$, we took the second-order derivative of $TVC_2(t_2,T)$ with respect to T.

$$\frac{d^2TVC_2(T)}{d^2T} = \frac{hD}{PB}(-P^2(a+cDI_e) + hDP(a+DI_e(c+s)) - sD^3I_e) \quad (25)$$

Lemma 2: The conditions sufficient to ensure that $TVC_2(t_2,T)$ is convex are $P > \frac{sD}{c}$ and $I_e > \frac{a(s-ch)}{cD(s+c)}$.

Proof: Let $P > \frac{sD}{c}$ and $B > 0$.

To show that $TVC_2(t_2,T)$ is a convex function, we need to prove that (25) > 0, which requires a condition to be found such that

$-P^2(a+cDI_e) + hDP(a+DI_e(c+s)) - sD^3I_e > 0$ Let $P = \frac{sD}{c}$ and substitute it into the formula above.

After rearranging, we have $I_e > \frac{a(s-ch)}{cD(s+c)}$

Thus, if $I_e > \frac{a(s-ch)}{cD(s+c)}$, the second-order derivative of $TVC_2(t_2,T)$ >0 □

Denote T_1^* and T_2^* as the extreme points of

$TVC_1(t_2^*, T_1^*)$ and $TVC_2(t_2^{**}, T_2^*)$, respectively. If Lemmas 1 and 2 hold, we have the following solutions:

$$TVC_1'(T) \begin{cases} < 0, & T \in (0, T_1^*) \\ = 0, & T = T_1^* \\ > 0, & T \in (T_1^*, \infty) \end{cases} \quad (26)$$

and

$$TVC_2'(T) \begin{cases} < 0, & T \in (0, T_2^*) \\ = 0, & T = T_2^* \\ > 0, & T \in (T_2^*, \infty) \end{cases} \quad (27)$$

IV. Numerical Example

To obtain sufficient usage of their production capacity, suppliers will offer retailers a promotion for delaying payments until a credit transaction deadline, *M*. Therefore, the retailers can apply the results found in this paper to build their inventory model. Furthermore, an optimal replenishment policy that dramatically reduces total cost can also be obtained. Data were collected from some companies located in Taiwan and processed with the model. To mimic reality, the parameters used in the proposed model were assigned as follows:

$P=50$	$D=30$	$h=0.1$	$c=1$	$M=1.2$
$a=0.3$	$s=8$	$I_e=0.01$	$I_k=0.12$	

After performing the calculations, we obtained $T_1^*=1.4625$ and $T_2^*=2.25$. When $M>t_2$, we found that the longer the optimal inventory cycle was, the greater the profit was that retailers could obtain.

As a result, we plotted TVC_1 and TVC_2 versus *T* in Figures 4 and 5, respectively. These two figures provide evidence that the aforementioned lemmas are valid. In addition, the delay in payments offered by certain suppliers would result in a negative inventory cost, which also leads to more products being purchased from the suppliers.

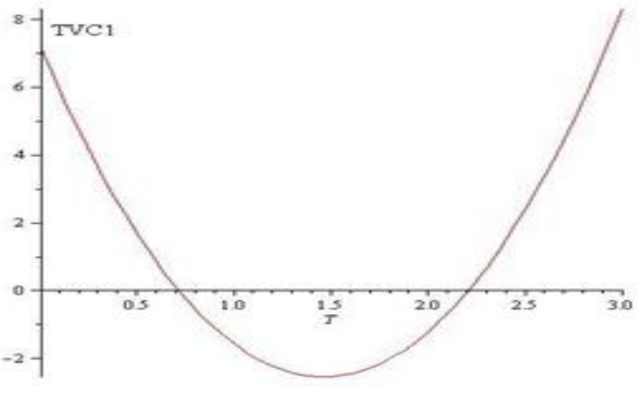

Figure 4: The value of TVC_1 versus *T* during $0 \le M \le t_2$.

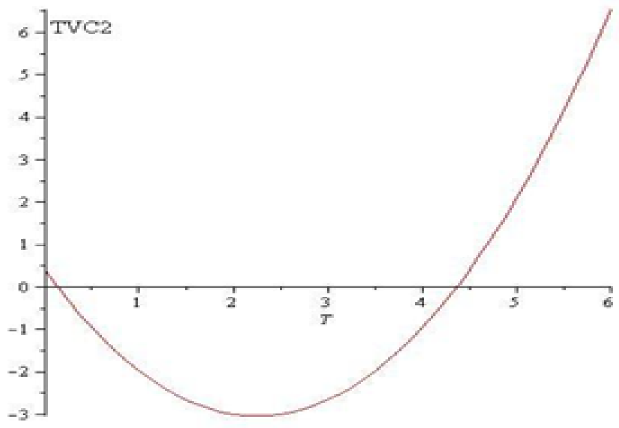

Figure 5: The value of TVC_2 versus *T* during $0 \le t_2 \le M$.

V. Conclusion

The aim of this paper was to develop an inventory model that incorporates delays in payments for permissible shortages. The proposed model for an optimal replenishment policy was identified by utilizing the global optimal conditions for the Maple-based numerical analysis. We found that both $TVC_1(T)$ and $TVC_2(T)$ were convex functions. This result assures the existence of the polar point that is unique under certain conditions. Moreover, as shown in Figures 4 and 5, adopting the promotion offered by the suppliers would benefit the retailers' businesses because the retailers would be more willing to purchase products from the suppliers. Therefore, a win-win scenario was achieved.

Accordingly, in this research, we assumed that the product was not deteriorating and that no defective items were produced during the replenishment process. For future analyses, we will relax the assumptions by considering deteriorating items and assuming that the demand rate and replenishment rate follow certain probability distributions (e.g., a Weibull distribution or GAMMA distribution) to provide solutions for replenishment issues in the real world. In addition, taking the factors of inflection and time preference into account would also be worth investigating

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