Hybrid Differential Evolution For Combined Heat And Power Economic Dispatch Problem

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Abstract—**This paper presents a hybrid differential evolution with multiplier updating (HDE-MU) to solve the complex combined heat and power economic dispatch (CHPED) problems. Transmission losses and valve-point effects of conventional thermal generators are taken into account. The hybrid differential evolution (HDE) has the ability to efficiently search and actively explore solutions. Multiplier updating (MU) is introduced to avoid deforming the augmented Lagrange function (ALF), which is adopted to manage system constraints of the CHPED problem. The proposed HDE-MU integrates the HDE with the MU. A practical CHPED system is employed to demonstrate that the proposed algorithm has the benefits of straightforwardness; ease of implementation; better effectiveness than the previous methods, and the requirement for only a small population when applied to the CHPED operation.**

Index Terms—**Combined heat and power, differential evolution, economic dispatch, multiplier updating.**

I. INTRODUCTION

The conversion of fossil fuel into electricity is not an efficient process. Even the energy efficiency of the most modern combined cycle plants are between 50 and 60% efficient [1]. Most of the energy wasted in the conversion process is heat. Recently, combined heat and power (CHP) units have played an increasingly important role in the utility industry [2~ 6]. Complexity arises if one or more units produce both power and heat. Such a case, both the power demand and the heat demand must be satisfied. Cogeneration units provide both electrical power and heat to customers. The heat production capacity of most cogeneration units, depend on the power generated, and vice versa. The mutual dependency of multiple-demand and Heat-Power capacity of those units introduce complexities in the integration of cogeneration units into the economic dispatch problem. The CHPED problem even may be more complicated if transmission losses and valve-point effects are taken into account. Solving such a complex optimization problem requires powerful techniques. Non-linear optimal algorithms, such as dual and quadratic programming [7], and gradient descent approaches, such as Lagrangian relaxation [8], have been applied for solving CHPED problems. However, these methods cannot handle non-smooth non-convex fuel cost function of the conventional thermal generator. The advent of stochastic search algorithms has overcome this problem for solving CHPED problems. Gravitational search algorithm (GSA) [2], cuckoo search algorithm [3], group search optimization (GSO) [4], exchange market algorithm [5], real coded genetic algorithm [6], opposition-based group search

optimization (OGSO) [9], Improved ant colony search algorithm [10], evolutionary programming [11], genetic algorithm [12], harmonic search algorithm [13], hybrid particle swarm optimization [14], self-adaptive real-coded genetic algorithm [15], novel selective particle swarm optimization [16], mesh adaptive direct search algorithm [17], oppositional teaching learning based optimization (OTLBO) [18], particle swarm optimization with time varying acceleration coefficients (TVAC-PSO) [19] and krill herd algorithm (KHA) [20] have been applied to solve the CHPED problems. The results obtained by the proposed algorithm, involving HDE and MU, have been compared with the existing methodologies reported in recent literature.

Storn and Price [21] developed the Differential evolution (DE) which immediately gained popularity as a robust evolutionary algorithm. DE has been widely applied to the optimization problems in the power systems [22~ 26]. Throughout the years, DE has been used extensively for optimization problems, many results of which are the best compared to other standard methodologies. However, it has problems of converging onto local optimal solutions. A migration [27] is embedded in the proposed HDE to overcome such drawbacks for solving CHPED problems. The migration operation is included in the HDE to regenerate a newly diverse population, which prevents individuals from gradually clustering and thus greatly increases the amount of search space explored for a small population.

The rest of this paper is organized as follows: in the next section, the basic equations governing CHPED have been briefly outlined. Section III explains the basic framework of the proposed algorithm involving HDE and MU. Section IV describes the test system, compares the results obtained by HDE-MU with other methods and provides the analysis of the obtained results. Finally, the conclusion is drawn in Section V.

II. PROBLEM FORMULATION

A. Objective function

The CHPED problem is to determine the unit power and heat production so that the system's production cost is minimized while the power and heat demands and other constraints are met. It can be mathematically stated as,

$$\text{Minimize} \quad \sum_{i=1}^{n_p} Cost_i(P_i) + \sum_{j=1}^{n_b} Cost_j(H_j, P_j) + \sum_{k=1}^{n_h} Cost_k(H_k) \quad (1)$$

Where

$$Cost_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \left| e_i \times \sin\left(f_i \times \left(P_i^{\min} - P_i\right)\right) \right| \quad (2)$$

$$Cost_j(H_j, P_j) = a_j + b_j P_j + c_j P_j^2 + d_j H_j^2 + r_j H_j + w_j P_j H_j \quad (3)$$

$$Cost_k(H_k) = a_k + b_k H_k + c_k H_k^2 \quad (4)$$

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and where P stands unit power generation; H is unit heat production; $Cost_i(P_i)$ *with i=1,2,..., n_p represents* the cost function of the ith power-only unit in \$/h and a_i, b_i, and c_i are cost coefficients of generator i, and e_i and f_i are fuel cost coefficients of unit i with valve-points effects; $Cost_j(H_j,P_j)$ with *j=1,2,..., n_b is* the fuel cost function of the jth CHP unit in \$/h and a_j, b_j, c_j, d_j, r_j, and w_j are cost coefficients of unit j; $Cost_k(H_k)$ with *k=1,2,..., n_h* represents the cost function of the kth heat-only unit in \$/h and a_k, b_k, and c_k are cost coefficients; n_p, n_b, and n_h denote the number of power-only units, co-generation units, and heat-only units, respectively; subscripts of i, j, and k are used for above mentioned unit; it should be mentioned that (P_i, P_j) and (H_j, H_k) are the output active power (in MW) and heat production (in MWth), respectively.

B. Constraints

Subject to the equilibrium constraints of electricity and heat production, and the capacity limits of each unit. Power and heat balance equations are represented by equality constraints (5) and (6), respectively.

$$\sum_{i=1}^{n_p} P_i + \sum_{j=1}^{n_b} P_j = P_d + PLm \qquad (5)$$

$$\sum_{j=1}^{n_b} H_j + \sum_{k=1}^{n_k} H_k = H_d \qquad (6)$$

$$P_i^{\min} \le P_i \le P_i^{\max}, \qquad i = 1, \cdots, n_p \qquad (7)$$

$$P_j^{min}(H_j) \le P_j \le P_j^{max}(H_j), \qquad j = 1, \cdots, n_b \qquad (8)$$

$$H_j^{min}(P_j) \le H_j \le H_j^{max}(P_j), \qquad j = 1, \cdots, n_b \qquad (9)$$

$$H_k^{min} \le H_k \le H_k^{max}, \qquad k = 1, \cdots, n_h \qquad (10)$$

Where P_d and H_d are system power and heat demands, respectively. In general, the transmission losses (PLm) of (5) can be calculated through the power generation of all units which is known as B-matrix approach (kron's loss formula [1]). P^{min} and P^{max} are unit power capacity limits; h^{min} and h^{max} are unit heat capacity limits.

The CHPED problem clearly introduces the complication of more constraints than in required pure power economic dispatch problem. The insufficiencies difficulties with conventional methods thus follow from the fact that CHPED is a highly constrained optimization problem.

III. THE PROPOSED HDE-MU

A. Differential evolution (DE)

The DE algorithm is one of the population-based optimization algorithms. The steps for implementing DE are as follows [21]:

Step 1: Initial population: A population of N_P initial solutions randomly distributed in the n_c dimensional search space of the optimization problem, are initiated. The DE uses

N_p vectors of variables x in the optimization problem, namely, $x^G = \{x_i^G, i = 1,...,N_p\}$, as a population in generation G. For convenience, the decision vector, x_i, is represented as $(x_{1i}...x_{ji}...x_{n_ci})$. Here, the decision variable, x_{ji} is directly coded as a real value within its bounds of ($x_j^{\min}, x_j^{\max}$). Each individual is generated as follows:

$$x_{ji}^G\big|_{G=0} = x_j^{\min} + rand(0,1)*(x_j^{\max} - x_j^{\min}) \qquad (11)$$
$$j = 1,2,...,n_c, \quad i = 1,2,...,N_P$$

Where $rand(0, 1)$ is a random number between 0 and 1.

Step 2: Mutation operator: In mutation step, for each individual x_i (target vector) of the new population, three different individuals x_{r1}, x_{r2}, and x_{r3} ($r1 \neq r2 \neq r3 \neq i$) are pseudo-randomly extracted from the population to generate a new vector as:

$$z_{ji} = x_{jr1} + F*(x_{jr2} - x_{jr3}) \qquad j = 1,2,...,n_c \qquad (12)$$

Where $F \in [0,2]$ is a uniformly distributed random number which controls the length of the population exploration vector (x_{r2} - x_{r3}).

Step 3: Crossover operator: After mutation step, the crossover operator, according to the following equation, is applied on the mutation vector Z_{ji} and the vector x_{ji} to generate the trial vector U_{ji}, for increasing population diversity of the mutation vector.

$$U_{ji} = \begin{cases} z_{ji}, & if \ rand(0,1) \le CR \\ x_{ji}, & otherwise \end{cases} \qquad (13)$$
$$j = 1,2,...,n_c, \quad i = 1,2,...,N_P$$

Where $CR \in [0,1]$ is known as the crossover rate which is a constant.

Step 4: Selection & evaluation operator: The selection & evaluation process is repeated for each pair of target/trial vectors using the evaluation function $F(U_{ji})$ to compare with the evaluation function value $F(x_{ji})$, and the better one will be selected to be a member of the DE population generation for the next iteration (x_{ji}^{G+1}).

B. HDE

In HDE [22~ 26], the one-to-one competition will have a faster convergence speed to give a higher probability toward a global optimum with much less computation time. It can use a small population in the evolutionary process to obtain a global solution. Generally, Evolutionary optimization involves two critical issues evolutionary direction and population diversity. As the evolutionary direction is effective in searching, the strong evolutionary direction can reduce the computational burden and increase the probability of rapidly finding an (possibly local) optimum. As population diversity is increased, the genotype of the offspring differs more from the parent. Accordingly, a highly diverse population can increase the probability of exploring the global optimum and prevent a premature convergence to a local optimum. These two important factors are here balanced by employing the

migration [27] into HDE that can determine an efficacious direction in which to search for a solution and simultaneously maintain an appropriate diversity for a small population. The migrant individuals are generated based on the best individual, $x_b^{G+1} = (x_{1b}^{G+1}, x_{2b}^{G+1}, ..., x_{n_c b}^{G+1})$, by non-uniformly random choice. Genes of the ith individual are regenerated according to

$$x_{ji}^{G+1} = \begin{cases} x_{jb}^{G+1} + \rho\left(x_j^{min} - x_{jb}^{G+1}\right), & if \quad r_1 < \dfrac{x_{jb}^{G+1} - x_j^{min}}{x_j^{max} - x_j^{min}} \\ x_{jb}^{G+1} + \rho\left(x_j^{max} - x_{jb}^{G+1}\right) & ,\text{otherwise} \end{cases} \quad (14)$$

where $j = 1, ..., n_C$; $i = 1, ..., N_p$, r_1 and ρ are random numbers in the range of [0,1]. The migration may be performed if only the best fitness has not been improved for over 500 generations running, and the migrant population will not only become a set of newly promising solutions but also easily escape the local extreme value trap. More details of the HDE have shown in [22~ 26].

C. MU

Michalewicz et al. [28] surveyed and compared several constraint-handling techniques used in evolutionary algorithms. Among these techniques, the penalty function method is one of the most popularly used to handle constraints. In this method, the objective function includes a penalty function that is composed of the squared or absolute constraint violation terms. Powell [29] noted that classical optimization methods include a penalty function have certain weaknesses that become most serious when penalty parameters are large. More importantly, large penalty parameters ill condition the penalty function so that obtaining a good solution is difficult. However, if the penalty parameters are too small, the constraint violation does not contribute a high cost to the penalty function. Accordingly, choosing appropriate penalty parameters is not trivial. Herein, the MU [30] is introduced to handle this constrained optimization problem. Such a technique can overcome the ill conditioned property of the objective function. Considering the nonlinear problem with general constraints as follows: Where $h_k(x)$ and $g_k(x)$ stand for equality and inequality

$$\begin{aligned} &\min_x f(x) \\ subject \quad to \quad &h_k(x) = 0, \quad k = 1, ..., m_e \\ &g_k(x) \le 0, \quad k = 1, ..., m_i \end{aligned} \quad (15)$$

constraints, respectively.

The augmented Lagrange function (ALF) [29] for constrained optimization problems is defined as:

$$\begin{aligned} L_a(x, v, \upsilon) = f(x) &+ \sum_{k=1}^{m_e} \alpha_k \left\{ \left[h_k(x) + v_k \right]^2 - v_k^2 \right\} \\ &+ \sum_{k=1}^{m_i} \beta_k \left\{ \left\langle g_k(x) + \upsilon_k \right\rangle_+^2 - \upsilon_k^2 \right\} \end{aligned} \quad (16)$$

where α_k and β_k are the positive penalty parameters, and the corresponding Lagrange multipliers $v = (v_1, ..., v_{m_e})$ and $\upsilon = (\upsilon_1, ..., \upsilon_{m_i}) \ge 0$ are associated with equality and inequality constraints, respectively.

The contour of the ALF does not change shape between generations while constraints are linear. Therefore, the contour of the ALF is simply shifted or biased in relation to the original objective function, $f(x)$. Consequently, small penalty parameters can be used in the MU. However, the shape of contour of L_a is changed by penalty parameters while the constraints are nonlinear, demonstrating that large penalty parameters still create computational difficulties. Adaptive penalty parameters of the MU are employed to alleviate the above difficulties. More details of the MU have shown in [30].

D. The proposed HDE-MU

Figure 1 displays the flow chart of the proposed algorithm, which has two iterative loops. The ALF is used to obtain a minimum value in the inner loop with the given penalty parameters and multipliers, which are then updated in the outer loop toward producing an upper limit of L_a. When both inner and outer iterations become sufficiently large, the ALF converges to a saddle-point of the dual problem [28]. Advantages of the proposed HDE-MU are that the HDE efficiently searches the optimal solution in the economic dispatch process and the MU effectively tackles system constraints.
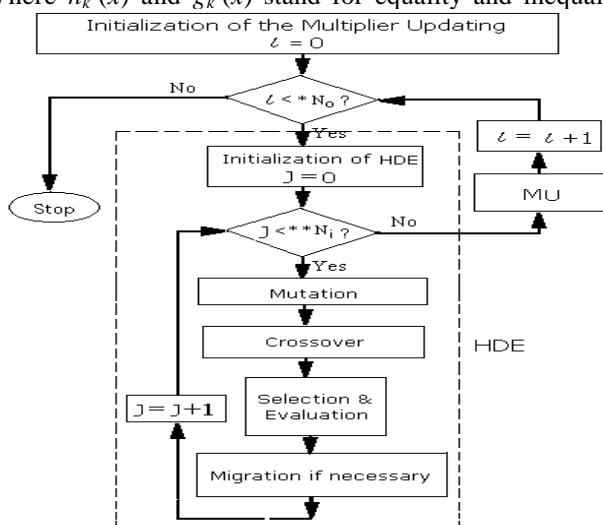

Fig.1 The flow chart of the HDE-MU
*No : maximum number of iterations of outer loop
**Ni : maximum number of iteartions of inner loop

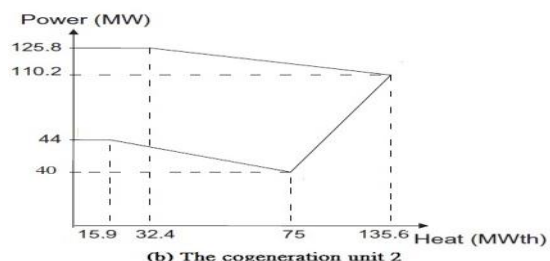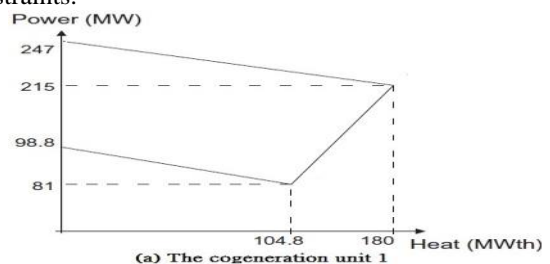(a) The cogeneration unit 1

(b) The cogeneration unit 2

Fig. 2 Heat-power feasible operation region

IV. System Simulations

This section considers a practical CHPED system which contains four conventional power units, two cogeneration units, and one heat-only unit. The system power demand (P_d) and heat demand (H_d) of this CHPED problem are 600MW and 150MWth, respectively. The valve-point effects and transmission loss are both considered, and this example used the linear boundary of the Heat-Power feasible region for cogeneration units. The associated data of this CHPED problem are the same as [19].

Figure 2 clear shows the feasible power and heat output region of cogeneration unit 1 and 2, and demonstrates correlations between the power and heat outputs.

The MU algorithm was used in HDE to hand the equality and inequality constraints. The computation was implemented on a personal computer (CPU clock 2.0GHz with 4G Ram) in FORTRAN-90 language. Setting factors used in this example are as follows; the population size N_p is set as 5 for HDE-MU. The iteration numbers of outer loop and inner loop are set to (outer, inner) as (20, 3000). The implementation of the proposed algorithm for this example can be described as follows:

$$L_a(x,v,\upsilon) = f(x) + \sum_{k=1}^{2} \alpha_k \left\{ [h_k(x)+v_k]^2 - v_k^2 \right\}$$
$$+ \sum_{k=1}^{6} \beta_k \left\{ \langle g_k(x)+\upsilon_k \rangle_+^2 - \upsilon_k^2 \right\} \quad (17)$$

$$objective: \min_{x=(P_1,P_2,P_3,P_4,P_5,P_6,H_5,H_6,H_7)} f(x)$$

$$f(x) = \sum_{i=1}^{4} Cost_i(P_i) + \sum_{j=5}^{6} Cost_j(H_j,P_j) + Cost_7(H_7) \quad (18)$$

where

$$Cost_1(P_1) = 0.008P_1^2 + 2P_1 + 25 + |100\sin(0.042*(10-P_1))|$$

$$Cost_2(P_2) = 0.003P_2^2 + 1.8P_2 + 60 + |140\sin(0.04*(20-P_2))|$$

$$Cost_3(P_3) = 0.0012P_3^2 + 2.1P_3 + 100 + |160\sin(0.038*(30-P_3))|$$

$$Cost_4(P_4) = 0.001P_4^2 + 2P_4 + 120 + |180\sin(0.037*(40-P_4))|$$

$$Cost_5(H_5,P_5) = 0.0345P_5^2 + 14.5P_5 + 0.03H_5^2 + 4.2H_5 + 0.031P_5H_5 + 2650$$

$$Cost_6(H_6,P_6) = 0.0435P_6^2 + 36P_6 + 0.027H_6^2 + 0.6H_6 + 0.011P_6H_6 + 1250$$

$$Cost_7(H_7) = 0.038H_7^2 + 2.0109H_7 + 950$$

Subject to;

$h_1:$ $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 - P_d - PLm = 0$

$h_2:$ $H_5 + H_6 + H_7 - H_d = 0$

$g_1:$ $1.78191489361H_5 - P_5 - 105.744680851 \le 0$

$g_2:$ $0.177777778H_5 + P_5 - 247.0 \le 0$

$g_3:$ $-0.16984732824H_5 - P_5 + 98.80 \le 0$

$g_4:$ $1.15841584158H_6 - P_6 - 46.8811881188 \le 0$

$g_5:$ $\begin{cases} -P_6 + 44.0 \le 0, & if \quad H_6 < 15.9 \\ -0.06768189509H_6 - P_6 + 45.0761421319 \le 0, & if \quad H_6 \ge 15.9 \end{cases}$

$g_6:$ $\begin{cases} P_6 - 125.8 \le 0, & if \quad H_6 < 32.4 \\ 0.15116279069H_6 + P_6 - 130.697674418 \le 0, & if \quad H_6 \ge 32.4 \end{cases}$

This minimum cost problem consists of one objective function with nine variable parameters, (P_1, P_2, P_3, P_4, P_5, P_6, H_5, H_6, and H_7), two equality constraints, (h_1 and h_2), and six inequality constraints, ($g_1 \sim g_6$), in the Heat-Power available region.

This CHPED example is used to illustrate the effectiveness of the proposed HDE-MU with respect to the quality of the solution obtained. This test shows the validly and practicability of general use for the CHPED problems. Table 1 compares nine computational results obtained from the

Table 1: Comparison of the proposed HDE-MU with previous methods

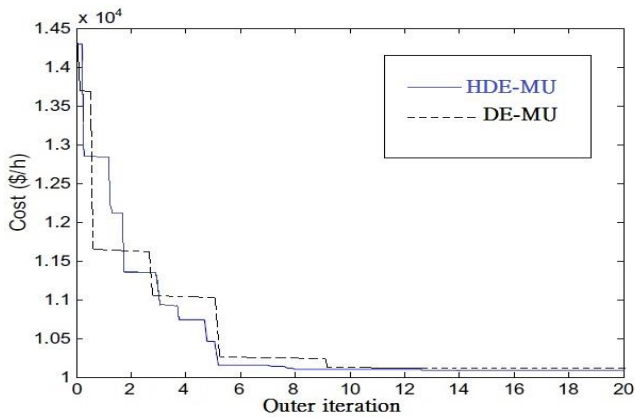| Methods / Items	HDE-MU	GSA [2]	OGSO [9]	GSO [4]	OBTLBO [18]	TLBO [18]	TVAC-PSO [19]	KHA [20]	CPSO [19]
$P(G1)$	45.56400	48.7638	45.6672	45.6188	45.8860	45.2660	47.3383	46.3835	75.0000
$P(G2)$	98.53982	98.7469	98.5401	98.5401	98.5398	98.5479	98.5398	104.1223	112.3800
$P(G3)$	112.67349	112.0000	112.6735	112.6727	112.6741	112.6786	112.6735	64.3729	30.0000
$P(G4)$	209.81582	208.5113	209.8158	209.8154	209.8141	209.8284	209.81582	246.1853	250.0000
$P(G5)$	94.14597	92.6909	94.0532	94.1027	93.8249	94.4121	92.3718	98.9736	93.2701
$P(G6)$	40.00000	40.0000	40.0000	40.0001	40.0002	40.0062	40.0000	40.7401	40.1585
$H(G5)$	27.40126	35.9704	27.9472	27.6600	29.2914	25.8365	37.8467	0.0000	32.5655
$H(G6)$	75.00000	75.0000	74.9996	74.9987	75.0002	74.9970	74.9999	66.7100	72.6738
$H(G7)$	47.59874	39.0000	47.0532	47.3413	45.7084	49.1666	37.1532	83.2900	44.7606
$\Sigma P(G)$ (MW)	600.73910	600.7128	600.7498	600.7498	600.7391	600.7392	600.73922	600.7777	600.8086
PLm (MW)	0.73910	0.7128	0.7498	0.7497	0.7394	0.7392	0.73922	0.7777	0.8086
$\Sigma H(G)$ (MWth)	150.00000	**149.9704**	150.0000	150.0000	150.0000	**150.0001**	**149.9998**	150.0000	**149.9999**
TC ($/h)	**10094.21766**	**9912.6928**	**10094.2407**	**10094.2670**	**10094.3529**	**10094.8384**	**10100.3164**	**10111.1501**	**10325.3339**
CPU time (Sec)	2.507	2.578	2.5681	2.4203	3.06	2.86	3.25	N/A	3.29

Fig. 3 Convergence characteristics of the proposed HDE-MU in comparison with DE-MU.

proposed HDE-MU, gravitational search algorithm (GSA) [2], opposition-based group search optimization (OGSO) [9], group search optimization (GSO) [4], oppositional teaching learning based optimization (OTLBO) [18], TLBO [18], particle swarm optimization with time varying acceleration coefficients (TVAC-PSO) [19], krill herd algorithm (KHA) [20] and classic PSO (CPSO) [19]. Although results of table 1 all located in the Heat-Power feasible operation region, the results of GSA [2], TLBO [18], TVAC-PSO [19] and CPSO [19] are infeasible solutions, because their heat outputs didn't match the heat demand (H_d = 150MWth) that will cause an unbalance of system's heat demand. By investigating the results presented in Table 1, it is observed that the best total cost utilizing HDE-MU is 10,094.21766
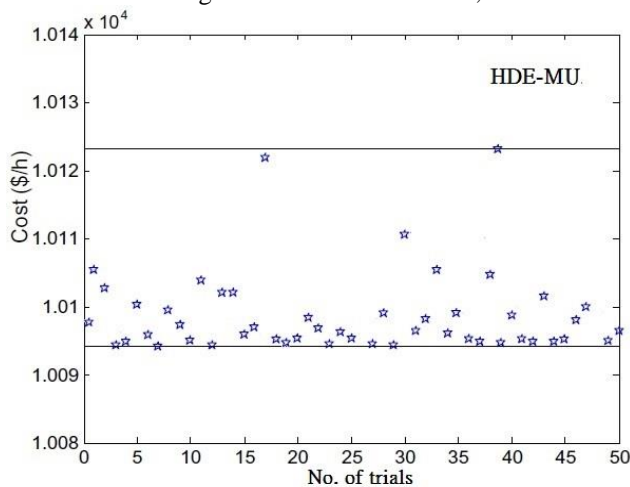

Fig. 4 Distribution of the minimum costs in 50 trials.

$/h, which is much less than the best results previously reported in TVAC-PSO [19], KHA [20] and CPSO [19], and also slightly better than the best results of OGSO [9], GSO [4], OBTLBO [18] and TLBO [18]. As seen from the best solution of HDE-MU listed in Table 1, the power output is 600.73910 MW, the heat output is 150 MWth and the transmission loss is 0.73910MW. Apparently, the equality constraints of heat and power demands are fully satisfied. Therefore, the result obtained from the proposed HDE-MU is an optimal and feasible solution. According to the absolute CPU time of different methods listed in the Table 1, the proposed HDE-MU is faster than most of the compared algorithms, but slightly slower than GSO [4]. Consequently, the HDE-MU is not only supplying the exact total of heat and power, but also has the less CPU time than most approaches

for the cogeneration system. Table 1 reveals that the HDE-MU is a satisfactory algorithm for solving the CHPED problem. The convergence characteristics of the proposed method in comparison with DE with MU (DE-MU) for this real example are depicted in Fig. 3, and the distribution of the minimum costs after 50 trials by the HDE-MU is plotted in Fig. 4. It can be observed that the worst, average and best costs are very close. The small standard deviation again confirms the stability of the proposed HDE-MU.

V. CONCLUSIONS

The HDE-MU for solving the CHPED problem has been proposed herein. Complication of the CHPED problem lies in the constraints imposed by the mutual dependencies of multi-demand and Heat-Power capacity. The HDE helps the proposed method efficiently search and refined exploit. The MU helps the proposed method avoid deforming the ALF and resulting in difficulty of solution searching. The proposed algorithm integrates the HDE and the MU that has the merits of taking a wide range of penalty parameters and a small population. A practical CHPED system with transmission losses and valve-point effects is used to compare the proposed HDE-MU with eight reported methods. Simulation results demonstrate that the proposed algorithm has more advantages for solving the CHPED problem than the previous methods. The contributions of this paper are the MU effectively handles the Heat-Power feasible region constraints, the HDE efficiently searches the optimal solutions in the economic dispatch process, and the author provides a valid and efficacious algorithm for the CHPED problem.

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