# $\beta - P$ Conectedness in $L$-Bitopological spaces

**FangLi Li , Hongkui Li, Tao Li**

***Abstract*— This paper introduce open sets and $p-$closed sets into $L$-Bitopological spaces, and based on this we introduce some related definitions and theorems about $\beta - P$ closed set and $\beta - P$ open sets . Furthermore, we give a new concept about $\beta - P$ local-connectivity .Then, it point out $\beta - P$ local-connectivity has two properties of topological invariance and finitely productive property and proves the other relevant theories.**

**Index Terms—** $L$- **bitopological spaces, $\beta - p$ local-connectivity, topological invariance, finitely productive property**

## I. INTRODUCTION

Since J. C. Kelly introduced the concept of double topological space, many scholars has a great interest about researching dual topology, then introduced $L$-bitopological spaces on the basis of $L$-topological space, and makes a long-term study of the separation .Connectivity is an important branch of fuzzy topology, the domestic scholars have studied the variety of connectivity, such as $\theta-$connectivity[3], connectivity[4], stratified connectedness [5], disconnectedness branches e .c. Due to the concept of connectivity is closely related to geometric closure , many connectivity is also defined based on the definition of different closure concepts. In this paper,we introduced $\beta - P$ open sets and $\beta - P$ closed sets in $L$-Bitopological spaces on the basis of $\beta - P$ open sets and $\beta - P$ closed sets .In this case ,we defined a new connectivity in $L$- bi-topological spaces which is called $\beta - P$ local-connectivity ,and the study of the connectivity has gotten some good properties.

## II. PRELIMINARY KNOWLEDGE

### A. $L$-bitopological spaces

**Definition 2.1**: Let $L$ be a F lattice ,that is a completely distributive lattice with the reverse involution ,let $X$ be a common set and let $L^X$ is a set that contains the whole $L-fuzzy$ sets on $X$ ,0 and 1 respectively expressed the minimum and maximum in $L$ , $M(L)$ and $M^*(L^X)$ respectively expressed all molecules of $L$ and $L^X$ ,we record $L-bts$ as $L-$bitopological space and $A_{\delta_1^-}$ as the closure of $A$ in $(L^X, \delta_1)$ [6].

### B. Final Stage The Related Defines And Conclusion About $p-$open set And $p-$close Set

**Definition 2.2**: Let $(L^X, \delta)$ be $L-bts$, $A \in L^X$ and record as $p-$ open set ,if and only if have open set $U$ ,makes $A \le U \le A^-$ ;If $A$ is a $p-$ open set, we call $A'$ is a $p-$ close set .The whole $p-$ open sets in $(L^X, \delta)$ are denoted by $LPO(L^{X^*})$,and the whole $p-$ close sets are denoted by $LPC(L^{X^*})$[7].

**Note**: The close set in $L-bts$ must be $p-$ close set, and on the contrary generally does not set up.

**Lemma 2.1**: Let $(L^X, \delta)$ be $L-bts$ , then $\delta \subset LPO(L^{X^*})$, $\delta' \subset LPC(L^{X^*})$.

**Definition 2.3**: Let $(L^X, \delta)$ be $L-bts$ , $A \in L^{X^*}$, $B \in L^{X^*}$ , then:

$(a)$ The union of all $p-$ open sets which contained by $A$ is called internal of $A$ 's $LE-p^*$,record as $A^{*\Delta}$,that is

$$A^{*\Delta} = \{B \in LPO(L^{X^*}) B \le A\}.$$

$(b)$ The union of all $p-$ close sets which contained by $A$ is called external of $A$ 's $LE-p^*$,record as $A^{*\leftarrow}$,that is

$$A^{*\leftarrow} = \{B \in LPC(L^{X^*}) A \le B\}$$

**Lemma 2.2**: Let $(L^X, \delta)$ be $L-bts$, $A, B \in L^{X^*}$,then

(a) $A^\circ \le A^{*\Delta} \le A \le A^{*\leftarrow} \le A^-$.

(b) If $A \in \delta \cap LPC(L^{X^*})$,then $A \in \delta'$.

(c) If $A \in \delta \cap LPO(L^{X^*})$,then $A \in \delta$.

(d) If $A \le B$,then $A^{*\leftarrow} \le B^{*\leftarrow}$, $A^{*\Delta} \le B^{*\Delta}$.

(e) $(A \vee B)^{*\leftarrow} = A^{*\leftarrow} \vee B^{*\leftarrow}$, $(A \vee B)^{*\Delta} = A^{*\Delta} \vee B^{*\Delta}$.

(f) The arbitrary intersection of $p-$ closed sets is $p-$ closed set,and the arbitrary intersection of $p-$ open sets is $p-$ open set.

**FangLi Li**, School of Science, Shandong University of Technology, ZiBo, China
**Hongkui Li**, School of Science, Shandong University of Technology, ZiBo, China.
**Tao Li**, Innovation and entrepreneurship College, Shandong University of Technology, Zibo, China.

*C. Figures The Introduction of $\beta-p$ open set And $\beta-p$ close set*

According to the preparation,we introduce $\beta-p$ open set and $\beta-p$ close set into $L-$bitopological spaces,and give some related properties and theorems [7] about $\beta-p$ open set and $\beta-p$ close set on the basis of $p-$closed set and $p-$open set,and give the corresponding proof.

**Definition 3.1**: Let $(L^X,\delta)$ be $L-bts, \beta\in L, A\in L^X$,record as $l_\alpha(A)=\{x\in X|A(x)>\beta\}$,let $\beta\in L-\{0\}$,if $l_\beta(A^{*\leftarrow})=l_\beta(A)$, then call $A$ as $\beta-p$ open set;If $A'$ is $\beta-p$ close set,then we call $A$ as $\beta-p$ open set .

**Lemma 3.1**:Let $(L^x,\delta)$ be $L-bts, A\in L^X$,if A is $\beta-$close set,then A is $\beta-p$ close set.

**Proof:** If $A$ is $\beta-$close set,then about $\beta\in L-\{0\}, A\in L^{X^*}$,when $\gamma\geq\beta$,there is $l_\beta(A^-)=l_\beta(A)$,due to $A^{*\leftarrow}\leq A^{*\leftarrow}$,then there is $l_\beta(A^{*\leftarrow})=l_\beta(A)$,so A is a $\beta-p$ close set.

**Lemma 3.2**:Let $(L^x,\delta)$ be $L-bts, A\in L^X$,if $A$ is a $\beta-$open set,then A is $\beta-p$ open set.

**Lemma 3.3**:Let $(L^x,\delta)$ be $L-bts, A\in L^X$,If $A$ is a $p-$ close set is the necessary and sufficient condition of $A$ is $1-p$ close set.

**Proof** :

"$\Leftarrow$" Clearly established.

"$\Rightarrow$" Due to "$A$" is $1-p$ close set,so $\forall\beta\geq 1'=0, l_\beta(A^{*\leftarrow})=l_\beta(A)$

If $A^{*\leftarrow}\neq A$,then there is $x\in X$,cause $A^{*\leftarrow}(x)>A(X)$,marked $\beta_0=A(x)\geq 0$,then $x\notin l_{\beta_0}(A^{*\leftarrow})$is conflict with ( * ) ,so $A^{*\leftarrow}=A$,that is $A$ is $\beta-p$ close set.

**Lemma 3.4**:Let $(L^x,\delta)$ be $L-bts, A,B\in L^X, \beta\in L-\{0\}$,then

(a) If $A\leq B$,then $l_\alpha(A)\subseteq l_\alpha(B)$.

(b) $l_\alpha(A\wedge B)=l_\alpha(A)\cup l_\alpha(B)$.

(c) $l_\alpha((A\wedge B)^{*\Delta})=l_\alpha(A^{*\Delta})\cup l_\alpha(B^{*\Delta})$.

(d) $l_\alpha((A\vee B)^{*\Delta})=l_\alpha(A^{*\Delta})\cap l_\alpha(B^{*\Delta})$.

(e) $l_\alpha(A^{*\leftarrow}\wedge B^{*\leftarrow})=l_\alpha(A^{*\leftarrow})\cap l_\alpha(B^{*\leftarrow})$.

(f) $l_\alpha(A^{*\leftarrow}\vee B^{*\leftarrow})=l_\alpha(A^{*\leftarrow})\cup l_\alpha(B^{*\leftarrow})$.

*D. $\beta-p$ local-connectivity*

1.Definitions about $\beta-p$ local-connectivity

**Definition 4.1.1**:Let $(L^X,\delta)$be $L-bts, A,B\in L^X, \alpha\in L-\{0\}$ , if $A^{*\leftarrow}\wedge B\leq\beta'$ and $A\wedge B^{*\leftarrow}\leq\beta'$,then call $A$ and $B$ are $\beta-p$ insular.

**Definition 4.1.2**:Let $(L^X,\delta)$ be $L-bts$, $S\in L^X$ ,if there is no $A,B\in L^X$ ,to make $A$ and $B$ are $\beta-p$ insular,and $A\vee B=S$, $A>\beta'$, $B>\beta'$,then call $S$ is Connected set in $(L^X,\delta)$.Particularly, when $1_X$ which is the Maximum element in $L^X$ is $\beta-p$ Connected set;Otherwise,call $(L^X,\delta)$ is $\beta-p$ disconnected space.

**Definition 4.1.3**: Let $(L^X,\delta)$ be $L-bts$, $x\in L^X$ ,if every neighborhood of A contains a $\beta-p$ connected neighborhood $V$ ,then call $x$ is $\beta-p$ local-connectivity;Otherwise,call $x$ is not $\beta-p$ local-connectivity [8].

**Definition 4.1.4**: If every point of $(L^X,\delta)$ is $\beta-p$ local-connectivity,then call $(L^X,\delta)$ is $\beta-p$ local-connected space.

2 .Basic properties of $\beta-p$ local-connectivity

**Theorem 4.2.1**: The local connected space must be $\beta-p$ local-connected space.

**Proof** : From the definition of local-connected spaces ,we can know that every neighborhood of A contains a $\beta-p$ connected neighborhood $V$ ,the $V$ is open set .By definition 4.2 in the literature [3] ,we can know that the connected open set must be $\beta-p$ open set of $V$ .As for $\forall x\in L^X$ ,if every neighborhood of A contains a $\beta-p$ connected neighborhood $V$ ,then $(L^x,\delta)$ is $\beta-p$ local-connected space according to definition 4.1.4.

On the contrary, does not necessarily set up.

**Theorem 4.2.2**: Let $(L^X,\delta)$ be $L-bts$ , then the following conditions are equivalent:

(a) $L^X$ is a $\beta-p$ local-connected space.

(b)Arbitrary $\beta-p$ connected branch of $L^X$ 's arbitrary $\beta-p$ open set is a $\beta-p$ open set.

(c) $L^X$ is a connected base.

Proof :

(a)$\Rightarrow$(b):

Let $U$ is a arbitrary $\beta-p$ open set of $\beta-p$ connected space,c is arbitrary $\beta-p$ connected branch of $U$ ,as for arbitrary $x\in c\in U$ ,then $U_X\in U$ .And because

$L^X$ is $\beta-p$ local-connected,there is a connected neighborhood $V$ ,and $V$ also is $\beta-p$ connected neighborhood of subspace $U$ [9].

$\Rightarrow$(c) :

Let $\alpha$ is a set family that formed by all $\beta-p$ open set of $L^X$ ,then $\alpha$ is a base of $L^X$ ,also because $\beta-p$ connected branch is $\beta-p$ connected subset,then $\alpha$ is connected base of $L^X$ .

$\Rightarrow$(a) :

Let $L^X$ has a $\beta-p$ connected base $\alpha$ ,then each member of $\alpha$ are all $\beta-p$ connected sets,as for $\forall x\in L^X$ ,make $\alpha_x=\{A\in\beta|x\in B\}$,then $x\in\alpha_x\subset\alpha$ ,that $\alpha_x$ is $\beta-p$ connected,we can know $L^X$ is $\beta-p$ local-connected by the definition 4.1.3.

**Theorem 4.2.3**: Let $(L^X{}_1,\delta_1)$ is a $\beta-p$ local-connected space, $(X,T)$ is a topological space, $f:L^X\to X$ is a continuous open mapping,then we can construct that $(L^X{}_2,\delta_2)$ is a $\beta-p$ local-connected space by $(X,T)$.

**Proof** :Let $L^X$ is a $\beta-p$ local-connected space, then we can know there is a $\beta-p$ connected base in $L^X$ by the definition 4.2.2.As for $f$ is a continuous open mapping,to make $\beta_0=\{f(B)|B\in\beta\}$ ,then $\forall B\in\beta, f(B), \forall B\in\beta, f(B)$ is $\beta-p$ connected open set of $X$ ,so $\beta_0$ is $\beta-p$ connected open set family of $X$ .As for arbitrary $\beta-p$ open set of $X$ , $f$ is $\beta-p$ connected and surjection,so:

$$A=f(f^-(A))=f\left(\bigcup_{B\in\beta_1}B\right)=\bigcup_{B\in\beta_1}f(B)$$

Then $\beta_0$ is a $\beta-p$ open connected base of $X$ .

By the theorem 4.2.2,we can know $(L^X{}_2,\delta_2)$ can structured by $(X,T)$ is $\beta-p$ connected space .

We call the nature is topological invariance which is can keep the same nature under $\beta-p$ continuous mapping in topological space.

Theorem 4.2.4: If $L^X{}_1$ , $L^X{}_2$ , ... $L^X{}_n$ are also $\beta-p$ local-connected space,then $L^X=L^X{}_1\times L^X{}_2\times....\times L^X{}_n$ is $\beta-p$ local-connected space too.

Proof :Let $L^X{}_i(i=1,2,...,n)$ is $\beta-p$ connected space,from theorem 4.2.2, we can know that there is a $\beta-p$ connected base $V_i(i=1,2,...,n)$ in $L^X{}_i$ ,then make $V=\{V_1\times V_2\times...\times V_n|V_i\in V(i=1,2,...,n)\}$ ,and from literature [10],we can know $\beta-p$ connected nature is finitely productive property ,so $V$ is a $\beta-p$ connected base of product space $L^X$ .Also,from theorem4.2.2,we can know that product space $L^X=L^X{}_1\times L^X{}_2\times....\times L^X{}_n$ is also a $\beta-p$ local-connected space.

Some properties $P$ of topological spaces are called finite integrable properties, if there are $n\geq1$ arbitrary topological space $L^X{}_1$ , $L^X{}_2$ , ... $L^X{}_n$ have property $P$ ,and contained product space $L^X=L^X{}_1\times L^X{}_2\times....\times L^X{}_n$ also have property $P$ ,then $\beta-p$ local-connected nature is finitely productive property.

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**Corresponding Author:** Hongkui Li

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