Study of Damped Vibration of Non Homogeneous Rectangular Plate of Variable Thickness

U. S. Rana, Robin

***Abstract*— In the present paper damped vibrations of non-homogeneous rectangular plate of parabolically varying thickness resting on elastic foundation has been studied. The non- homogeneity of the plate material due to the variation in Young's modulus and density is assumed, which varies exponentially. The governing equation of motion of plate of varying thickness in one direction is solved by numerical technique quintic spline for clamped-clamped and clamped-simply supported boundary conditions. The effect of damping, non-homogeneity, elastic foundation and taperness is discussed with permissible range of parameters. It is observed that nodal lines are shifted towards edge *X*=1 as thickness of edge *X*=0 increases and no change appeared in the pattern of nodal lines for non-homogeneity and elastic foundation.**

***Index Terms*—Damping, Elastic Foundation, Nonhomogeneity, Taperness.**

I. INTRODUCTION

As technology is growing rapidly, the importance of study of vibration is increasing. Non- homogeneous elastic plates have acquired great importance as structural components in various engineering fields such as missile technology, aerospace industry, naval ship design and telephone industry etc; require a phenomenal increase in the development of fiber-reinforced materials due to desirability of high strength, light weight, corrosion resistance and high temperature performance. Due to appropriate variation of plate thickness these plates have significantly greater efficiency for bending, buckling and vibration as compared to plate with uniform thickness and also provide the advantage of reduction in weight and size essential for economy. Plates resting on elastic foundation have applications in pressure vessels technology such as petrochemical, marine and aerospace industry, building activities in cold regions and aircraft landing in arctic operations [1]-[2]. In a series of papers, Lal et al. [3] have studied the transverse vibrations of a rectangular plate of exponentially varying thickness resting on an elastic foundation. Transverse vibration of non-homogeneous orthotropic rectangular plate with variable thickness was discussed by Lal and Dhanpati [4]. In many applications of vibration and wave theory the magnitudes of the damping forces are small in comparison with the elastic and inertia forces but these small forces may have very great influence under some special situations. Recently O'Boy [5] has analyzed the damping of flexural vibration and Alisjahbana and Wangsadinata [6] discussed the realistic vibrational problem incorporating dynamic analysis of rigid roadway pavement under moving traffic loads. In reality all the vibrations are damped vibration and foundations of underlying vibrational problems are elastic in nature, so no vibration can be thought of being in existence without damping and elastic foundation. Keeping this in view and practicality of problem, Robin and Rana [7]-[9] studied the damped vibration of rectangular/infinite elastic plates with variable thickness resting on elastic foundation. In this paper, damped vibrations of non- homogeneous isotropic rectangular plate of parabolically varying thickness along one direction and resting on elastic foundation is analyzed by employing Lévy approach and assuming exponential variation in Young's modulus and density along with constant Poisson ratio.

Various numerical techniques such as Frobenious method, finite difference method, simple polynomial approximation, Galerkin's method, Rayleigh-Ritz method, finite element method and Chebyshev collocation method, difference quadrature method etc, have been employed to analyze the modes of vibration of plates with different geometries. As finite difference and finite element require fine mesh size to obtain accurate results but are computationally expensive .The results due to Frobenious method are in the form of series and to achieve the high level of accuracy of results, large number of terms required which includes round off truncation errors. However, quintic splines interpolation technique has the capability of producing highly accurate results with minimum computational efforts for initial and boundary value problems also this method of solution is preferred over other methods for the reasons as a chain of lower order approximations may yield a better accuracy than a global higher order approximation and natural boundary conditions can be considered easily. Therefore in the present paper, quintic spline method is used to obtain first three modes of vibration for two different combinations of clamped and simply-supported boundary conditions.

II. MATHEMATICAL FORMULATION

Consider a non- homogeneous isotropic rectangular plate of length 'a', breath 'b', thickness '$h(x, y)$' and density 'ρ', with resting on a Winkler- type elastic foundation 'k_f' occupying the domain $0 \le x \le a, 0 \le y \le b$ in x-y plane. The middle surface being z=0 and the origin is at one of the corners of the plate. The two parallel edges (y=0, y=b) are assumed to be simply supported while the other two edges are differently restrained (clamped and simply supported). The x-and y axes are taken along the principal directions and z –axes is perpendicular to the x-y plane (Fig. 1).

U.S. Rana, Department of Mathematics, D.A.V. College, Dehradun, India, Phone/ Mobile No.+91- 9412988626.

Robin, Department of Applied science and Humanities, Quantum school of Technology, Roorkee, India, Phone/ Mobile No.+91-9761323214.

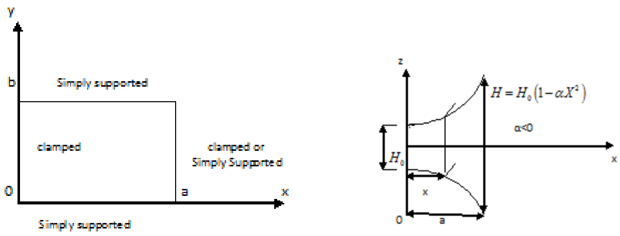

Fig. 1. Boundary conditions and vertical cross-section of the plate.

The differential equation which governs the damped transverse vibration of such plates is given by

$$\nabla^2(D\nabla^2 w)-(1-\upsilon)\left[\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]$$
$$+\rho h\frac{\partial^2 w}{\partial t^2}+K\frac{\partial w}{\partial t}+K_f w=0, \tag{1}$$

where $D = Eh^3(x,y)/12(1-\upsilon^3)$, is the flexural rigidity at any point in the middle plane of the plate, K is the damping constant, $w(x,y,t)$ is the transverse deflection.

Let the two opposite edges y=0 and y=b of the plate be simply supported and thickness $h = h(x,y)$ varies parabolically along the length *i.e.* in the direction of x-axis. Thus, 'h' is independent of y *i.e.* $h = h(x)$. For a harmonic solution, it is assumed that the deflection function w satisfying the condition

$$w(x,y,t) = W(x)\sin\frac{m\pi y}{b}e^{-\gamma t}\cos pt$$ at y=0 and y=b, (2)

where p is the circular frequency of vibration and m is a positive integer.

Thus (1) becomes

$$Eh^3\frac{d^4W}{dx^4}\cos pt+\left\{6Eh^2\frac{dh}{dx}+2h^3\frac{dE}{dx}\right\}\frac{d^3W}{dx^3}\cos pt$$
$$+\left\{\begin{array}{l}6Eh\left(\frac{dh}{dx}\right)^2+3Eh^2\frac{d^2h}{dx^2}+6h^2\frac{dh}{dx}\frac{dE}{dx}\\ +h^3\frac{d^2E}{dx^2}-2Eh^3\frac{m^2\pi^2}{b^2}\end{array}\right\}\frac{d^2W}{dx^2}\cos pt$$
$$-\left\{6Eh^2\frac{dh}{dx}+2h^3\frac{dE}{dx}\right\}\frac{dW}{dx}\cos pt$$
$$+\left[\frac{m^4\pi^4}{b^4}Eh^3+\upsilon\frac{m^2\pi^2}{b^2}\left\{\begin{array}{l}3Eh^2\frac{d^2h}{dx^2}+6Eh\left(\frac{dh}{dx}\right)^2\\ +6h^2\frac{dh}{dx}\frac{dE}{dx}+h^3\frac{d^2E}{dx^2}\end{array}\right\}\right]W\cos pt$$
$$+\left[\begin{array}{l}12(1-\upsilon^2)\rho h\left\{\begin{array}{l}(\gamma^2-p^2)\cos pt\\ +2p\gamma\sin pt\end{array}\right\}\\ +12(1-\upsilon^2)k\{-p\sin pt-\gamma\cos pt\}\end{array}\right]W=0. \tag{3}$$

Following non-dimensional variables are introduced

$$H=\frac{h}{a},\quad X=\frac{x}{a},\quad \overline{E}=\frac{E}{a},\quad \overline{W}=\frac{W}{a},\quad \overline{\rho}=\frac{\rho}{a},$$
$$\lambda^2=m^2\pi^2\left(\frac{a}{b}\right)^2$$

and equating the coefficient of *sin(pt)* and *cos(pt)* independently to zero, (3) reduces to

$$\overline{E}H^4\frac{\partial^4\overline{W}}{\partial X^4}+\left\{6\overline{E}H^3\frac{dH}{dX}+2H^4\frac{d\overline{E}}{dX}\right\}\frac{d^3\overline{W}}{dX^3}$$
$$+\left\{\begin{array}{l}3\overline{E}H^3\frac{d^2H}{dX^2}+6\overline{E}H^2\left(\frac{dH}{dX}\right)^2+6H^3\frac{dH}{dX}\frac{d\overline{E}}{dX}\\ +H^4\frac{d^2\overline{E}}{dX^2}-2\lambda^2\overline{E}H^4\end{array}\right\}\frac{d^2\overline{W}}{dX^2}$$
$$-\lambda^2\left\{6\overline{E}H^3\frac{\partial H}{\partial X}+2H^4\frac{d\overline{E}}{dX}\right\}\frac{d\overline{W}}{dX}$$
$$+\left[\begin{array}{l}\lambda^4\overline{E}H^4-\lambda^2\upsilon\left\{\begin{array}{l}H^4\frac{d^2\overline{E}}{dX^2}\\ +6H^3\frac{dH}{dX}\frac{d\overline{E}}{dX}+6\overline{E}H^2\left(\frac{dH}{dX}\right)^2\\ +3\overline{E}H^3\frac{d^2H}{dX^2}\end{array}\right\}\\ +12(1-\upsilon^2)\left\{\frac{-k^2}{4a^2\overline{\rho}}+HK_f-a^2p^2\overline{\rho}H^2\right\}\end{array}\right]\overline{W}=0. \tag{4}$$

The physical quantities of interest are taperness, non-homogeneity, which are defined as,

$H = H_0\left(1-\alpha X^2\right)$, $\overline{E}=\overline{E}_0 e^{\beta X}$, $\overline{\rho}=\overline{\rho}_0 e^{\beta X}$, where $H_0=(H)_{X=0}$, $\overline{E}_0=\left(\overline{E}\right)_{X=0}$, $\overline{\rho}_0=\left(\overline{\rho}\right)_{X=0}$ and 'α' is the taper constant.

On equating the coefficient the following equation is formed

$$A_0\frac{d^4\overline{W}}{dX^4}+A_1\frac{d^3\overline{W}}{dX^3}+A_2\frac{d^2\overline{W}}{dX^2}+A_3\frac{d\overline{W}}{dX}+A_4\overline{W}=0, \tag{5}$$

where

$$A_0=(1-\alpha X^2)^4,$$
$$A_1=2\beta(1-\alpha X^2)^4-12\alpha X(1-\alpha X^2)^3,$$
$$A_2=24\alpha^2X^2(1-\alpha X^2)^2-6\alpha(1+2\beta X)(1-\alpha X^2)^3$$
$$+(\beta^2-2\lambda^2)(1-\alpha X^2)^4,$$
$$A_4=\lambda^4(1-\alpha X^2)^4-\upsilon\lambda^2\left\{\begin{array}{l}\beta(1-\alpha X^2)^4\\ -6\alpha(1+2\beta X)(1-\alpha X^2)^3\\ +24\alpha^2X^2(1-\alpha X^2)^2\end{array}\right\}$$
$$-\left\{d_k^2I^{*2}e^{-2\beta X}+\Omega^2I^*(1-\alpha X^2)^2-E_f(1-\alpha X^2)e^{-\beta X}C^*\right\},$$
$$d_K{}^2=\frac{3(1-\upsilon^2)K^2}{a^2\overline{E}_0\overline{\rho}_0},\ I^*=\frac{1}{H_0{}^2},\ C^*=\frac{1}{H_0{}^3},\ E_f=\frac{12(1-\upsilon^2)K_f}{\overline{E}_0},$$
$$\Omega^2=\frac{12(1-\upsilon^2)a^2\overline{\rho}_0p^2}{\overline{E}_0},$$

and Ω, d_K, E_f are frequency, damping and elastic foundation parameters respectively.

In order to determine smooth and best approximation of solution of (5) together with boundary conditions at the edge X=0 and X=1, quintic spline interpolation technique is used.

According to the spline technique, suppose *W(x)* be a function with continuous derivatives in [0, 1] and interval [0, 1] be divided into 'n' subintervals by means of points X_i such that $0 = X_0 < X_1 < X_2 < ... < X_n = 1$, where $\Delta X = 1/n, X_i = i\Delta X (i = 0,1,2,...,n)$. Let the approximating function $\overline{W}(X)$ for the *W(x)* be a quintic spline with the following properties:

(i) $\overline{W}(X)$ is a quintic polynomial in each interval (X_k, X_{k+1}).

(ii) $\overline{W}(X) = W(X_k), k = 0,1,2,...,n.$

(iii) $\frac{d\overline{W}}{dX}, \frac{d^2\overline{W}}{dX^2}, \frac{d^3\overline{W}}{dX^3}$ *and* $\frac{d^4\overline{W}}{dX^4}$ are continuous.

In view of above axioms, the quintic spline takes the form

$$\overline{W}(X) = a_0 + \sum_{i=0}^{4} a_i (X - X_0)^i + \sum_{j=0}^{n-1} b_j (X - X_j)_+^{\,5}, \tag{6}$$

where $(X - X_j)_+ = \begin{cases} 0, & if\ X \le X_j \\ (X - X_j), & if\ X > X_j \end{cases}$, and

$a_i's, b_j's$ are constants.

Thus for the satisfaction at the n[th] knot, (5) reduced to

$$\begin{aligned} & A_4 a_0 + [A_4(X_m - X_0) + A_3]a_1 + [A_4(X_m - X_0)^2 \\ & +2A_3(X_m - X_0) + 2A_2]a_2 \\ & +[A_4(X_m - X_0)^3 + 3A_3(X_m - X_0)^2 \\ & +6A_2(X_r - X_0) + 6A_1]a_3 + [A_4(X_m - X_0)^4 \\ & +4A_3(X_m - X_0)^3 + 12A_2(X_m - X_0)^2 \\ & +24A_1(X_m - X_0) + 24A_0]a_4 + \sum_{j=0}^{n-1} b_j [A_4(X_m - X_j)_+^{\,5} \\ & +5A_3(X_m - X_j)_+^{\,4} + 20A_2(X_m - X_j)_+^{\,3} \\ & +60A_1(X_m - X_j)_+^{\,2} + 120A_0(X_m - X_j)_+] = 0. \end{aligned} \tag{7}$$

For m=0 (1) n, above system contains (n+1) homogeneous equation with (n+5) unknowns, a_i, i=0(1)4, and b_j, j=0, 1, 2… (n-1), can be represented in matrix form as

$$A][B] = [0], \tag{8}$$

where [A] is a matrix of order (n+1) $\times$(n+5), while [B] and [0] are column matrices of order (n×5).

III. Boundary Conditions and Frequency Equation

The following two cases of boundary conditions have been considered: (C-C): clamped at both the edge X=0 and X=1.

(i) (C-SS): clamped at X=0 and simply supported at X=1.

The relations that should be satisfied at clamped and simply supported respectively are

$$W = \frac{dW}{dX} = 0,\ W = \frac{d^2W}{dX^2} = 0. \tag{9}$$

Applying the boundary conditions C-C to the displacement function by (9) one obtains a set of four homogeneous equations in terms of (n+5) unknown constants which can be written as

$$[B^{cc}][B] = [0], \tag{10}$$

where B^{cc} is a matrix of order 4×(n+5).

Therefore the (8) together with (10) gives a complete set of (n+5) homogeneous equations having (n+5) unknowns which can be written as

$$\left[\frac{A}{B^{cc}}\right][B] = [0]. \tag{11}$$

For a non-trivial solution of (11), the characteristic determinant must vanish, *i.e.*

$$\left|\frac{A}{B^{cc}}\right| = 0. \tag{12}$$

Similarly for (C-SS) plate the frequency determinant can be written as

$$\left|\frac{A}{B^{ss}}\right| = 0. \tag{13}$$

where B^{ss} is a matrix of order 4×(n+5).

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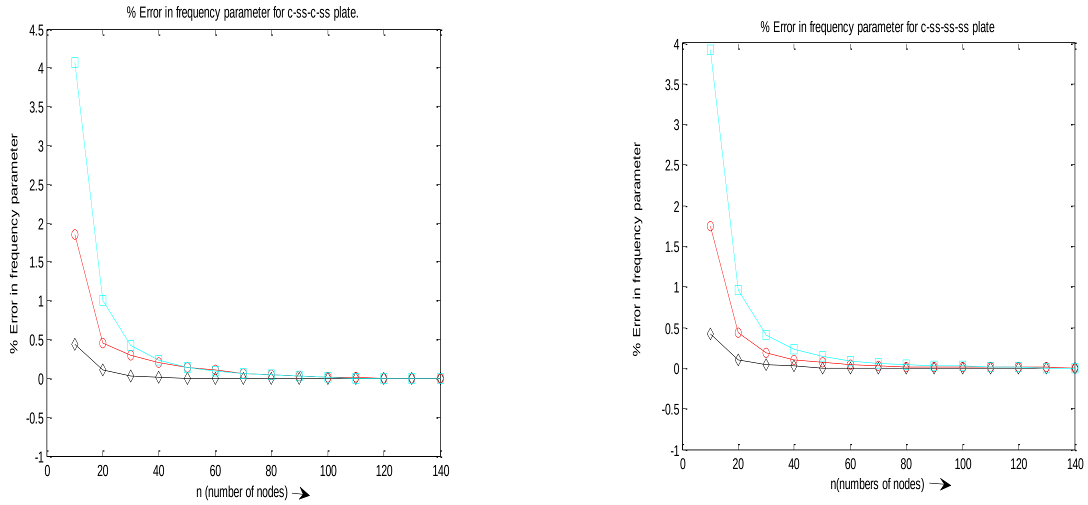
IV. Numerical Results and Discussion

The frequency equations (12), (13) provide the values of frequency parameter Ω for various values of plate parameters. In the present paper, first three frequency modes of vibration have been computed for the above mentioned two boundary conditions for different values of foundation parameter E_f=0.0(0.005)0.02, damping parameter d_k=0.0(0.025)0.01 and taper parameter α=0.0(0.1)0.4 for β=0.0, 0.4, Poisson ratio's ν=0.3, thickness of plate h=0.03 and aspect ratio a/b=0.25. The numerical method provides approximate values therefore in order to minimize the error; there is an urgent need to determine the optimum size of interval length ΔX. To choose appropriate number of nodes n, convergence studies have been carried out for different sets of plate parameter. Table I presents the convergence of frequency parameter with increasing number of nodes for specified plate *i.e.* α=0.4, β=0.4, E_f=0.02, d_k=0.01. In the present problem, a computer program was developed and executed for n=10(10)150 and observed that, frequency.

Table I Number of nodes for convergence of frequency parameter Ω for isotropic C-C and C-SS plates for *h=0.03, ν=0.3, m=1, β=0.04,α=0.4, d_k=0.01, E_f=0.02, a/b=0.25.*

value of n	C-C plate			C-SS plate		
	Mode			Mode		
10	0.908	1.7233	3.2343	0.8326	1.4919	2.8563
20	0.9105	1.7189	3.1752	0.8338	1.4875	2.7999
30	0.9109	1.718	3.1639	0.834	1.4866	2.79
40	0.9111	1.7176	3.1599	0.8341	1.4863	2.7866
50	0.9111	1.7175	3.1581	0.8341	1.4862	2.785
60	0.9112	1.7174	3.1571	0.8342	1.4861	2.7842
70	0.9112	1.7173	3.1565	0.8342	1.4861	2.7836
80	0.9112	1.7173	3.1561	0.8342	1.4861	2.7833
90	0.9112	1.7173	3.1559	0.8342	1.4861	2.7831
100	0.9112	1.7172	3.1557	0.8342	1.486	2.7829
110	0.9112	1.7172	3.1555	0.8342	1.486	2.7828
120	0.9112	1.7172	3.1554	0.8342	1.486	2.7827
130	0.9112	1.7172	3.1554	0.8342	1.486	2.7826
140	0.9112	1.7172	3.1553	0.8342	1.486	2.7826
150	0.9112	1.7172	3.1553	0.8342	1.486	2.7826



(a) (b)

Fig. 2. Percentage error in frequency parameter Ω: (a) C-C plate (b) C-SS plate, for a/b=0.25, α=0.04,β=0.4,E_f =0.02,d_k=0.01;◊First mode, ○ Second mode; □ Third mode; Percentage error=[(Ω_n- Ω_{140})/ Ω_{140}]x100; n=10(10)140.

Table II Comparison of frequency parameter Ω for isotropic, homogeneous (β=0), without damping parameter (dk=0), C-C and C-SS plates of uniform thickness (α=0) for m=1 and υ=0.3.

Boundary Conditions		E_f=0.0				E_f=0.01			
	value of a/b	0.5		1		0.5		1	
	Ref./Mode	I	II	I	II	I	II	I	II
C-C	Liessa	---	---	28.946	69.320	---	---	---	---
	Lal et al.	23.816	63.635	28.951	69.327	26.214	64.472	30.954	70.187
	Jain & Soni	23.816	63.535	28.951	69.327	---	---	---	---
	Sharma et al.	23.815	63.5345	28.950	69.327	26.214	64.472	30.954	70.187
	Present	23.8161	63.5401	28.9508	69.3314	26.2146	64.4775	30.9540	70.1914
C-SS	Liessa	---	---	23.646	58.641	---	---	---	---
	Lal et al.	17.332	52.098	23.646	58.646	20.503	53.237	26.060	59.661
	Jain & Soni	17.332	52.097	23.647	58.688	20.506	53.288	26.061	59.702
	Sharma et al.	17.332	---	23.636	58.646	20.503	53.237	26.060	59.661
	Present	17.3321	52.1022	23.6464	58.6498	20.5037	53.2413	26.0606	59.6641

parameter converges with increasing number of nodes and for convergence of frequency parameter in higher modes more nodes are needed than for the lower ones. The convergence of frequency with increasing number of nodes is monotonic for C-C and C-SS plate. Percentage error in frequency parameter for first three modes of vibration are presented in Fig. 2 (a, b) for clarity. Therefore the numerical results are obtained for n=140 and depicted through tables (II-V) and graphs (3-7) since no consistent improvement in results while $n \geq 140$. It is found that frequency parameter for clamped plate is greater than that of simply supported plate whatever the values of other parameters are.

A comparison of results with those available in the literature obtained by other methods with in permissible range of plate parameters has been presented in Table II. Table II shows a comparison of results for homogeneous (β=0.0) isotropic plates of uniform thickness (α=0.0) taken as h=0.1 with exact solution [10], with those obtained by Chebyshev collocation technique [11], Frobeneous method [12], and Differential quadrature method [13] for $m=1$, two values of aspect ratio a/b=0.5, 1.0 and E_f =0.0, 0.01. Excellent agreement of the results shows the versatility of present technique.

Table III(a) and III(b) show the numerical values of frequency parameter Ω with the increasing value of damping parameter d_k for homogeneous (β=0.0) and non-homogeneous (β=0.04) respectively, for both the boundary conditions C-C and C-SS. These results are also presented in Fig. 3(a), 3(b) and 3(c) for the fixed value of taper constant α and foundation parameter E_f for first three modes of vibration of C-C and C-SS plates. Fig. 3(a) shows the behavior of frequency parameter Ω decreases with the increasing values of damping parameter d_k for two different values of taper parameter α=0.0, 0.4, foundation parameter E_f =0.0, 0.01 and non homogeneity parameter β=0.0, 0.4 for both the plates. It is observed that the rate of decrease of Ω with damping parameter d_k for C-SS is higher than that for C-C plate keeping all other plate parameters fixed. This rate decreases with the increase in the value of non homogeneity parameter β. A similar inference can be seen from Fig. 3 (b) and (c), when the plate is vibrating in the second mode as well as in the third mode of vibration except that the rate of decrease of Ω with d_k is lesser as compared to the first mode.

Table IV(a) and IV(b) provide the inference of foundation parameter E_f on frequency parameter Ω for two values of damping parameter d_k=0.0, and 0.01 respectively, for the fixed value of taper parameter α=0.0, 0.4 and non homogeneity parameter β=0.0, 0.4. It is noticed that the frequency parameter Ω increases continuously with the increasing value of foundation parameter E_f for C-C and C-SS plates, whatever be the value of other plate parameters. It is found that the rate of increases of frequency parameter Ω for C-SS plate is higher than C-C plate for three modes. Fig. 4(a) gives the inference of foundation parameter E_f on frequency parameter Ω for the first mode of vibration. This rate increases with the increase in the value of foundation parameter E_f, it decreases with the increases in the number of modes, as clear from 4(b) and (c) when the plate is vibrating in the second and third mode of vibration. From Fig. 4(b), the effect of foundation parameter is found to increase the frequency parameter Ω, however the rate of increase gets reduced to more than half of the first mode for both the boundary conditions. In case of third mode, this rate of increase further decreases and becomes nearly half of the second mode as is evident from Fig. 4(c). The results show that presence of an elastic foundation increases the frequency parameter in all the cases.

Table V(a) and V(b) show the effect of taper parameter α on frequency parameter Ω for two different value of damping parameter d_k=0.0 and 0.01 respectively, for the fixed value of foundation parameter E_f =0.0, 0.01 and β=0.0, 0.4. Fig. 5(a) provides the graphs of frequency parameter Ω verses taper parameter α for the first mode of vibration. It is observed in the presence of damping parameter d_k *i.e.* for d_k =0.1 the frequency parameter Ω decreases continuously with increasing values of taper parameter α for both the boundary condition, whatever be the value of other plate parameters. But in the absence of damping parameter d_k .i.e for d_k=0.0 it has been observed that the frequency parameter Ω decreases with increasing values of taper parameter α for C-C plates, whatever be the value of other parameters and it has also been observed that for C-SS plate, there is a continuously decrement in the value of frequency parameter Ω for fixed values of E_f =0.0, and β=0.0, 0.4. In case of E_f =0.01 and β=0.0 i.e homogeneous plate, there is continuously increment, however for non-homogeneous plate (β=0.4), there is a local minima occurs at 0.2. When the plate is vibrating in the second mode (Fig. 5(b)), the frequency parameter Ω is found to decrease with increasing value of α for both the boundary conditions in all the cases. The frequency parameter Ω is found to increase with the value of E_f (other parameter being fixed). However, the rate of increase of Ω is less when compared to that for the fundamental mode. It is also observed that frequency parameter Ω is found to decrease with the value of β for d_k =0.0 but in the presence of damping parameter .i.e for d_k =0.01, frequency parameter Ω is found to decrease with the value of β for E_f =0.1 and increase for E_f =0.0. As far as the behavior of the plate vibrating in the third mode (Fig. 5(c)) is concerned, it is the same as for the second mode with the difference that the rate of decrease of frequency parameter Ω with taper constant α is much higher when compared to the first two modes.

The normalized displacements for the two boundary conditions C-C and C-SS, considered in this paper are shown in Fig..6 and Fig..7 respectively. The plate thickness varies parabolically in X-direction and the plate is considered resting on elastic foundations E_f=0.02 with damping parameter d_k =0.01. Mode shapes for a rectangular plate i.e, a/b=0.25 have been computed and observed that the nodal

Table III (a) Values of frequency parameter Ω for different values of damping parameter d_k *.h=0.03, ν=0.3, m=1, β=0.0, a/b=0.25.*

		α=0.0 , E_f=0.0		α=0.0 , E_f=0.01		α=0.4 , E_f=0.0		α=0.4 , E_f=0.01	
	Mode	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**
	I	0.6816	0.4766	0.8933	0.7487	0.5597	0.4242	0.8331	0.7567
d_k=0.0	**II**	1.8643	1.515	1.9516	1.6213	1.5605	1.3074	1.6806	1.451
	III	3.6431	3.1448	3.6885	3.1974	3.075	2.6982	3.1383	2.7708
	I	0.6765	0.4692	0.8894	0.744	0.5515	0.4126	0.8276	0.7503
d_k=0.0025	**II**	1.8624	1.5128	1.9498	1.6192	1.5574	1.3036	1.6777	1.4476
	III	3.6421	3.1437	3.6876	3.1963	3.0735	2.6964	3.1367	2.769
	I	0.6609	0.4465	0.8776	0.7299	0.5261	0.3754	0.811	0.7307
d_k=0.005	**II**	1.8568	1.5059	1.9445	1.6127	1.5481	1.2921	1.6691	1.4372
	III	3.6392	3.1404	3.6848	3.193	3.0687	2.6908	3.132	2.7635
	I	0.6341	0.4058	0.8576	0.7057	0.4808	0.3033	0.7824	0.6967
d_k=0.0075	**II**	1.8474	1.4943	1.9355	1.6019	1.5326	1.2727	1.6547	1.4197
	III	3.6345	3.1349	3.68	3.1876	3.0607	2.6814	3.1242	2.7544
	I	0.5945	0.3406	0.8288	0.6703	0.4086	0.1529	0.7406	0.646
d_k=0.01	**II**	1.8342	1.4779	1.9229	1.5867	1.5105	1.2451	1.6342	1.3949
	III	3.6278	3.1271	3.6734	3.1799	3.0494	2.6683	3.1131	2.7416

Table III (b) Values of frequency parameter Ω for different values of damping parameter d_k ,$h=0.03$, $v=0.3$, $m=1$, $\beta=0.4$, $a/b=0.25$.

		α=0.0 , E_f=0.0		α=0.0 , E_f=0.01		α=0.4 , E_f=0.0		α=0.4 , E_f=0.01	
	Mode	**c-c**	**c-ss**	**c-c**	**c-ss**	**c-c**	**c-ss**	**c-c**	**c-ss**
	I	0.6823	0.4645	0.8595	0.6938	0.5564	0.4137	0.7857	0.6941
d_k=0.0	**II**	1.8652	1.5039	1.9373	1.5918	1.5557	1.2949	1.6534	1.4115
	III	3.644	3.1337	3.6815	3.177	3.0698	2.6849	3.1208	2.7434
	I	0.6788	0.4597	0.8568	0.6906	0.5511	0.4064	0.782	0.6898
d_k =0.0025	**II**	1.8639	1.5023	1.936	1.5903	1.5537	1.2925	1.6515	1.4093
	III	3.6434	3.1329	3.6808	3.1763	3.0688	2.6837	3.1198	2.7422
	I	0.6684	0.445	0.8486	0.681	0.5347	0.3834	0.7705	0.6765
d_k =0.005	**II**	1.8601	1.4976	1.9324	1.5859	1.5478	1.2852	1.6459	1.4026
	III	3.6414	3.1307	3.6789	3.1741	3.0657	2.6802	3.1168	2.7388
	I	0.6506	0.4193	0.8347	0.6645	0.5063	0.3418	0.751	0.6538
d_k =0.0075	**II**	1.8537	1.4898	1.9262	1.5785	1.5378	1.273	1.6366	1.3915
	III	3.6382	3.1269	3.6757	3.1703	3.0606	2.6743	3.1118	2.733
	I	0.6249	0.3804	0.8148	0.6408	0.4635	0.273	0.7229	0.6207
d_k =0.01	**II**	1.8447	1.4788	1.9176	1.5681	1.5238	1.2558	1.6234	1.3757
	III	3.6336	3.1216	3.6711	3.1651	3.0535	2.666	3.1048	2.7249

Table IV (a) Values of frequency parameter Ω for different values of Elastic foundation parameter E_f .$h=0.03$, $v=0.3$, $m=1$, $d_k=0.0$, $a/b=0.25$.

		α=0.0 , β=0.0		α=0.0 , β=0.4		α=0.4 , β=0.0		α=0.4 , β=0.4	
	Mode	**c-c**	**c-ss**	c-c	c-ss	**c-c**	**c-ss**	**c-c**	**c-ss**
	I	0.6816	0.4766	0.6823	0.4645	0.5597	0.4242	0.5564	0.4137
E_f=0.0	**II**	1.8643	1.515	1.8652	1.5039	1.5605	1.3074	1.5557	1.2949
	III	3.6431	3.1448	3.644	3.1337	3.075	2.6982	3.0698	2.6849
	I	0.7945	0.6275	0.776	0.5904	0.7098	0.6136	0.6808	0.5714
E_f=0.005	**II**	1.9084	1.5691	1.9015	1.5484	1.6216	1.3811	1.6053	1.3544
	III	3.6658	3.1712	3.6628	3.1554	3.1068	2.7347	3.0954	2.7143
	I	0.8933	0.7487	0.8595	0.6938	0.8331	0.7567	0.7857	0.6941
E_f=0.01	**II**	1.9516	1.6213	1.9373	1.5918	1.6806	1.451	1.6534	1.4115
	III	3.6885	3.1974	3.6815	3.177	3.1383	2.7708	3.1208	2.7434
	I	0.9822	0.8527	0.9356	0.7837	0.9403	0.8766	0.8782	0.7982
E_f=0.015	**II**	1.9939	1.6719	1.9723	1.634	1.7376	1.5179	1.7001	1.4664
	III	3.7111	3.2233	3.7001	3.1985	3.1694	2.8064	3.1461	2.7722
	I	1.0636	0.9454	1.006	0.8642	1.0364	0.9817	0.9618	0.8902
E_f=0.02	**II**	2.0352	1.7211	2.0068	1.6752	1.7928	1.5819	1.7456	1.5192
	III	3.7334	3.2491	3.7186	3.2198	3.2003	2.8416	3.1711	2.8007

Table IV(b) Values of frequency parameter Ω for different values of Elastic foundation parameter E_f .$h=0.03$, $v=0.3$, $m=1$, $d_k=0.01$, $a/b=0.25$.

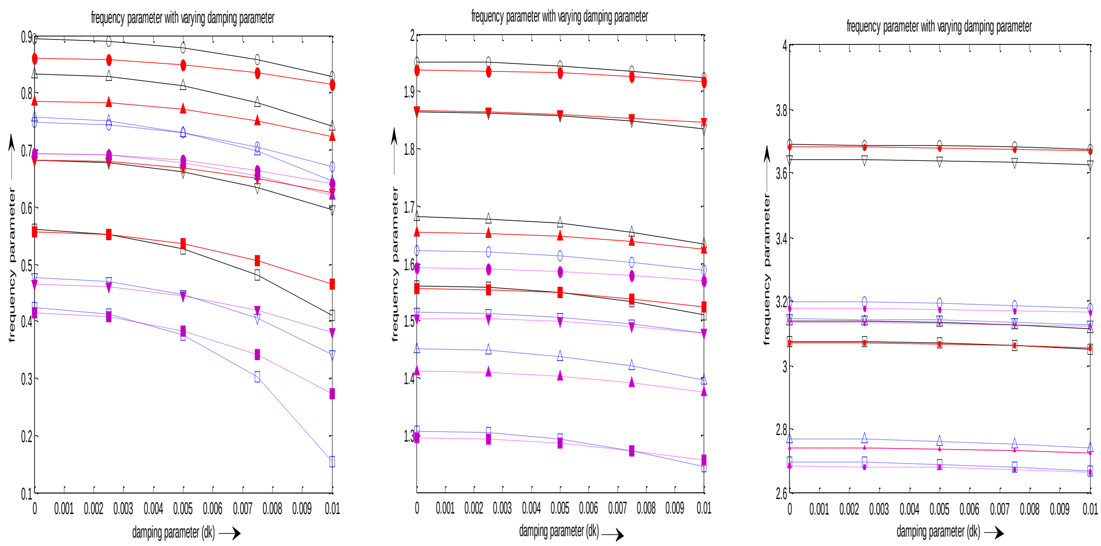
		α=0.0 , β=0.0		α=0.0 , β=0.4		α=0.4 , β=0.0		α=0.4 , β=0.4	
	Mode	**c-c**	**c-ss**	c-c	c-ss	**c-c**	**c-ss**	**c-c**	**c-ss**
	I	0.5945	0.3406	0.6249	0.3804	0.4086	0.1529	0.4635	0.273
E_f=0.0	**II**	1.8342	1.4779	1.8447	1.4788	1.5105	1.2451	1.5238	1.2558
	III	3.6278	3.1271	3.6336	3.1216	3.0494	2.6683	3.0535	2.666
	I	0.7212	0.5317	0.7261	0.527	0.5981	0.4696	0.6072	0.4795
E_f=0.005	**II**	1.8791	1.5333	1.8815	1.5241	1.5736	1.3221	1.5743	1.3171
	III	3.6507	3.1536	3.6524	3.1434	3.0814	2.7052	3.0792	2.6956
	I	0.8288	0.6703	0.8148	0.6408	0.7405	0.646	0.7229	0.6207
E_f=0.01	**II**	1.9229	1.5867	1.9176	1.5681	1.6342	1.3949	1.6234	1.3757
	III	3.6734	3.1799	3.6711	3.1651	3.1131	2.7416	3.1048	2.7249
	I	0.9239	0.7849	0.8947	0.7371	0.8595	0.7834	0.8225	0.7352
E_f=0.015	**II**	1.9658	1.6384	1.953	1.6109	1.6927	1.4641	1.6709	1.4319
	III	3.6961	3.206	3.6898	3.1866	3.1445	2.7775	3.1301	2.7539
	I	1.01	0.8847	0.968	0.8223	0.9638	0.8998	0.9112	0.8342
E_f=0.02	**II**	2.0078	1.6885	1.9878	1.6526	1.7493	1.5303	1.7172	1.486
	III	3.7185	3.2319	3.7083	3.208	3.1756	2.8131	3.1553	2.7826

Table V(a) Values of frequency parameter Ω for different values of taper parameter *α.*, *h=0.03, ν=0.3, m=1, d_k=0.0, a/b=0.25.*

		β=0.0 , E_f=0.0		β=0.0 , E_f=0.01		β=0.4 , E_f=0.0		β=0.4 , E_f=0.01	
	Mode	**c-c**	**c-ss**	c-c	c-ss	**c-c**	**c-ss**	**c-c**	**c-ss**
	I	0.6816	0.4766	0.8933	0.7487	0.6823	0.4645	0.8595	0.6938
α=0.0	**II**	1.8643	1.515	1.9516	1.6213	1.8652	1.5039	1.9373	1.5918
	III	3.6431	3.1448	3.68	3.1974	3.644	3.1337	3.6815	3.177
	I	0.6527	0.4647	0.877	0.7494	0.6524	0.453	0.8402	0.693
α=0.1	**II**	1.7931	1.4665	1.8866	1.5798	1.7926	1.455	1.8696	1.5483
	III	3.5106	3.0405	3.55	3.0967	3.51	3.0288	3.55	3.075
	I	0.623	0.4522	0.8613	0.7509	0.6216	0.4408	0.8213	0.6926
α=0.2	**II**	1.7191	1.4159	1.8199	1.5375	1.7172	1.4041	1.7999	1.5039
	III	3.3725	2.9318	3.4252	2.9925	3.3704	2.9196	3.4134	2.9691
	I	0.592	0.4387	0.8466	0.7532	0.5897	0.4278	0.8031	0.693
α=0.3	**II**	1.6418	1.3631	1.7513	1.4946	1.6385	1.3509	1.7279	1.4583
	III	3.2278	2.8181	3.2852	2.8841	3.2242	2.8054	3.2708	2.8589
	I	0.5597	0.4242	0.8331	0.7567	0.5564	0.4137	0.7857	0.6941
α=0.4	**II**	1.5605	1.3074	1.6806	1.451	1.5557	1.2949	1.6534	1.4115
	III	3.075	2.6982	3.1383	2.7708	3.0698	2.6849	3.1208	2.7434

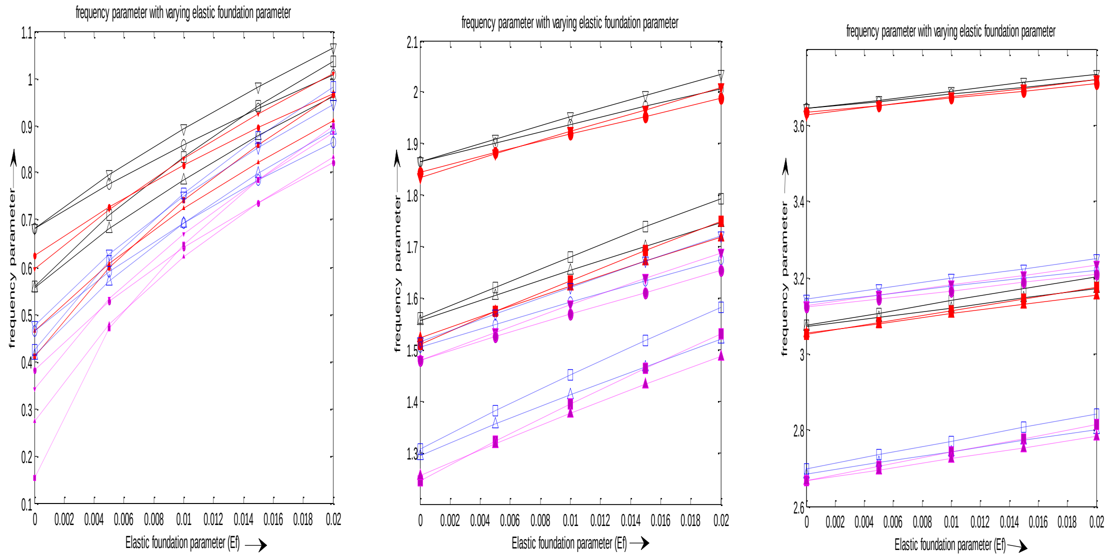
Table V(b) Values of frequency parameter Ω for different values of taper parameter *α,h=0.03, ν=0.3, m=1, d_k=0.01, a/b=0.25.*

		β=0.0 , E_f=0.0		β=0.0 , E_f=0.01		β=0.4 , E_f=0.0		β=0.4 , E_f=0.01	
	Mode	**c-c**	**c-ss**	c-c	c-ss	**c-c**	**c-ss**	**c-c**	**c-ss**
	I	0.5945	0.3406	0.8288	0.6703	0.6249	0.3804	0.8148	0.6408
α=0.0	**II**	1.8342	1.4779	1.9229	1.5867	1.8447	1.4788	1.9176	1.5681
	III	3.6278	3.1271	3.6734	3.1799	3.6336	3.1216	3.6711	3.1651
	I	0.5554	0.3106	0.8071	0.6649	0.5889	0.3597	0.792	0.636
α=0.1	**II**	1.7597	1.4252	1.855	1.5416	1.7702	1.4275	1.8481	1.5226
	III	3.4936	3.0208	3.5426	3.0774	3.4986	3.0156	3.5387	3.062
	I	0.5125	0.2738	0.7852	0.6592	0.5507	0.3359	0.7691	0.6311
α=0.2	**II**	1.6816	1.3695	1.7845	1.4949	1.6924	1.3737	1.7762	1.4756
	III	3.3534	2.9097	3.4064	2.9708	3.3578	2.905	3.4009	2.9548
	I	0.4645	0.2257	0.7631	0.6529	0.5093	0.3077	0.7461	0.626
α=0.3	**II**	1.599	1.3099	1.7112	1.4461	1.6106	1.3168	1.7015	1.4267
	III	3.2059	2.7927	3.2637	2.8592	3.2099	2.7889	3.2567	2.8428
	I	0.4086	0.1529	0.7405	0.646	0.4635	0.273	0.7229	0.6207
α=0.4	**II**	1.5105	1.2451	1.6342	1.3949	1.5238	1.2558	1.6234	1.3757
	III	3.0494	2.6683	3.1131	2.7416	3.0535	2.666	3.1048	2.7249



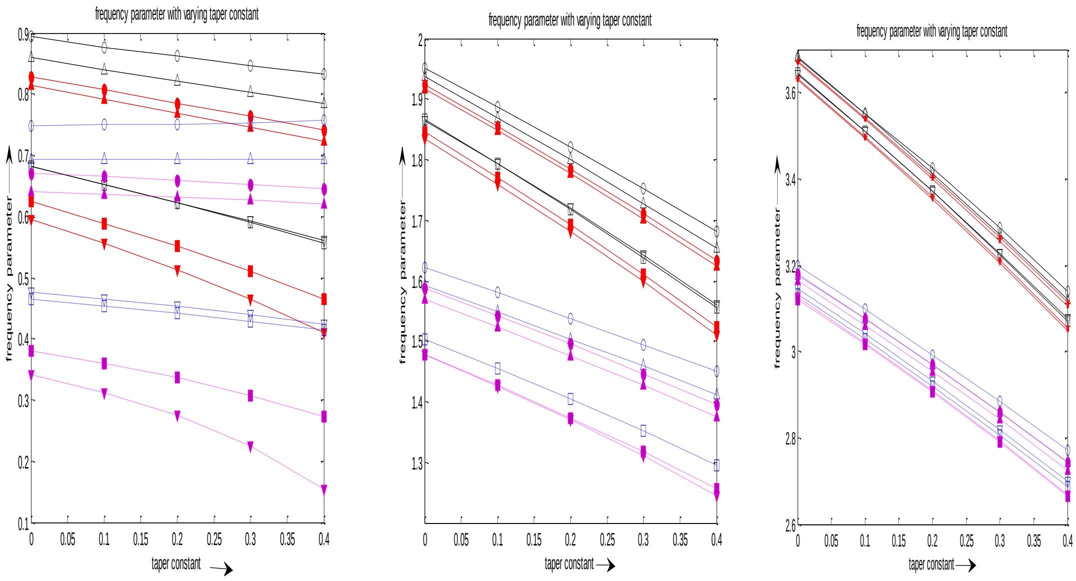
(a) (b) (c)

Fig. 3. Natural frequencies for C-C and C-SS plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=0.25 —, C-C; --, C-SS;▽, α=0.0, E_f=0.0, β=0.0; ▼, α=0.0 , E_f=0.0, β=0.04; ○, α=0.0 , E_f=0.01,β=0.0; ●, α=0.0 , E_f=0.01, β=0.01; □, α=0.4, E_f=0.0, β=0.0; ■, α=0.4 , E_f=0.0, β=0.01; △, α=0.4 , E_f=0.01, β=0.0; ▲ α=0.4 , E_f=0.01, β=0.01



(a) (b) (c)

Fig. 4. Natural frequencies for C-C and C-SS plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=0.25 . —, C-C; ----, C-SS;▽, α=0.0, d_k=0.0, β=0.0; ▼, α=0.0 , d_k =0.01, β=0.0; ○, α=0.0 , d_k =0.0,β=0.4; ●, α=0.0 , d_k =0.01, β=0.4; □, α=0.4, d_k =0.0, β=0.0; ■ , α=0.4 , d_k =0.1, β=0.0; △, α=0.4 , d_k =0.0, β=0.4; ▲ , α=0.4 , d_k =0.01, β=0.4



(a) (b) (c)

Fig. 5. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode, for a/b=0.25. —, C-C; ----, C-SS;▽, d_k=0.0, E_f=0.0, β=0.0; ▼, d_k=0.01 , E_f=0.0, β=0.0; ○, d_k =0.0 , E_f=0.01,β=0.0; ●, d_k =0.01 , E_f=0.01, β=0.0; □, d_k =0.0, E_f=0.0, β=0; ■, d_k =0.01 , E_f=0.0, β=0.4; Δ, d_k =0.0 , E_f=0.01, β=0.4; ▲ , d_k =0.01 , E_f=0.01, β=0.4;

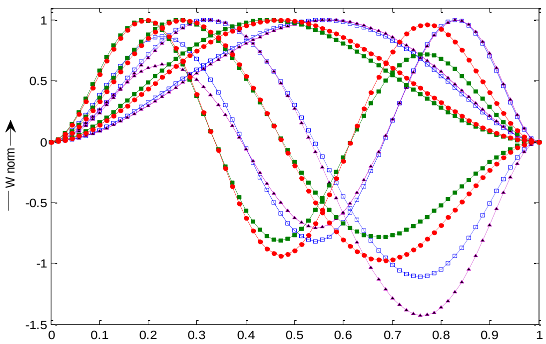


Fig. 6. Normalized displacements for C-C plate, for a/b=0.25; h=0.03,d_k=0.01, E_f=0.02;
—, First mode;——,Second mode;……,Third mode;
●, α=-0.5, β=-0.5; □, α=0.5, β=0.5; ▲, α=0.5, β=-0.5; ■, α=-0.5,β=0.5

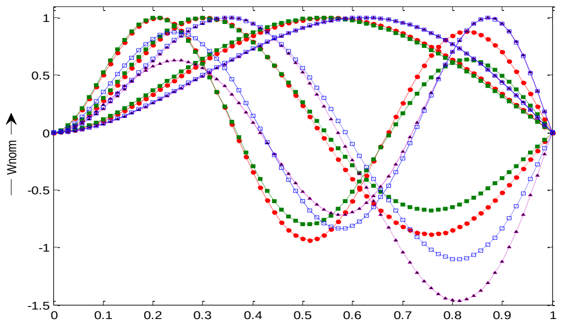


Fig. 7. Normalized displacements for C-SS plate, for a/b=0.25; h=0.03, d_k=0.01, E_f=0.02;
—, First mode; ——, Second mode; ……, Third mode;
●,α=-0.5,β=-0.5;□, α=0.5,β=0.5;▲,α=0.5,β=-0.5; ■, α=-0.5,β=0.5

lines are seen to shift towards the edge, *i.e.* $X=1$ as the edge $X=0$ increases in thickness for both the plates. No special change was seen in the pattern of nodal lines by taking different values of β and E_f. As normalized displacements were differing only at the third or fourth place after decimal for both the boundary conditions.

V. CONCLUSION

The results of present study are computed using MATLAB within the permissible range of parameters up to the desired accuracy (10^{-8}), which validates the actual phenomenon of vibrational problem. Variation in thickness, elastic foundation, damping parameter and non- homogeneity parameter are of great interest since it provides reasonable approximation to linear vibrations. One of the major causes of plate failures in industrial machines like turbine blades is from undamped/damped vibration, which results in high cyclic fatigue. Determination of vibration frequencies is of utmost importance for assessment of failure life. The results of present study suggests that the external damping (e.g. friction damping or lacquer damping) may easily be determined corresponding to inherent damping of plate in underlying situation. Thus the present study may be helpful in designing of plates which requires an accurate determination of their natural frequencies and mode shapes.

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