Dynamics analysis of a novel limited-DOF parallel manipulator with two planar limbs

Canguo Zhang, Yi Lu, Jianming Liang, Mingchao Geng

Abstract—It is significant to develop a limited-DOF parallel manipulator (PM) with high rigidity. However, the existing limited-DOF PMs include so many spherical joints which has less capability of pulling force bearing, less rotation range and lower precision under alternately heavy loads. A novel 5-DOF PM with two planar limbs is proposed and its dynamics are analysed systematically. A 3-dimension simulation mechanism of the proposed manipulator is constructed and its structure characteristics is analysed. The kinematics formulae for solving the displacement, velocity, acceleration of the platform, the active legs are established. An analytic example is given for solving the dynamics of the proposed manipulator and the analytic solved results are verified by the simulation mechanism. It provide the theoretical and technical foundations for its manufacturing, control and application.

Index Terms—dynamics; limited-DOF; parallel manipulator; planar limbs

1. INTRODUCTION

Currently, various limited-DOF PMs are attracting much attention due to their fewer active legs, large workspace, simpler structure, easy control and simple kinematic solutions [1-2]. Various limited-DOF parallel manipulators (PMs) have been applied in fields of rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recovery and training of human neck or waist and micro-Nano operation of bio-medicine [3-4]. In the aspects, Xie et al. [3] synthesized a class of limited-DOF PMs with several spherical joints(S). He and Gao [4] synthesized a class of 4-DOF PMs with 4 limbs, several S. S has the following disadvantages due to its structure: (1) the drag load capability is lower; (2) the rotation range is limited; (3) precision is lowered under alternately heavy loads. For this reason, The PMs with planar limbs have attracted many attentions because the planar limb only include revolute joints R and prismatic joint P. Wu and Gosselin [5] designed a PM with 3 planar limbs which are formed by a four-bar linkage. Lu et al. [6] proposed a novel 6-DOF PM with three planar limbs. In the aspects of dynamics of PMs, Using Newton–Euler methods, Dasgupta B et al. [7] studied dynamic models of the Stewart platform manipulator. Gallardo et al. [8] derived dynamic models of a modular spatial hyper-redundant manipulator by screw theory. Based on the principle of virtual work, Lu and Li [9] solved the dynamics of a platform manipulator with three planer limbs. Using the Lagrange methods, Mendes et al. [10] Li et al. [11] derived dynamic models of the limited platform manipulator. Lu and Hu [12] derived unified and simple velocity and acceleration for some limited-DOF PMs with linear active legs.

Up to now, no effort towards the dynamics analysis of the limited-DOF PMs with planar limbs is found. For this reason, this paper proposed a novel 5-DOF parallel manipulator with two planar limbs. Its structure characteristics, kinematics and dynamics are studied systematically.

II. PROTOTYPE OF NOVEL 5-DOF PM AND ITS STRUCTURAL CHARACTERISTICS

A 5-DOF PM with 2 planar limbs includes a moving platform m, a fixed base B, 2 vertical rods, 2 identical planar limbs Q i (i = 1, 2) and a SPR (spherical joint S-active prismatic joint P-revolute joint R) type active leg, see Fig 1(a). Here, m is a regular triangle with 3 vertices b i (i = 1, 2, 3), 3 sides b i = b; B is a regular triangle, 3 sides L = L, and a central point O; see Fig 1(b). Each of Q i includes 1 upper beam g b, 1 lower beam g B and 2 linear active rods r b. Each of r b is composed 1 linear actuator, 1 cylinder q b and 1 piston rod p b. In each of Q i, the middle of G b connects with B by a horizontal revolute joint R b at B; the one end of vertical rod connects with m by a vertical revolute joint R b at b, the other end of the vertical rod connects with the middle of g b by a revolute joint R b; the two ends of r b connect with the two ends of g b; and G b by revolute joints R b; g b; G b; and 2 r b form a closed planar mechanism Q i. This PM is named as the 5-DOF PM with 2Q i for distinguishing other kinds of PM with different planar limbs.

Let ⊥, || be perpendicular, parallel, and collinear constraints respectively. Let {m} be a coordinate frame o-xyz fixed on m at o, (B) be a coordinate frame O-XYZ fixed on B at O. The PM includes the following geometric conditions: z ⊥ m, y | ob 2, x|| By | OB 2, R b || B, R b ⊥ δ b, R b ⊥ δ b, R b || z, g b[m, G b][{b i, g b, r i}], b i,b 2 = g b, B; b 2 = G b, ob e = e, OB = E. Comparing with the existing limited-DOF PMs, the proposed 5-DOF PM with 2Q i possess the merits as follows:
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(1) Each of planar limbs \( Q \) only includes revolute joints \( R \) and prismatic joint \( P \), therefore, it is simple in structure and is easy manufacturing.

(2) Since all \( R \) in each of \( 2Q \) are parallel mutually, each of \( r_i \) in \( Q \) is only subjected a linear force along its axis. Thus, the hydraulic translatational actuator can be used for increasing a capability of large load bearing. In addition, a bending moment and a rotational torque between the piston rod and the cylinder can be avoided.

(3) In each of planar limbs \( Q, R \) has higher precision than \( S \) under large cyclic loading because backlash of \( R \) can be eliminated more easily than that of \( S \). The workspace can be increased due to \( R \) having larger rotation range than \( S \) before interference.

III. DISPLACEMENT OF 5-DOF PM

The derivation of displacement formulae of the proposed PM is a prerequisite for solving velocity, acceleration and statics of the PM. The coordinates of \( b_i \) of \( m \) in \( \{ m \} \) and \( B_i \) of \( B \) in \( \{ B \} \) are expressed as follows:

\[
B_i = \begin{bmatrix}
\pm q \\
-1 \\
0
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
0 \\
E \\
0
\end{bmatrix}, \quad b_i^n = \begin{bmatrix}
\pm q \\
e/2 \\
0
\end{bmatrix}
\]

\[
b_2^n = \begin{bmatrix}
0 \\
e\sqrt{3} \\
E
\end{bmatrix}, \quad q = \sqrt{\frac{3}{3}} \quad e = \sqrt{\frac{3}{3}} \quad L
\]

Here \( E \) is the distance from \( B_i \) to \( O \), \( e \) is the distance from \( b_i \) to \( o, i = 1, 3 \). As \( i = 1, \pm \) is +; as \( i = 3, \pm \) is -. This condition is also available for Equations (3), (4) and (7).

Let \( X_o, Y_o, Z_o \) be the position components of \( m \) at \( o \) in \( \{ B \} \). Let \( \varphi \) be one of 3 Euler angles \( (\alpha, \beta, \gamma) \). Set \( \sin \varphi, c_\varphi = \cos \varphi \), \( b_i \) of \( m \) in \( \{ B \} \) are expressed as follows:

\[
b_i = R_i^m = b_i^n + o
\]

(2)

Figure 1 A 3D model of the PM with 2Q and its kinetostatics model

\[
o = \begin{bmatrix}
X_o \\
Y_o \\
Z_o
\end{bmatrix}, \quad R_m^B = \begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3
\end{bmatrix} \begin{bmatrix}
c_{\alpha}c_{\beta}c_{\gamma} - s_{\alpha}s_{\gamma} & -c_{\alpha}s_{\beta} & c_{\alpha}c_{\gamma} \\
c_{\alpha}s_{\beta} & c_{\alpha}c_{\beta} & -s_{\alpha}
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix}
\]

Here \( R_m^B \) is a rotation matrix from \( \{ m \} \) to \( \{ B \} \) in order ZYZ (about \( Z_o \) by \( \alpha \), \( Y \) by \( \beta \), \( Z \) by \( \gamma \));

\[
x_{1,2,3}, x_o, y_o, z_o, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3, \omega_1, \omega_2, \omega_3, \phi_1, \phi_2, \phi_3, \theta_1, \theta_2, \theta_3 \] are nine orientation parameters of \( \{ m \} \).

The coordinates of \( b_i \) in \( \{ B \} \) are expressed based on Equations (1) and (2) as follows:

\[
b_i = \frac{1}{2} \begin{bmatrix}
\pm q x_i - e y_i + 2X_o \\
\pm q x_i - e y_i + 2X_o
\end{bmatrix}, \quad b_2 = \begin{bmatrix}
e y_i + X_o \\
\pm q x_i - e y_i + 2X_o + 2Z_o
\end{bmatrix}
\]

Let \( r_i (i = 1, 2, 3) \) be the vector from \( B \) to \( b_i \); \( e_i (i = 1, 2, 3) \) be the vector from \( o \) to \( b_i \). They are derived from Equations (1) and (3) as:

\[
r_1 = \frac{1}{2} \begin{bmatrix}
\pm(q x_i - q E) - e y_i + 2X_o \\
\pm q x_i - e y_i + 2Y_o + E
\end{bmatrix}, \quad r_2 = \begin{bmatrix}
e y_i + X_o \\
\pm q x_i - e y_i + 2Z_o
\end{bmatrix}
\]

\[
e_1 = \frac{e}{2} \begin{bmatrix}
\pm q x_i - y_i \\
\pm q x_m - y_m
\end{bmatrix}, \quad e_2 = e \begin{bmatrix}
\pm q x_i - y_i \\
\pm q x_m - y_m
\end{bmatrix}
\]

Let \( n_{10} \) and \( n_i \) be the vector of \( G_i \) and its unit vector. Based on the geometric condition, there are \( n_{01} = [B_1B_20]B_3, n_{10}, n_i \), \( n_i \) can be derived by Equation (1) as follows:

\[
n_{01} = B_3 - B_1 = \frac{E}{2} \begin{bmatrix}
3 \\
0
\end{bmatrix}, \quad n_{02} = B_3 - B_1 = \begin{bmatrix}
qE \\
0
\end{bmatrix}
\]
\[ n_i = \frac{n_i}{n_1} \quad (i=1,2) \]  
(5)

Let \( u_0 \) and \( u_i \) be the vector and the unit vector of the upper beam \( g_1 \). It is known that both \( u_0 \) and \( r_i \) locate in the same plane \( Q \), and let \( F \) be the vector which is perpendicular to \( Q \). Based on the geometric condition, \( u_0 \), \( u_i \) can be derived as follows.

\[ F = n_i \times r_i, \quad \mu_k = \pm n_i \times F, \quad n_i = \begin{bmatrix} z_i & z_m & z_\nu \end{bmatrix}^T \]

\[ \mu_k = \frac{\mu_{0k}}{\mu_0} \quad (i=1,2) \]  
(6)

Let \( B_i \) be the vector from \( b_i \) to \( b_j \), \( r_{ij} \) are expressed as follows:

\[ B_i B_j = B_i B_j = D, \quad b_i = b_i + b_{ij} + b_{ij} \]

\[ e_{ij} = b_i b_j + d \mu_k \]

\( r_{ij} \) \quad (i=1,2)

Let \( \delta_i \) be the unit vector of \( r_i \), let \( \delta_j \) be the unit vector of \( r_j \).

The formulae for solving \( r_i \), \( \rho_{ij} \), \( \delta_i \) and \( \delta_j \) are derived from Equations (4)-(7) as follows:

\[ \delta_i = \frac{r_i}{r_{ij}}, \quad \delta_j = \frac{r_j}{r_{ij}} \]

\( r_{ij} = r_i r_j \quad (i=1,2) \)

\( r_{ij}^2 = r_i^2 + r_j^2 + r_{ij}^2 \quad (i=1,2) \)

Thus, \( r_i \) is the vector of SPR active leg. \( r_{ij}(i=1,2;j=1,2) \) are the vectors of active leg in planer limbs.

IV. KINEMATICS ANALYSIS OF THE 5-DOF PM WITH 2Q, AND STATICS MODEL

A kinematics model of the planar limb \( Q \) are shown in Fig. 1(b). Let \( V, A, v, \omega, a, \alpha \), and \( \beta \) be the general forward velocity, the general forward acceleration, the linear velocity, the angular velocity, the linear acceleration and the angular accelerations of \( m \) at \( o \), respectively. They are expressed as:

\[ V = \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad A = \begin{bmatrix} a \\ \alpha \end{bmatrix}, \quad v = \begin{bmatrix} v_3 \\ v_z \end{bmatrix}, \quad \omega = \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}, \quad a = \begin{bmatrix} a_x \\ a_y \end{bmatrix}, \quad \alpha = \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix} \]

\( V \) and \( R_i \) be the scalar angular velocity of vertical rod about \( m \) at \( b_i \) and its unit vector. Let \( \omega_{ij} \) and \( R_{ij} \) be the scalar angular velocity of \( g_i \) about vertical rod at \( b_i \) and its unit vector and there are \( R_{ij} \perp R_{ij}, R_{ij} \perp R_{ij} \). They can be expressed as follows:

\[ R_i = n_i, \quad R_{ij} = \frac{R_i \times \delta_i}{|R_i \times \delta_i|}, \quad R_{ij} = R_{ij}, \]

\( R_{ij} = n_i, \quad R_{ij} = \mu_i, \quad v_{ij} = v + \omega \times e_i \)

\( v_{ij} = v + \omega \times e_i \)

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\( v_{ij} = v + \omega \times e_i \)

\( v_{ij} = v + \omega \times e_i \)
Here, \( J \) is a 6×6 Jacobian matrix of the 5–DOF PM with 2 planar limbs.

### B. Acceleration of the PM and statics model

The establishment of acceleration model of the proposed PM is a prerequisite to establish dynamics model of the proposed PM. Let \( a_{qi} \) be the input scalar acceleration along \( r_{qi} \). By differentiating Eq. (17) with respect to time, the acceleration matrix of the active legs equation is derived as:

\[
a_{rij} = J A + V^T HV
\]

(18)

Here, \( H \) is a \( 6 \times 6 \) Hessian matrix of the 5-DOF PM with 2Q.

Let \( F_{ri} \) be the active force which is applied on \( r_{ri} \), \( F_{ri} \) (i = 1, 2; j = 1, 2) be the active force which is applied on \( r_{jp} \). Let \( (F, T) \) be workload wrench which is applied on moving platform \( m \) at \( o \). When neglected mass and inertia moment of moving Links, based on the principle of virtual work, the statics formula of the 5–DOF PM with 2 planer limbs is derived as follow:

\[
F^T_r V_r + [F^T_r \ T_s^T] V = 0
\]

\[
F_r = [F_{r1}, F_{r2}, F_{r3}]^T
\]

\[
V_r = [V_{r11}, V_{r12}, V_{r21}, V_{r22}, V_{r31}, 0]^T
\]

(19)

\[
F_r = J_s \begin{bmatrix} F_s \\ T_s \end{bmatrix}, \ J_s = -(J^{-1})^T
\]

Here \( J \) is a 6×6 Jacobian matrix of the 5–DOF PM. \( J \) has been solved in the Equations (17). Given the workload that applied on the moving platform, the driving force \( F_{ri} \) (i = 1, 2; j = 1, 2) and \( F_{ri} \) along active legs can be solved using Equations (19).

### V. DYNAMICS OF 5-DOF PM

#### A. Kinematics of the 5-DOF PM

The kinematic models of the active leg \( r_{ij} \) in planar limbs and active leg \( r_{ij} \) in SPR limb, are shown in Fig. 2(a). The active leg \( r_{ij} \) in SPR limbs is composed of a piston rod \( p_{ij} \) and a cylinder \( q_{ij} \). Let \( l_{ij} \) be the distance from the mass center of \( q_{ij} \) to \( B_i \). Let \( l_{ij} \) be the distance from the mass center of \( p_{ij} \) to \( B_i \).

Let \( \pi \) be one of \( q_{ij}, p_{ij}, q_{ij}, p_{ij}, q_{ij}, p_{ij} \), \( G_{ij} \). Let \( V_\alpha, A_\alpha, v_\alpha, \omega_\alpha, a_\alpha \), \( \varepsilon_\alpha \), be the general velocity, the general acceleration, the linear velocity, the angular velocity, the linear acceleration and angular acceleration of \( \pi \) at its mass center, respectively. They are derived as follows:

\[
v_{p3} = v_{r3} + \omega_{r3} \times (r_{ij} - l_{pr}) \delta_{ij} = v_{r3} \delta_{ij} + (r_{ij} - l_{pr}) \omega_{r3} \times \delta_{ij}
\]

\[
J_{vpr3} = \delta_{3j} J_{vpr3} = -(r_{ij} - l_{pr}) \delta_{ij} J_{vpr3} V = J_{vpr3} V = J_{vpr3} V
\]

(20)

Differentiating Equation (20) with respect to time, it leads to:

\[
a_{pr3} = (v_{r3} + \omega_{r3} \times (r_{ij} - l_{pr}) \delta_{ij})'
\]

(21)

\[
V_{p3} = \begin{bmatrix} v_{pr3} \\ \omega_{pr3} \end{bmatrix}, A_{p3} = \begin{bmatrix} a_{pr3} \\ \varepsilon_{pr3} \end{bmatrix}
\]

(22)

Each of the linear legs is composed of a piston and a cylinder. The piston does not spin about the cylinder’s axis. So the angular velocity and angular acceleration of piston is equivalent to that of cylinder. This condition is also available for linear legs in planar limbs. Similarly, \( V_{qij}, A_{qij} \), are derived as follows:

\[
v_{qij} = v_{qij} + \omega_{qij} \times (r_{ij} - l_{ij}) \delta_{ij} = v_{qij} \delta_{ij} + (r_{ij} - l_{ij}) \omega_{qij} \times \delta_{ij}
\]

(24)

\[
V_{q3} = \begin{bmatrix} v_{qij} \\ \omega_{qij} \end{bmatrix}, A_{q3} = \begin{bmatrix} a_{qij} \\ \varepsilon_{qij} \end{bmatrix}
\]

(25)

In the planar limbs, Let \( l_{ij} \) be the distance from the mass center of \( p_{ij} \) to \( b_i \). The formulae for solving \( V_{p3} \) and \( A_{p3} \) have been derived as follows:

\[
v_{p3} = (v_{r3} + \omega_{r3} \times (r_{ij} - l_{pr}) \delta_{ij})'
\]

(26)

\[
V_{qij}, A_{qij} \) can be expressed from Equations (26) and (27) as follows:

\[
V_{p3} = \begin{bmatrix} v_{p3} \\ \omega_{p3} \end{bmatrix}, A_{p3} = \begin{bmatrix} a_{p3} \\ \varepsilon_{p3} \end{bmatrix}
\]

(28)

Let \( l_{qij} \) be the distance from the mass center of \( q_{ij} \) to \( B_i \). \( V_{qij} \) and \( A_{qij} \) are solved as follows:

\[
V_{qij} = (v_{qij} + \omega_{qij} \times (r_{ij} - l_{qij}) \delta_{ij})'
\]

(29)

\[
V_{qij}, A_{qij} \) can be expressed from Equations (29) and (30) as follows:
\[ V_{qij} = \begin{bmatrix} v_{qij} \\ \omega_{qij} \end{bmatrix}, A_{qij} = \begin{bmatrix} a_{qij} \\ \epsilon_{qij} \end{bmatrix} \] (31)

Figure 2 Kinematics model of active links (a) and dynamic model of the 5-DOF PM (b)

The mass center of upper beam \( g_i \) is coincident to the vertices \( b_i \) of the moving platform, so the linear velocity of upper beam is equivalent to that of the vertices \( b_i \), \( v_{gi} \), \( \omega_{gi} \) are represented and solved as follows:

\[ v_{gi} = v + \omega \times \epsilon_i = J_{vgi} V, \quad J_{vgi} = [E_{b3x3} - \hat{e}]_{3x6} \] (32)

\[ \omega_{gi} = \omega + \omega_{r4} R_{r4} + \omega_{r5} R_{r5} \] (33)

\[ V_{gi} = \begin{bmatrix} v_{gi} \\ \omega_{gi} \end{bmatrix}, A_{gi} = \begin{bmatrix} a_{gi} \\ \epsilon_{gi} \end{bmatrix} \] (34)

The mass center of \( G_i \) is in the same location with \( B_i \) so the linear velocity and linear acceleration of \( G_i \) are zero. \( V_{Gi}, A_{Gi} \) are solved as follows:

\[ \omega_{Gi} = \omega_{r1} R_{pr1}, V_{Gi} = J_{oGi} V \] (35)

\[ V_{Gi} = \begin{bmatrix} v_{Gi} \\ \omega_{Gi} \end{bmatrix}, A_{Gi} = \begin{bmatrix} a_{Gi} \\ \epsilon_{Gi} \end{bmatrix} \] (36)

\[ \omega_{Gi} = \omega_{r1} R_{pr1} \]

\[ V_{Gi} = J_{oGi} V \]

\[ A_{Gi} = \begin{bmatrix} 0 \\ \epsilon_{Gi} \end{bmatrix} \]

\[ \epsilon_{Gi} = \begin{bmatrix} \epsilon_{Gi} \end{bmatrix} \]

\[ G_{m} = m_{o} g \]

\[ F_{m} = -m_{a} \omega \times (I_{o} \omega) \]

\[ G_{qi} = m_{qi} g \]

\[ F_{qi} = -m_{qi} a_{qi} \omega_{qi} \times (I_{qi} \omega_{qi}) \] (37)

When ignoring the friction of all the joints in the 5-DOF PM, the dynamic workload wrench \((F, T)\) includes the static wrench \((F_1, T_1)\), the inertia wrench \((F_m, T_m)\) and the gravity \(G_m\) of the platform, the equivalent inertia wrench and the gravity of active legs, the equivalent inertia wrench and the gravity of lower beam \(G_i\) and upper beam \(g_i\). Thus, based on the principle of virtual principle work, a power equation is derived as follows:

\[ F^T V + \sum_{i=1}^{2} \left[ F_{pr3} + G_{pr3} \right]^T T_{pr3} V_{pr3} + \sum_{i=1}^{2} \left[ F_{pr1} + G_{pr1} \right]^T T_{pr1} V_{pr1} = 0 \] (38)

Based on the above established equation, the dynamic workload can be mapped into a part of \((F, T)\). When considering the friction of all the joints in the 5-dof PM, the damping loads of the joints should be transformed into a part of the dynamic workload wrench \((F, T)\) by counting the efficiency \(\eta\) of the PM. Thus, a formula is derived for solving the dynamic workload wrench applied on active links from Equations (19) and (38) as bellow:
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\[ F_r = -\frac{1}{\eta} (J^{-1})^T \begin{bmatrix} F \\ T \end{bmatrix} = -\frac{1}{\eta} (J^{-1})^T \ldots \]

\[
\begin{bmatrix}
F_{r1} & F_{r2} & F_{r3} \\
T_{r1} & T_{r2} & T_{r3}
\end{bmatrix} + \sum_{i=1}^{3} \sum_{j=1}^{3} \begin{bmatrix}
\frac{f_{ij} + g_{ij}}{T_{ij}} \\
\frac{f_{ij} + g_{ij}}{T_{ij}}
\end{bmatrix}
\]

VI. SOLVED EXAMPLES OF 5-DOF PM

Some given dimensions of the 5-DOF PM with the inside active legs and a force applied on the moving platform are listed in Table 1. The velocity, acceleration of active legs are given in Table 1. A program is compiled in Matlab based on relative derived equations. The statics and dynamics are solved using the compiled program in order to verify all derived equations. The displacement and Euler angles of themoving platform are solved, see Fig 3(a)-(b). The static active forces of active legs are solved, see Figure 3(c). The analytic solutions of the dynamic active forces of active legs are obtained when considering \( (F, T) \) and all inertia wrench and the gravity, see Figure 3(d). All analytical solutions are verified using a simulation mechanism constructed in an advanced CAD software.

Table 1. Given parameters of the mechanism, input velocity of PM and workloads applied on \( m \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>L/mm</td>
<td>240</td>
<td>( F_r/N )</td>
<td>(0.1,1000)</td>
</tr>
<tr>
<td>( m )/mm</td>
<td>120</td>
<td>( T_{r1}/(N\cdot m) )</td>
<td>(0.0,10)</td>
</tr>
<tr>
<td>( D )/mm</td>
<td>40</td>
<td>( L_1/kg\cdot mm^2 )</td>
<td>2000</td>
</tr>
<tr>
<td>( g/(m/s^2) )</td>
<td>9.8</td>
<td>( L_2/kg\cdot mm^2 )</td>
<td>500</td>
</tr>
<tr>
<td>( d/mm )</td>
<td>12.5</td>
<td>( L_3/kg\cdot mm^2 )</td>
<td>500</td>
</tr>
<tr>
<td>( m_1/kg )</td>
<td>10</td>
<td>( l_{11}, l_{12}/mm )</td>
<td>100</td>
</tr>
<tr>
<td>( m_2/kg )</td>
<td>5</td>
<td>( l_{21}, l_{22}/mm )</td>
<td>100</td>
</tr>
<tr>
<td>( m_3/kg )</td>
<td>5</td>
<td>( \nu_{12}/(m/s) )</td>
<td>2.5/2*t^2</td>
</tr>
<tr>
<td>( m_4/kg )</td>
<td>3</td>
<td>( \nu_{13}/(m/s) )</td>
<td>2.8/2*t^2</td>
</tr>
<tr>
<td>( m_5/kg )</td>
<td>3</td>
<td>( \nu_{23}/(m/s) )</td>
<td>3.3/2*t^2</td>
</tr>
<tr>
<td>( \nu_{11}/(m/s) )</td>
<td>1000</td>
<td>( \nu_{12}/(m/s) )</td>
<td>3.6/2*t^2</td>
</tr>
<tr>
<td>( \nu_{13}/(m/s) )</td>
<td>1000</td>
<td>( \nu_{23}/(m/s) )</td>
<td>0.8/2*t^2</td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

A novel 5-DOF parallel manipulator is proposed. The standard Jacobian matrix, the standard Hessian matrix, the dynamics formulae are established for the proposed 5-DOF PM. When given the workload wrench applied on the moving platform the coordinative dynamic active force applied on active legs can be solved by considering inertia wrench and mass of the PM. The analytic solutions of coordinative dynamics for the manipulator are verified by its simulation solutions. This novel 5-DOF PM has potential applications for forging operator, rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recover and training of human neck or waist, and micro–Nano operation of bio-medicine, and assembly cells. Theoretical formulae and results provide foundation for its structure optimization, control, manufacturing and applications. The stiffness of this PM should be studied in the future.

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