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Dynamics analysis of a novel limited-DOF parallel manipulator with two planar limbs

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***Abstract*—It is significant to develop a limited-DOF parallel manipulator (PM) with high rigidity. However, the existing limited-DOF PMs include so many spherical joint which has less capability of pulling force bearing, less rotation range and lower precision under alternately heavy loads. A novel 5-DOF PM with two planar limbs is proposed and its dynamics are analysed systematically. A 3-dimension simulation mechanism of the proposed manipulator is constructed and its structure characteristics is analysed. The kinematics formulae for solving the displacement, velocity, acceleration of the platform, the active legs are established. An analytic example is given for solving the dynamics of the proposed manipulator and the analytic solved results are verified by the simulation mechanism. It provide the theoretical and technical foundations for its manufacturing, control and application.**

***Index Terms*—dynamics; limited-DOF; parallel manipulator; planar limbs**

I. INTRODUCTION

Currently, various limited-DOF PMs are attracting much attention due to their fewer active legs, large workspace, simpler structure, easy control and simple kinematic solutions [1-2].Various limited-DOF parallel manipulators (PMs) have been applied in fields of rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recover and training of human neck or waist and micro–Nano operation of bio-medicine [3–4]. In the aspects, Xie et al. [3] synthesized a class of limited-DOF PMs with several spherical joints(S). He and Gao [4] synthesized a class of 4-DOF PMs with 4 limbs, several S. S has the following disadvantages due to its structure: (1) the drag load capability is lower; (2) the rotation range is limited; (3) precision is lowed under alternately heavy loads. For this reason, The PMs with planar limbs have attracted many attentions because the planar limb only include revolute joints R and prismatic joint P. Wu and Gosselin [5] designed a PM with 3 planar limbs which are formed by a four-bar linkage. Lu et al. [6] proposed a novel 6-DOF PM with three planar limbs. In the aspects of dynamics of PMs, Using Newton–Euler methods, Dasgupta B et al. [7] studied dynamic models of the Stewart platform manipulator. Gallardo et al. [8] derived dynamic models of a modular spatial hyper-redundant manipulator by screw theory. Based on the principle of virtual work, Lu and Li [9] solved the dynamics of a platform manipulator with three planner limbs. Using the Lagrange methods, Mendes et al. [10] Li et al. [11] derived dynamic models of the limited platform manipulator. Lu and Hu [12] derived unified and simple velocity and acceleration for some limited-DOF PMs with linear active legs.

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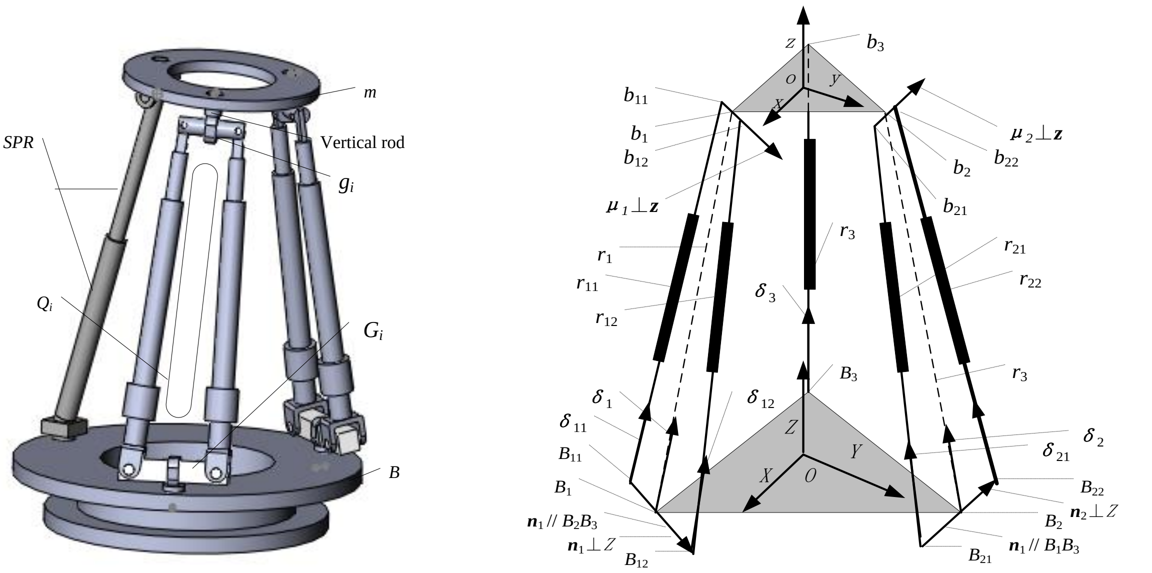
Up to now, no effort towards the dynamics analysis of the limited-DOF PMs with planar limbs is found. For this reason, this paper proposed a novel 5-DOF parallel manipulator with two planar limbs. Its structure characteristics, kinematics and dynamics are studied systematically.

II. PROTOTYPE OF NOVEL 5-DOF PM AND ITS STRUCTURAL CHARACTERISTICS

A 5-DOF PM with 2 planar limbs includes a moving platform m, a fixed base B, 2 vertical rods, 2 identical planar limbs Q_i (i = 1, 2) and a SPR (spherical joint S-active prismatic joint P-revolute joint R) type active leg, see Fig 1(a). Here, m is a regular triangle with 3 vertices b_i (i = 1, 2, 3), 3 sides $l_i = l$, and a central point o; B is a regular triangle, 3 sides $L_i = L$, and a central point O, see Fig 1(b).Each of Q_i includes 1 upper beam g_i, 1 lower beam G_i and 2 linear active rods r_{ij}. Each of r_{ij} is composed 1 linear actuator, 1 cylinder q_{ij} and 1 piston rod p_{ij}. In each of Q_i, the middle of G_i connects with B by a horizontal revolute joint R^{i1} at B_i; the one end of vertical rod connects with m by a vertical revolute joint R^{i4} at b_i, the other end of the vertical rod connects with the middle of g_i by a revolute joint R^{i5}; the two ends of r_{ij} connect with the two ends of g_i and G_i by revolute joints R^{i2}. g_i, G_i, and 2 r_{ij} form a closed planar mechanism Q_i. This PM is named as the 5-DOF PM with 2Q_i for distinguishing other kinds of PM with different planar limbs.

Let ⊥, ||, | be perpendicular, parallel, and collinear constraints respectively. Let {m} be a coordinate frame o-xyz fixed on m at o, {B} be a coordinate frame O-XYZ fixed on B at O. The PM includes the following geometric conditions: $z \perp m$, $y \mid ob_2$, $x \parallel b_1b_3$, $Z \perp B$, $Y \mid OB_2$, $\boldsymbol{R}^{i1} \parallel B$, $\boldsymbol{R}^{i2} \perp \boldsymbol{\delta}_i$, $\boldsymbol{R}^{i2} \perp \boldsymbol{\delta}_{ij}$, $\boldsymbol{R}^{i4} \perp \boldsymbol{R}^{i5}$, $\boldsymbol{R}^{i4} \parallel z$, $g_i \parallel m$, $G_i \parallel B$, (g_i, G_i, r_i, r_{ij}) being in Q_i, $b_{i1}b_{i2} = g_i$, $B_{i1}B_{i2} = G_i$, $ob_i = e$, $OB_i = E$. Comparing with the existing limited-DOF PMs, the proposed 5-DoF PM with 2Q_i possess the merits as follows:



（a） 3D model of the PM with $2Q_i$ （b） kinematics model of the PM with $2Q_i$

Figure 1 A 3D model of the PM with $2Q_i$ and its kinetostatics model

(1) Each of planar limbs Q_i only includes revolute joints R and prismatic joint P, therefore, it is simple in structure and is easy manufacturing.

(2) Since all R in each of $2Q_i$ are parallel mutually, each of r_{ij} in Q_i is only subjected a linear force along its axis. Thus, the hydraulic translational actuator can be used for increasing a capability of large load bearing. In addition, a bending moment and a rotational torque between the piston rod and the cylinder can be avoided.

(3) In each of planar limbs Q_i, R has higher precision than S under large cyclic loading because backlash of R can be eliminated more easily than that of S. The workspace can be increased due to R having larger rotation range than S before interference.

III. DISPLACEMENT OF 5-DOF PM

The derivation of displacement formulae of the proposed PM is a prerequisite for solving velocity, acceleration and statics of the PM. The coordinates of b_i of m in $\{m\}$ and B_i of B in $\{B\}$ are expressed as follows:

$$\boldsymbol{B}_i=\frac{E}{2}\begin{bmatrix}\pm q\\-1\\0\end{bmatrix},\boldsymbol{B}_2=\begin{bmatrix}0\\E\\0\end{bmatrix},\boldsymbol{b}_i^m=\frac{e}{2}\begin{bmatrix}\pm q\\-1\\0\end{bmatrix}$$

$$\boldsymbol{b}_2^m=\begin{bmatrix}0\\e\\0\end{bmatrix},\quad q=\sqrt{3},e=\frac{\sqrt{3}}{3}l,E=\frac{\sqrt{3}}{3}L \tag{1}$$

Here E is the distance from B_i to O, e is the distance from b_i to o, i = 1, 3. As i = 1, ± is +; as i = 3, ± is −. This condition is also available for Equations (3), (4) and (7).

Let X_o, Y_o, Z_o be the position components of m at o in $\{B\}$. Let φ be one of 3 Euler angles (α, β, γ). Set $s_\varphi = \sin\varphi$, $c_\varphi = \cos\varphi$, b_i of m in $\{B\}$ are expressed as follows:

$$\boldsymbol{b}_i=\boldsymbol{R}_m^B\boldsymbol{b}_i^m+\boldsymbol{o} \tag{2}$$

$$\boldsymbol{o}=\begin{bmatrix}X_o\\Y_o\\Z_o\end{bmatrix},\ \boldsymbol{R}_m^B=\begin{bmatrix}x_l & y_l & z_l\\x_m & y_m & z_m\\x_n & y_n & z_n\end{bmatrix}$$

$$=\begin{bmatrix}c_\alpha c_\beta c_\gamma-s_\alpha s_\gamma & -c_\alpha c_\beta s_\gamma-s_\alpha c_\gamma & c_\alpha s_\beta\\s_\alpha c_\beta c_\gamma+c_\alpha s_\gamma & s_\alpha c_\beta s_\gamma+c_\alpha c_\gamma & s_\alpha s_\beta\\-s_\alpha c_\gamma & s_\alpha s_\gamma & c_\beta\end{bmatrix}$$

Here $\boldsymbol{R}_m^B$ is a rotation matrix from $\{m\}$ to $\{B\}$ in order ZYZ (about Z_1 by α, Y by β, Z_2 by γ); x_l, x_m, x_n, y_l, y_m, y_n, z_l, z_m, z_n are nine orientation parameters of $\{m\}$.

The coordinates of b_i in $\{B\}$ are expressed based on Equations (1) and (2) as follows:

$$\boldsymbol{b}_i=\frac{1}{2}\begin{bmatrix}\pm qex_l-ey_l+2X_o\\\pm qex_m-ey_m+2Y_o\\\pm qex_n-ey_n+2Z_o\end{bmatrix},\ \boldsymbol{b}_2=\begin{bmatrix}ey_l+X_o\\ey_m+Y_o\\ey_n+Z_o\end{bmatrix} \tag{3}$$

Let $\boldsymbol{r}_i$ (i = 1, 2, 3) be the vector from B_i to b_i, $\boldsymbol{e}_i$ (i = 1, 2, 3) be the vector from o to b_i. They are derived from Equations (1) and (3) as:

$$\boldsymbol{r}_i=\frac{1}{2}\begin{bmatrix}\pm(qex_l-\text{q}E)-ey_l+2X_o\\\pm qex_m-ey_m+2Y_o+E\\\pm qex_n-ey_n+2Z_o\end{bmatrix}\ \boldsymbol{r}_2=\begin{bmatrix}ey_l+X_o\\ey_m+Y_o-E\\ey_n+Z_o\end{bmatrix}$$

$$\boldsymbol{e}_i=\frac{e}{2}\begin{bmatrix}\pm qx_l-y_l\\\pm qx_m-y_m\\\pm qx_n-y_n\end{bmatrix}\ \boldsymbol{e}_2=e\begin{bmatrix}y_l\\y_m\\y_n\end{bmatrix} \tag{4}$$

Let $\boldsymbol{n}_{0i}$ and $\boldsymbol{n}_i$ be the vector of G_i and its unit vector. Based on the geometric condition, there are $\boldsymbol{n}_{01} \| B_2B_3$, $\boldsymbol{n}_{02} \| B_1B_3$, $\boldsymbol{n}_{0i}$, $\boldsymbol{n}_i$ can be derived by Equation (1) as follows:

$$\boldsymbol{n}_{01}=\boldsymbol{B}_2-\boldsymbol{B}_3=\frac{E}{2}\begin{bmatrix}q\\3\\0\end{bmatrix},\boldsymbol{n}_{02}=\boldsymbol{B}_1-\boldsymbol{B}_3=\begin{bmatrix}qE\\0\\0\end{bmatrix}$$

$$\boldsymbol{n}_i=\frac{\boldsymbol{n}_{0i}}{|\boldsymbol{n}_{0i}|}\quad(i=1,2) \tag{5}$$

Let $\boldsymbol{u}_{0i}$ and $\boldsymbol{u}_i$ be the vector and the unit vector of the upper beam g_i. It is known that both $\boldsymbol{u}_{0i}$ and $\boldsymbol{r}_i$ locate in the same plane Q_i and let $\boldsymbol{F}$ be the vector which is perpendicular to Q_i. Based on the geometric condition, $\boldsymbol{u}_{0i}$, $\boldsymbol{u}_i$ can be derived as follows:

$$\boldsymbol{F}=\boldsymbol{n}_{0i}\times\boldsymbol{r}_i,\ \boldsymbol{\mu}_{0i}=\pm\boldsymbol{n}_z\times\boldsymbol{F},\quad \boldsymbol{n}_z=\begin{bmatrix}z_l & z_m & z_n\end{bmatrix}^T$$

$$\boldsymbol{\mu}_i=\frac{\boldsymbol{\mu}_{0i}}{|\boldsymbol{\mu}_{0i}|}\quad(i=1,2) \tag{6}$$

Let $B_{i1}B_i = B_iB_{i2} = D$, $b_ib_{i1} = b_ib_{i2} = d$, $\boldsymbol{r}_{ij}$ be the vector from B_{ij} to b_{ij}. $\boldsymbol{r}_{ij}$ are expressed as follows:

$$B_{i1}B_i=B_iB_{i2}=D\boldsymbol{n}_i,\ \boldsymbol{e}_{i1}=\boldsymbol{b}_i\boldsymbol{b}_{i1}=d\boldsymbol{\mu}_i$$

$$\boldsymbol{e}_{i2}=\boldsymbol{b}_i\boldsymbol{b}_{i2}=-d\boldsymbol{\mu}_i$$

$$\begin{cases}\boldsymbol{r}_{i1}=\boldsymbol{B}_{i1}\boldsymbol{B}_i+\boldsymbol{B}_i\boldsymbol{b}_i+\boldsymbol{b}_i\boldsymbol{b}_{i1}\\ \boldsymbol{r}_{i2}=\boldsymbol{B}_i\boldsymbol{b}_i-\boldsymbol{B}_i\boldsymbol{B}_{i2}+\boldsymbol{b}_i\boldsymbol{b}_{i2}\end{cases}$$

$$\Rightarrow\begin{cases}\boldsymbol{r}_{i1}=\boldsymbol{r}_i+d\boldsymbol{\mu}_i-D\boldsymbol{n}_i\\ \boldsymbol{r}_{i2}=\boldsymbol{r}_i-d\boldsymbol{\mu}_i+D\boldsymbol{n}_i\end{cases}\quad(i=1,2) \tag{7}$$

Let $\boldsymbol{\delta}_i$ be the unit vector of r_i, let $\boldsymbol{\delta}_{ij}$ be the unit vector of r_{ij}. The formulae for solving $\boldsymbol{r}_i$, $\boldsymbol{r}_{ij}$, $\boldsymbol{\delta}_i$, and $\boldsymbol{\delta}_{ij}$ are derived from Equations (4)–(7) as follows:

$$\boldsymbol{\delta}_i=\frac{\boldsymbol{r}_i}{r_i},\ \boldsymbol{\delta}_{ij}=\frac{\boldsymbol{r}_{ij}}{r_{ij}} \tag{8}$$

$$r_i^2=r_{ix}^2+r_{iy}^2+r_{iz}^2,\ r_{ij}^2=r_{ijx}^2+r_{ijy}^2+r_{ijz}^2$$

Thus, $\boldsymbol{r}_3$ is the vector of *SPR* active leg. $\boldsymbol{r}_{ij}$(i=1,2,j=1,2) are the vectors of active leg in planer limbers.

IV. Kinematics Analysis of the 5-DoF PM with 2Q_I and Statics Model

A kinematics model of the planar limb Q_i are shown in Fig .1(b). Let $\boldsymbol{V}, \boldsymbol{A}, \boldsymbol{v}, \boldsymbol{\omega}, \boldsymbol{a}$, and $\boldsymbol{\varepsilon}$ be the general forward velocity, the general forward acceleration, the linear velocity, the angular velocity, the linear acceleration and the angular accelerations of m at o, respectively. They are expressed as:

$$\boldsymbol{V}=\begin{bmatrix}\boldsymbol{v}\\ \boldsymbol{\omega}\end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{\varepsilon}\end{bmatrix}$$

$$\boldsymbol{v}=\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix},\boldsymbol{\omega}=\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix},\boldsymbol{a}=\begin{bmatrix}a_x\\ a_y\\ a_z\end{bmatrix},\boldsymbol{\varepsilon}=\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{bmatrix} \tag{9}$$

Let $\boldsymbol{v}_{bi}$ be a velocity vector of m at b_i, $\boldsymbol{v}_{bij}$ be a velocity vector of the upper beam g_i at b_{ij}, $\boldsymbol{\omega}_{bi}$ be the angular velocity of g_i, v_{ri} be a scalar velocity along r_i, v_{rij} be the input scalar velocity along r_{ij}, $\boldsymbol{\omega}_{ri}$ be the angular velocity of r_i; $\boldsymbol{\omega}_{rij}$ be the angular velocity of r_{ij}, Let ω_{i1} and $\boldsymbol{R}_{i1}$ be a scalar angular velocity of the lower beam G_i about B at B_i and its unit vector; ω_{i2} and $\boldsymbol{R}_{i2}$ be a scalar angular velocity of r_i about G_i at B_i and its unit vector;ω_{i3} and $\boldsymbol{R}_{i3}$ be the scalar angular velocity of r_i about g_i at b_i and its unit vector and there is $\boldsymbol{R}_{i3}\parallel\boldsymbol{R}_{i2}$. Let ω_{i4} and $\boldsymbol{R}_{i4}$ be the scalar angular velocity of vertical rod about m at b_i and its unit vector. Let ω_{i5} and $\boldsymbol{R}_{i5}$ be the scalar angular velocity of g_i about vertical rod at b_i and its unit vector and there are $\boldsymbol{R}_{i3}\perp\boldsymbol{R}_{i4}$, $\boldsymbol{R}_{i3}\perp\boldsymbol{R}_{i5}$ .They can be expressed as follows:

$$\begin{gathered}\boldsymbol{R}_{i1}=\boldsymbol{n}_i,\boldsymbol{R}_{i2}=\frac{\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i}{|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|},\boldsymbol{R}_{i3}=\boldsymbol{R}_{i2},\\ \boldsymbol{R}_{i4}=\boldsymbol{n}_z,\boldsymbol{R}_{i5}=\boldsymbol{\mu}_i,\boldsymbol{v}_{bi}=\boldsymbol{v}+\boldsymbol{\omega}\times\boldsymbol{e}_i\\ \boldsymbol{v}_{bij}=\boldsymbol{v}_{bi}+\boldsymbol{\omega}_{bi}\times\boldsymbol{e}_{ij}=\boldsymbol{v}_{rij}+\boldsymbol{\omega}_{rij}\times\boldsymbol{r}_{ij}\\ \boldsymbol{\omega}_{bi}=\boldsymbol{\omega}+\omega_{i4}\boldsymbol{R}_{i4}+\varphi_{i5}\boldsymbol{R}_{i5}=\boldsymbol{\omega}_{ri}+\omega_{i3}\boldsymbol{R}_{i3}\\ \boldsymbol{\omega}_{ri}=\omega_{i1}\boldsymbol{R}_{i1}+\omega_{i2}\boldsymbol{R}_{i2}\\ v_{ri}=\boldsymbol{v}_{bi}\cdot\boldsymbol{\delta}_i,v_{rij}=\boldsymbol{v}_{bij}\cdot\boldsymbol{\delta}_{ij}(i=1,2;j=1,2)\end{gathered} \tag{10}$$

A. General input velocity V_{rij} and angular velocity ω_{rij}.

v_{rjj} (i=1, 2, j=1, 2) and $\boldsymbol{V}_{rij}$ be the input velocity along r_{jj} and the general velocity input of the planer limbs. Let $\boldsymbol{\omega}_{rij}$ be the angular velocity of r_{ij}. The formulae for solving $\boldsymbol{\omega}_{rij}$ and v_{rjj} can be derived as follows:

$$\boldsymbol{\omega}_{rij}=\boldsymbol{J}_{\omega ij}\boldsymbol{V}\quad(i=1,2;j=1,2) \tag{11}$$

$$\begin{aligned}v_{rij}&=\boldsymbol{v}_{bij}\cdot\boldsymbol{\delta}_{ij}=\left(\boldsymbol{v}_{bi}+\boldsymbol{\omega}_{bi}\times\boldsymbol{e}_{ij}\right)\cdot\boldsymbol{\delta}_{ij}\\ &=(\boldsymbol{v}+\boldsymbol{\omega}\times\boldsymbol{e}_i)\cdot\boldsymbol{\delta}_{ij}+(\boldsymbol{J}_{\omega bi}\boldsymbol{V}\times\boldsymbol{e}_{ij})\cdot\boldsymbol{\delta}_{ij}=\boldsymbol{J}_{vij}\boldsymbol{V}\end{aligned} \tag{12}$$

$$\begin{gathered}\boldsymbol{V}_{rij}=\boldsymbol{J}_{rij}\boldsymbol{V},\ \boldsymbol{V}_{rij}=\begin{bmatrix}v_{r11} & v_{r12} & v_{r21} & v_{r22}\end{bmatrix}^T\\ \boldsymbol{J}_{rij}=\begin{bmatrix}\boldsymbol{J}_{v11} & \boldsymbol{J}_{v12} & \boldsymbol{J}_{v21} & \boldsymbol{J}_{v22}\end{bmatrix}^T\end{gathered} \tag{13}$$

Here, $\boldsymbol{J}_{\omega ij}$ is a 3×6 matrix; $\boldsymbol{J}_{vij}$ is a 1×6 matrix; $\boldsymbol{J}_{rij}$ is a 4×6 matrix.

In the *SPR* type active leg, let v_{r3} be the input velocity along r_3, Let $\boldsymbol{\omega}_{r3}$ be the angular velocity of r_3.The formulae for solving v_{r3} and $\boldsymbol{\omega}_{r3}$ have been derived in [10] as follows:

$$v_{r3}=(\boldsymbol{v}+\boldsymbol{\omega}\times\boldsymbol{e}_3)\cdot\boldsymbol{\delta}_3=\boldsymbol{J}_{r3}\boldsymbol{V},\ \boldsymbol{J}_{r3}=[\boldsymbol{\delta}_3^T\quad \boldsymbol{e}_3\times\boldsymbol{\delta}_3]_{1\times6} \tag{14}$$

$$\begin{gathered}\boldsymbol{\omega}_{r3}=\frac{1}{r_3}\left(\hat{\boldsymbol{\delta}}_3\boldsymbol{v}-\hat{\boldsymbol{\delta}}_3\hat{\boldsymbol{e}}_3\boldsymbol{\omega}+r_3\boldsymbol{\delta}_3\boldsymbol{\delta}_3^T\boldsymbol{\omega}\right)=\boldsymbol{J}_{\omega r3}\boldsymbol{V}\\ \boldsymbol{J}_{\omega r3}=\begin{bmatrix}\hat{\boldsymbol{\delta}}_3 & -\hat{\boldsymbol{\delta}}_3\hat{\boldsymbol{e}}_3+r_3\boldsymbol{\delta}_3\boldsymbol{\delta}_3^T\end{bmatrix}_{3\times6}\end{gathered} \tag{15}$$

Here, $\boldsymbol{J}_{r3}$ is a 1×6 matrix; $\boldsymbol{J}_{\omega3}$ is a 3×6 matrix.

In the 5–DOF PM there are constrained wrench ($\boldsymbol{F}_y$, $\boldsymbol{T}_c$) in the *SPR* type active leg limited the movement of the PM. The constrained wrench do not do any power during the movement of PM. Let $\boldsymbol{f}_3$ be the unit vector of $\boldsymbol{F}_y$, $\boldsymbol{d}_3$ is the vector of the arm from o to $\boldsymbol{F}_y$, thus the constrained wrench have been derived in [12].An auxiliary velocity equation is derived as:

$$0=\boldsymbol{J}_{vy}\boldsymbol{V}\quad \boldsymbol{J}_{vy}=\begin{bmatrix}\boldsymbol{f}_3^T & (\boldsymbol{d}_3\times\boldsymbol{f}_3)^T\end{bmatrix}_{1\times6} \tag{16}$$

Here, $\boldsymbol{J}_{vy}$ is a 1×6 matrix. By combining Eq. (13), (14) with Eq. (16), a general inverse velocity $\boldsymbol{v}_r$ can be derived as:

$$\begin{gathered}\boldsymbol{V}_r=\boldsymbol{J}\boldsymbol{V}\ ,\boldsymbol{V}_r=\begin{bmatrix}v_{r11} & v_{r12} & v_{r21} & v_{r22} & v_{r3} & 0\end{bmatrix}^T\\ \boldsymbol{J}=\begin{bmatrix}\boldsymbol{J}_{v11} & \boldsymbol{J}_{v12} & \boldsymbol{J}_{v21} & \boldsymbol{J}_{v22} & \boldsymbol{J}_{r3} & \boldsymbol{J}_{vy}\end{bmatrix}^T\end{gathered} \tag{17}$$

Here, $\boldsymbol{J}$ is a 6×6 Jacobian matrix of the 5–DOF PM with 2 planer limbers.

B. Acceleration of the PM and statics model

The establishment of acceleration model of the proposed PM is a prerequisite to establish dynamics model of the proposed PM. Let a_{rij} be the input scalar acceleration along r_{ij}. By differentiating Eq. (17) with respect to time, the acceleration matrix of the active legs equation is derived as:

$$\boldsymbol{a}_{rij} = \boldsymbol{J}\boldsymbol{A} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{V} \tag{18}$$

$$\boldsymbol{a}_{rij} = \begin{bmatrix} a_{11} & a_{12} & a_{21} & a_{22} & a_{r3} & 0 \end{bmatrix}^T$$

Here, $\boldsymbol{H}$ is a 6 × 6 × 6 Hessian matrix of the 5-DOF PM with $2Q_i$.

Let F_{r3} be the active force which is applied on r_3, F_{rij} (i = 1, 2; j = 1, 2) be the active force which is applied on r_{ij}. Let (F, T) be workload wrench which is applied on moving platform m at o. When neglected mass and inertia moment of moving Links, based on the principle of virtual work, the statics formula of the 5–DOF PM with 2 planer limbers is derived as follow:

$$\begin{gathered} \boldsymbol{F}_r^{\mathrm{T}}\boldsymbol{V}_r + [\boldsymbol{F}_s^{\mathrm{T}} \quad \boldsymbol{T}_s^{\mathrm{T}}]\boldsymbol{V} = 0 \\ \boldsymbol{F}_r = [F_{11} \quad F_{12} \quad F_{21} \quad F_{22} \quad F_{r3} \quad F_y]^{\mathrm{T}} \\ \boldsymbol{V}_r = [v_{r11} \quad v_{r12} \quad v_{r21} \quad v_{r22} \quad v_{r31} \quad 0]^{\mathrm{T}} \\ F_r = \boldsymbol{J}_s \begin{bmatrix} F_s \\ T_s \end{bmatrix}, \ \boldsymbol{J}_s = -\left(\boldsymbol{J}^{-1}\right)^{\mathrm{T}} \end{gathered} \tag{19}$$

Here $\boldsymbol{J}$ is a 6×6 Jacobian matrix of the 5–DOF PM. $\boldsymbol{J}$ has been solved in the Equations (17). Given the workload that applied on the moving platform, the driving force F_{rij} (i = 1, 2; j = 1, 2) and F_{r3} along active legs can be solved using Equations (19).

V. Dynamics of 5-DoF PM

A. Kinematics of the 5-DoF PM

The kinematics models of the active leg r_{ij} in planar limbs and active leg r_{r3} in *SPR* limb, are shown in Fig. 2(a).The active leg r_3 in *SPR* limbs is composed of a piston rod p_{r3} and a cylinder q_{r3}. Let l_{qr3} be the distance from the mass center of q_{r3} to B_3, Let l_{pr3} be the distance from the mass center of p_{r3} to b_3. Let π be one of q_{ij}, p_{ij}, q_{r3}, p_{r3}, g_i, G_i. Let $\boldsymbol{V}_\pi$, $\boldsymbol{A}_\pi$, $\boldsymbol{v}_\pi$, $\boldsymbol{\omega}_\pi$, $\boldsymbol{a}_\pi$, $\boldsymbol{\varepsilon}_\pi$ be the general velocity, the general acceleration, the linear velocity, the angular velocity, the linear acceleration and angular acceleration of π at its mass center, respectively. They are derived as follows:

$$\begin{gathered} \boldsymbol{v}_{pr3} = \boldsymbol{v}_{r3} + \boldsymbol{\omega}_{r3} \times (r_3 - l_{pr3})\boldsymbol{\delta}_3 = v_{r3}\boldsymbol{\delta}_3 + (r_3 - l_{pr3})\boldsymbol{\omega}_{r3} \times \boldsymbol{\delta}_3 \\ = \boldsymbol{\delta}_3\boldsymbol{J}_{r3}\boldsymbol{V} - (r_3 - l_{pr3})\hat{\boldsymbol{\delta}}_3\boldsymbol{J}_{\omega r3}\boldsymbol{V} = \boldsymbol{J}_{vpr3}\boldsymbol{V} \\ \boldsymbol{J}_{vpr3} = \boldsymbol{\delta}_3\boldsymbol{J}_{vr3} - (r_3 - l_{pr3})\hat{\boldsymbol{\delta}}_3\boldsymbol{J}_{\omega r3} \end{gathered} \tag{20}$$

Differentiating Equation (20) with respect to time, it leads to:

$$\boldsymbol{a}_{pr3} = (\boldsymbol{v}_{r3} + \boldsymbol{\omega}_{r3} \times (r_{r3} - l_{mr3})\boldsymbol{\delta}_3)' \tag{21}$$

$\boldsymbol{V}_{pr3}$, $\boldsymbol{A}_{pr3}$ are solved from Equations (20) and (21) as follows:

$$\boldsymbol{V}_{pr3} = \begin{bmatrix} \boldsymbol{v}_{pr3} \\ \boldsymbol{\omega}_{pr3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_{pr3} \\ \boldsymbol{\omega}_{r3} \end{bmatrix}, \boldsymbol{A}_{pr3} = \begin{bmatrix} \boldsymbol{a}_{pr3} \\ \boldsymbol{\varepsilon}_{pr3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{pr3} \\ \boldsymbol{\varepsilon}_{r3} \end{bmatrix} \tag{22}$$

Each of the linear legs is composed of a piston and a cylinder. The piston does not spin about the cylinder's axis. So the angular velocity and angular acceleration of piston is equivalent to that of cylinder. This condition is also available for linear legs in planner limbers. Similarly, $\boldsymbol{V}_{qr3}$, $\boldsymbol{A}_{qr3}$, are derived as follows:

$$\begin{gathered} \boldsymbol{v}_{qr3} = \boldsymbol{\omega}_{r3} \times l_{qr3}\boldsymbol{\delta}_3 = l_{qr3}\boldsymbol{\omega}_{r3} \times \boldsymbol{\delta}_3 \\ = -l_{qr3}\hat{\boldsymbol{\delta}}_3\boldsymbol{J}_{\omega r3}\boldsymbol{V} = \boldsymbol{J}_{vqr3}\boldsymbol{V} \\ \boldsymbol{J}_{vr3} = -l_{qr3}\hat{\boldsymbol{\delta}}_3\boldsymbol{J}_{\omega r3} \end{gathered} \tag{23}$$

Differentiating Equation (20) with respect to time, $\boldsymbol{a}_{qr3}$ are solved as follow:

$$\boldsymbol{a}_{qr3} = (l_{qr3}\boldsymbol{\omega}_{r3} \times \boldsymbol{\delta}_3)' \tag{24}$$

$\boldsymbol{V}_{pr3}$, $\boldsymbol{A}_{pr3}$ can be expressed from Equations (23) and (24) as follows:

$$\boldsymbol{V}_{qr3} = \begin{bmatrix} \boldsymbol{v}_{qr3} \\ \boldsymbol{\omega}_{qr3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_{qr3} \\ \boldsymbol{\omega}_{r3} \end{bmatrix}, \boldsymbol{A}_{qr3} = \begin{bmatrix} \boldsymbol{a}_{qr3} \\ \boldsymbol{\varepsilon}_{qr3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{qr3} \\ \boldsymbol{\varepsilon}_{r3} \end{bmatrix} \tag{25}$$

In the planar limbs, Let l_{pij} be the distance from the mass center of p_{ij} to b_{ij}, The formulae for solving $\boldsymbol{V}_{pij}$ and $\boldsymbol{A}_{pij}$ have been derived as follows:

$$\begin{gathered} \boldsymbol{v}_{pij} = \boldsymbol{v}_{rij} + \boldsymbol{\omega}_{rij} \times (r_{ij} - l_{pij})\boldsymbol{\delta}_{ij} = v_{rij}\boldsymbol{\delta}_{ij} + (r_{ij} - l_{pij})\boldsymbol{\omega}_{rij} \times \boldsymbol{\delta}_{ij} \cdots \\ = \boldsymbol{\delta}_{ij}\boldsymbol{J}_{vij}\boldsymbol{V} - (r_{ij} - l_{pij})\hat{\boldsymbol{\delta}}_{ij}\boldsymbol{J}_{\omega ij}\boldsymbol{V} = \boldsymbol{J}_{vpij}\boldsymbol{V} \end{gathered} \tag{26}$$

Differentiating Equation (26) with respect to time, $\boldsymbol{a}_{pij}$ are solved as follow:

$$\boldsymbol{a}_{pij} = (\boldsymbol{v}_{rij} + \boldsymbol{\omega}_{rij} \times (r_{ij} - l_{pij})\boldsymbol{\delta}_{ij})' \tag{27}$$

$\boldsymbol{V}_{pij}$, $\boldsymbol{A}_{pij}$ can be expressed from Equations (26) and (27) as follows:

$$\boldsymbol{V}_{pij} = \begin{bmatrix} \boldsymbol{v}_{pij} \\ \boldsymbol{\omega}_{pij} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_{pij} \\ \boldsymbol{\omega}_{rij} \end{bmatrix}, \ \boldsymbol{A}_{pij} = \begin{bmatrix} \boldsymbol{a}_{pij} \\ \boldsymbol{\varepsilon}_{pij} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{pij} \\ \boldsymbol{\varepsilon}_{rij} \end{bmatrix} \tag{28}$$

Let l_{qij} be the distance from the mass center of q_{ij} to B_{ij}. $\boldsymbol{V}_{pij}$ and $\boldsymbol{A}_{pij}$ are solved as follows:

$$\boldsymbol{v}_{qij} = \boldsymbol{\omega}_{rij} \times l_{qij}\boldsymbol{\delta}_{ij} = l_{qij}\boldsymbol{\omega}_{rij} \times \boldsymbol{\delta}_{ij} = -l_{qij}\hat{\boldsymbol{\delta}}_{ij}\boldsymbol{J}_{\omega ij}\boldsymbol{V} = \boldsymbol{J}_{vqij}\boldsymbol{V} \tag{29}$$

Differentiating Equation (29) with respect to time, $\boldsymbol{a}_{qij}$ are solved as follow:

$$\boldsymbol{a}_{qij} = (l_{qij}\boldsymbol{\omega}_{rij} \times \boldsymbol{\delta}_{ij})' \tag{30}$$

$\boldsymbol{V}_{qij}$, $\boldsymbol{A}_{qij}$ can be expressed from Equations (29) and (30) as follows:

$$\boldsymbol{V}_{qij}=\begin{bmatrix}\boldsymbol{v}_{qij}\\ \boldsymbol{\omega}_{qij}\end{bmatrix}=\begin{bmatrix}\boldsymbol{v}_{qij}\\ \boldsymbol{\omega}_{rij}\end{bmatrix},\boldsymbol{A}_{qij}=\begin{bmatrix}\boldsymbol{a}_{qij}\\ \boldsymbol{\varepsilon}_{qij}\end{bmatrix}=\begin{bmatrix}\boldsymbol{a}_{qij}\\ \boldsymbol{\varepsilon}_{rij}\end{bmatrix} \tag{31}$$

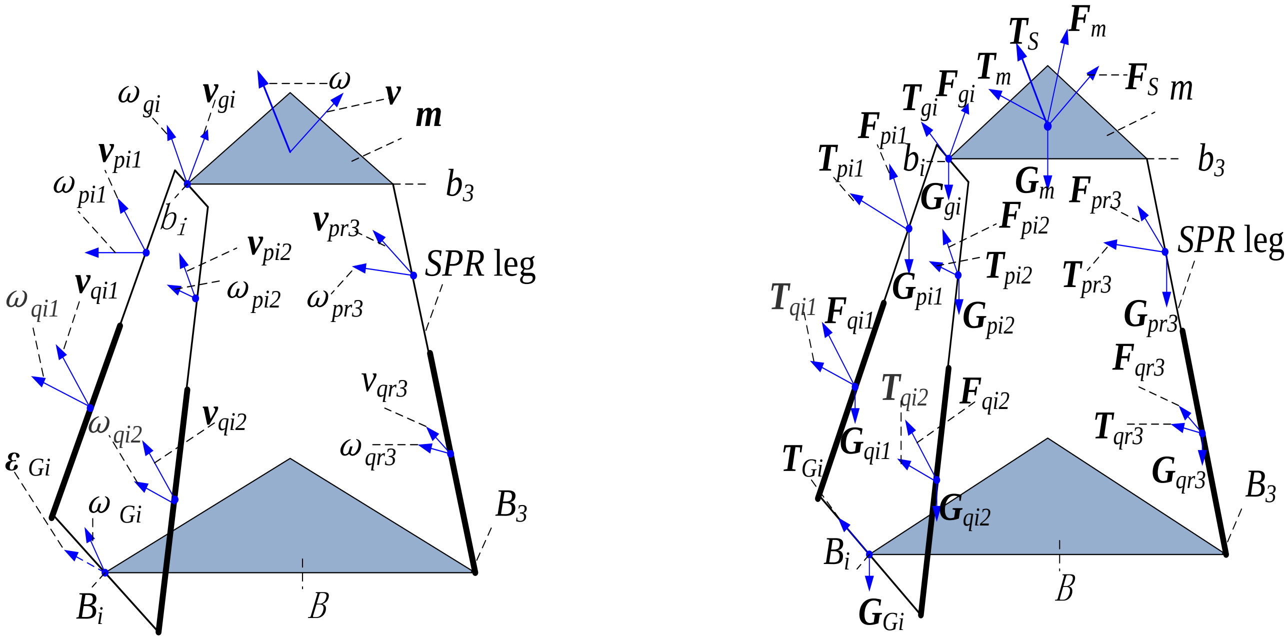


Figure 2 Kinematics model of active links (a) and dynamic model of the. 5-DOF PM (b)

The mass center of upper beam g_i is coincident to the vertices b_i of the moving platform, so the linear velocity of upper beam is equivalent to that of the vertices b_i. $\boldsymbol{v}_{gi}$, $\boldsymbol{\omega}_{gi}$ are represented and solved as follows:

$$\boldsymbol{v}_{gi}=\boldsymbol{v}_{bi}=\boldsymbol{v}+\boldsymbol{\omega}\times\boldsymbol{e}_i=\boldsymbol{J}_{vgi}\boldsymbol{V} \quad , \quad \boldsymbol{J}_{vgi}=\begin{bmatrix}\boldsymbol{E}_{3\times3} & -\hat{\boldsymbol{e}}_i\end{bmatrix}_{3\times6} \tag{32}$$

$$\boldsymbol{\omega}_{gi}=\boldsymbol{\omega}+\omega_{i4}\boldsymbol{R}_{i4}+\omega_{i5}\boldsymbol{R}_{i5} \tag{33}$$

$$\boldsymbol{V}_{gi}=\begin{bmatrix}\boldsymbol{v}_{gi}\\ \boldsymbol{\omega}_{gi}\end{bmatrix}=\begin{bmatrix}\boldsymbol{v}_{gi}\\ \boldsymbol{\omega}_{gi}\end{bmatrix},\boldsymbol{A}_{gi}=\begin{bmatrix}\boldsymbol{a}_{gi}\\ \boldsymbol{\varepsilon}_{gi}\end{bmatrix}=\begin{bmatrix}\boldsymbol{a}_{gi}\\ \boldsymbol{\varepsilon}_{gi}\end{bmatrix} \tag{34}$$

The mass center of $\boldsymbol{G}_i$ is in the same location with B_i .so the linear velocity and linear acceleration of $\boldsymbol{G}_i$ are zero. $\boldsymbol{V}_{Gi}$, $\boldsymbol{A}_{Gi}$ are solved as follows:

$$\boldsymbol{\omega}_{Gi}=\omega_{i1}\boldsymbol{R}_{i1}=\boldsymbol{R}_{i1}\boldsymbol{J}_{\omega i1}\boldsymbol{V}=\boldsymbol{J}_{\omega Gi}\boldsymbol{V} \tag{35}$$

$$\boldsymbol{V}_{Gi}=\begin{bmatrix}\boldsymbol{v}_{Gi}\\ \boldsymbol{\omega}_{Gi}\end{bmatrix}=\begin{bmatrix}\boldsymbol{0}_{1\times3}\\ \boldsymbol{\omega}_{Gi}\end{bmatrix},\quad \boldsymbol{A}_{Gi}=\begin{bmatrix}\boldsymbol{a}_{Gi}\\ \boldsymbol{\varepsilon}_{Gi}\end{bmatrix}=\begin{bmatrix}\boldsymbol{0}_{1\times3}\\ \boldsymbol{\varepsilon}_{Gi}\end{bmatrix} \tag{36}$$

B. Dynamics of the 5-DoF PM with two planner limbers

The dynamics models of the active rod r_{ij} in planar limbs and active rod r_3 in *SPR* limb, are shown in Fig. 2(b). Let ($\boldsymbol{F}_{qi}$, $\boldsymbol{T}_{qi}$) and $\boldsymbol{G}_{qi}$ be the inertia wrench and gravity of the qi (qi $=q_{ij}$, p_{ij},q_{r3}, p_{r3}, g_i, G_i). Respectively, Let m_{qi} and $\boldsymbol{I}_{qi}$ be the mass and the inertia moment tensor matrix of the qi at its mass center. Let ($\boldsymbol{F}_s$, $\boldsymbol{T}_s$) be the operating wrench exerted on m at o in {m }. Let m_o be the mass of the moving platform m ,($\boldsymbol{F}_m$, $\boldsymbol{T}_m$) and $\boldsymbol{G}_m$ be the inertia wrench and the gravity of the platform m. $\boldsymbol{I}_m$ be the mass and inertia moment tensor matrix of the moving platform m about point o; $\boldsymbol{g}$ be a gravity acceleration. These dynamic parameters can be expressed as follows:

$$\begin{cases}\boldsymbol{G}_m=m_o\,\boldsymbol{g}\\ \boldsymbol{F}_m=-m_o\,\boldsymbol{a},\boldsymbol{T}_m=-\boldsymbol{I}_o\boldsymbol{\varepsilon}-\boldsymbol{\omega}\times(\boldsymbol{I}_o\boldsymbol{\omega})\\ \boldsymbol{G}_{qi}=m_{qi}\,\boldsymbol{g}\\ \boldsymbol{F}_{qi}=-m_{qi}\,\boldsymbol{a}_{qi},\boldsymbol{T}_{qi}=-\boldsymbol{I}_{qi}\boldsymbol{\varepsilon}_{qi}-\boldsymbol{\omega}_{qi}\times(\boldsymbol{I}_{qi}\boldsymbol{\omega}_{qi})\end{cases} \tag{37}$$

When ignoring the friction of all the joints in the 5-DOF PM, the dynamic workload wrench ($\boldsymbol{F}$,$\boldsymbol{T}$) includes the statics wrench ($\boldsymbol{F}_s$, $\boldsymbol{T}_s$), the inertia wrench ($\boldsymbol{F}_m$, $\boldsymbol{T}_m$) and the gravity $\boldsymbol{G}_m$ of the platform, the equivalent inertia wrench and the gravity of active legs, the equivalent inertia wrench and the gravity of lower beam G_i and upper beam g_i . Thus, based on the principle of virtual work, a power equation is derived as follows:

$$\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{T}\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}+\begin{bmatrix}\boldsymbol{F}_m+\boldsymbol{F}_s+\boldsymbol{G}_m\\ \boldsymbol{T}_m+\boldsymbol{T}_s\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}+\sum_{i=1}^{2}\sum_{j=1}^{2}\begin{bmatrix}\boldsymbol{F}_{pij}+\boldsymbol{G}_{pij}\\ \boldsymbol{T}_{pij}\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}_{pij}+\cdots$$
$$\sum_{i=1}^{2}\sum_{j=1}^{2}\begin{bmatrix}\boldsymbol{F}_{qij}+\boldsymbol{G}_{qij}\\ \boldsymbol{T}_{qij}\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}_{qij}+\sum_{i=1}^{2}\begin{bmatrix}\boldsymbol{F}_{gi}+\boldsymbol{G}_{gi}\\ \boldsymbol{T}_{gi}\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}_{gi}+\sum_{i=1}^{2}\begin{bmatrix}\boldsymbol{F}_{Gi}+\boldsymbol{G}_{Gi}\\ \boldsymbol{T}_{Gi}\end{bmatrix}^{\mathrm{T}}\boldsymbol{V}_{Gi}$$
$$+\begin{bmatrix}\boldsymbol{F}_{pr3}+\boldsymbol{G}_{pr3}\\ \boldsymbol{T}_{pr3}\end{bmatrix}^{T}\boldsymbol{V}_{pr3}+\begin{bmatrix}\boldsymbol{F}_{qr3}+\boldsymbol{G}_{qr3}\\ \boldsymbol{T}_{qr3}\end{bmatrix}\boldsymbol{V}_{qr3}=0 \tag{38}$$

Based on the above established equation, the dynamic workload can be mapped into a part of ($\boldsymbol{F}$, $\boldsymbol{T}$). When considering the friction of all the joints in the 5-dof PM, the damping loads of the joints should be transformed into a part of the dynamic workload wrench ($\boldsymbol{F}$, $\boldsymbol{T}$) by counting the efficiency η of the PM. Thus, a formula is derived for solving the dynamic workload wrench applied on active links from Equations (19) and (38) as bellow:

$$\boldsymbol{F}_r = -\frac{1}{\eta}\left(\boldsymbol{J}^{-1}\right)^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{T}\end{bmatrix} = \frac{1}{\eta}\left(\boldsymbol{J}^{-1}\right)^{\mathrm{T}}\cdots$$

$$\left\{\begin{matrix}\begin{bmatrix}\boldsymbol{F}_m+\boldsymbol{F}_s+\boldsymbol{G}_m\\ \boldsymbol{T}_m+\boldsymbol{T}_s\end{bmatrix}+\sum_{i=1}^{2}\sum_{j=1}^{2}\boldsymbol{J}_{pij}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{pij}+\boldsymbol{G}_{pij}\\ \boldsymbol{T}_{pij}\end{bmatrix}+\sum_{i=1}^{2}\sum_{j=1}^{2}\boldsymbol{J}_{qij}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{qij}+\boldsymbol{G}_{qij}\\ \boldsymbol{T}_{qij}\end{bmatrix}+\\ \sum_{i=1}^{2}\boldsymbol{J}_{si}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{si}+\boldsymbol{G}_{si}\\ \boldsymbol{T}_{si}\end{bmatrix}+\sum_{i=1}^{2}\boldsymbol{J}_{ui}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{ui}+\boldsymbol{G}_{ui}\\ \boldsymbol{T}_{ui}\end{bmatrix}+\boldsymbol{J}_{pr3}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{pr3}+\boldsymbol{G}_{pr3}\\ \boldsymbol{T}_{pr3}\end{bmatrix}+\boldsymbol{J}_{qr3}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{F}_{qij}+\boldsymbol{G}_{qij}\\ \boldsymbol{T}_{qij}\end{bmatrix}\end{matrix}\right\}$$

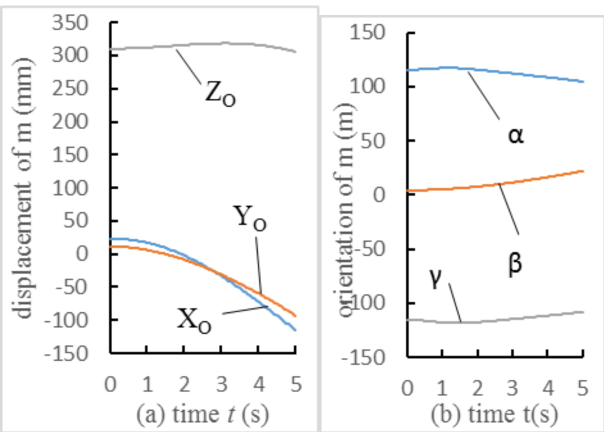
(39)

VI. SOLVED EXAMPLES OF 5-DOF PM

Some given dimensions of the 5-DOF PM with the inside active legs and a force applied on the moving platform are listed in Table 1. The velocity, acceleration of active legs are given in Table 1.A program is compiled in Matlab based on relative derived equations. The statics and dynamics are solved using the compiled program in order to verify all derived equations. The displacement and Euler angles of themoving platform are solved, see Fig 3(a)-(b). The static active forces of active legs are solved, see Figure 3(c). The analytic solutions of the dynamic active forces of active legs are obtained when considering ($\boldsymbol{F}_s$, $\boldsymbol{T}_s$) and all inertia wrench and the gravity, see Figure 3(d). All analytical solutions are verified using a simulation mechanism constructed in an advanced CAD software.

Table 1.Given parameters of the mechanism, input velocity of PM and workloads applied on *m*

parameter	value	parameter	value
L/mm	240	$\boldsymbol{F}_s$ / N	(0,0,1000)
l/mm	120	$\boldsymbol{T}_s$/ (N• m)	(0,0,10)
D/mm	40	$\mathbf{I}_o$/kg mm^2	2000
g/(m/s^2)	9.8	$\mathbf{I}_{gi}$/kg mm^2	500
d/mm	12.5	$\mathbf{I}_{Gi}$/kg mm^2	500
m_o/kg	10	l_{qr3} l_{qij}/ mm	100
m_{qij} m_{qr3} / kg	5	l_{pr3} l_{pij}/ mm	100
m_{pij} m_{pr3} / kg	5	v_{r11}(mm/s)	2.5/2*t^2
m_{gi} / kg	3	v_{r12}(mm/s)	2.8 /2*t^2
m_{Gi} / kg	3	v_{r21}(mm/s)	3.3 /2*t^2
$\mathbf{I}_{pij}$ $\mathbf{I}_{pr3}$ /kg mm^2	1000	v_{r22}(mm/s)	3.6/2*t^2
$\mathbf{I}_{qij}$ $\mathbf{I}_{qr3}$ /kg mm^2	1000	v_{r3}(mm/s)	0.8/2*t^2



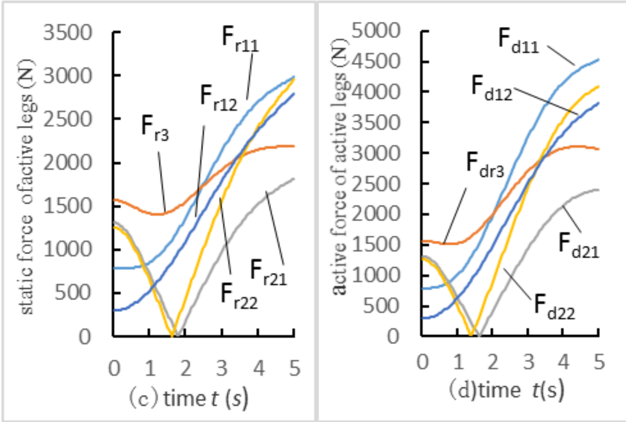


Figure 3 Analytic solutions of dynamics of 5-DOF PM

VII. CONCLUSIONS

A novel 5-DoF parallel manipulator is proposed. The standard Jacobian matrix, the standard Hessian matrix, the dynamics formulae are established for the proposed 5-DoF PM. When given the workload wrench applied on the moving platform the coordinative dynamic active force applied on active legs can be solved by considering inertia wrench and mass of the PM. The analytic solutions of coordinative dynamics for the manipulator are verified by its simulation solutions. This novel 5-DOF PM has potential applications for forging operator, rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recover and training of human neck or waist, and micro–Nano operation of bio-medicine, and assembly cells. Theoretical formulae and results provide foundation for its structure optimization, control, manufacturing and applications. The stiffness of this PM should be studied in the future.

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