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A Numerical Study of Mixed Convection in Square Lid-Driven with Internal Elliptic Body and Constant Flux Heat Source on the Bottom Wall

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***Abstract*— A numerical study of mixed convection in square lid- driven with internal elliptic body and constant flux heat source on the bottom wall. Mixed convection heat transfer in a two- dimensional square cavity of length (L), containing eccentric adiabatic elliptic body. The right wall heated and moving in the vertical direction while the left wall cold upward and downward lid- directions. Both the upper and lower wall adiabatic while the constant heat flux from partially heated bottom wall. Four different cases have been studied based on the location of the elliptic body. Case I, II, III and IV refer to the elliptic is located near the top, right, bottom and left walls of the cavity respectively. Results are presented for upward (+ y) and downward (- y) directions of the left lid in vertical axis and for different values of Richrdson numbers. This study is done for constant Prandtl number, *Pr* = 0.71. Fluid flow and thermal fields and the local Nusselt number are presented for all four case studies. The results shows that the behavior of temperature, streamlines and velocity profile is sensitive to the location of the elliptic body and to the direction of lid and this results explain also, that the maximum values of local Nusselt number occurs when the left lid moving downward (- y) refers to higher heat transfer.**

***Index Terms*— Laminar mixed convection; Lid-driven; Thermal and mass diffusion**

I. INTRODUCTION

Mixed convection in enclosures is encountered in many engineering systems such as cooling of electronic components, ventilation in building and fluid movement in solar energy collectors etc.

The combined temperature and concentration buoyancy force is called double-diffusion. Double-diffusion occurs in a very wide range of applications such as oceanography, astrophysics, geology, biology, and chemical processes, as well as in many engineering applications such as solarponds, natural asstoragetanks, crystal manufacturing, and metal solidifications processes. Al-Amiri et al. [1] investigated numerically steady mixed convection in a square lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion. The heat and mass transfer rates were examined using several operational dimensionless parameters, such as the Richardson number *Ri*, Lewis number *Le* and buoyancy ratio parameter *N*. The average Nusselt and Sherwood numbers are obtained at the bottom wall for some values of the parameters considered in this investigation. The results demonstrate the range where high heat and mass transfer rates can be attained for a given Richardson number. Sharif [2] studied numerically laminar mixed convective heat transfer in two-dimensional shallow rectangular driven cavities of aspect ratio 10. The top moving lid of the cavity is at a higher temperature than the bottom wall. The effects of inclination of the cavity on the flow and thermal fields are investigated. The stream line and isotherm plots and the variation of the local and average Nusselt numbers at the hot and cold walls are presented. Chen and Cheng [3] investigated numerically the Periodic behavior of the mixed convective flow in a rectangular cavity with a vibrating lid. The periodic flow patterns and heat transfer characteristics found are discussed with attention being focused on the interaction between the frequency of the lid velocity vibration and the frequency of the natural periodic flow. Khanafer et al. [4] investigated numerically unsteady laminar mixed convection heat transfer in a lid driven cavity. The forced convective flow inside the cavity is attained by a mechanically induced sliding lid, which is set to oscillate horizontally in a sinusoidal fashion. The natural convection effect is sustained by subjecting the bottom wall to a higher temperature than its top counterpart. In addition, the two vertical walls of the enclosure are kept insulated. Fluid flow and heat transfer characteristics are examined in the domain of the Reynolds number, Grashof number and the dimensionless lid oscillation frequency. Rahman et al. [5] investigated numerically the conjugate effect of joule heating and magnetic force, acting normal to the left vertical wall of an obstructed lid-driven cavity saturated with an electrically conducting fluid. The cavity is heated from the right vertical wall isothermally. Temperature of the left vertical wall, which has constant flow speed, is lower than that of the right vertical wall. Horizontal walls of the cavity are adiabatic. Results were presented in terms of streamlines, isotherms, average Nusselt number at the hotwall and average fluid temperature in the cavity for the magnetic parameter, *Ha* and Joule heating parameter *J*. The results showed that the obstacle has significant effects on the flow field at the pure mixed convection region and on the thermal field at the pure forced convection region. Rahman et al. [6] made a numerical work effects of Reynolds and Prandtl numbers on mixed convection in an obstructed vented cavity. Rahman et al.[7] Optimization of mixed convection in a lid-driven enclosour with a heat generating circular body. Rahman *et al.* [8] studied the effects of magnetic field on mixed convective flow in a horizontal channel with a bottom heated open enclosure with rectangular horizontal lower surface and vertical side surfaces using Galarkin weighted residual finite-element technique. M. M. Billah et al.[9] have performed the numerical simulation of magneto-hydrodynamics mixed convection in an open channel having a

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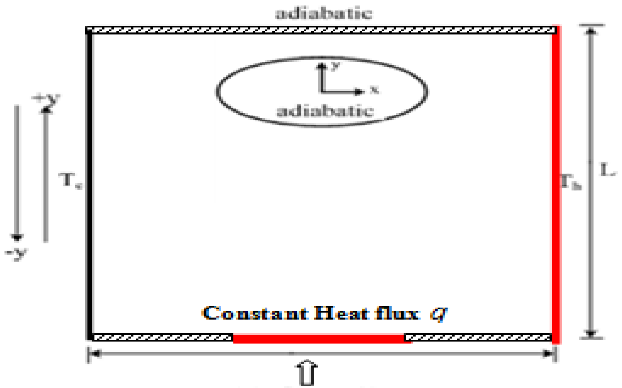
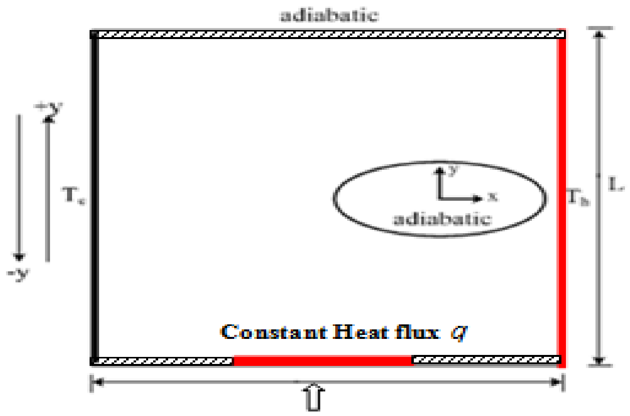
semi-circular heater. They observed that Hartmann numbers have a significant effect on average Nusselt number, average fluid temperature and drag force. M. A. Teamah et al. [10] analyzed the study numerical simulation of double-diffusive mixed convective flow in rectangular enclosure with insulated moving lid. A. Al-Amiri et al. [11] they found that numerical simulation of unsteady mixed convection in a driven cavity using an externally excited sliding lid. Sivasankaran et al. [12] numerically examined the mixed convection in a lid-driven cavity with sinusoidal boundary conditions at the sidewalls with magnetic field. They observed that the flow behavior and heat transfer rate inside the cavity strongly depend on the magnetic field. Saha et al. [13] have performed the numerical simulation of the mixed convection flow and heat transfer in a lid-driven cavity with wavy bottom surface. They observed that the heat transfer mechanisms and flow characteristics inside the cavity noticeably depend on the number of undulations, Grashof number and Reynolds number. Rahman *et al.* [14] analyzed mixed convection in a rectangular cavity with a heat conducting horizontal circular cylinder by using finite element method. T. Hayat et al. [15], presented the Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid. Abbasian et al. [16] investigated mixed convection flow in a lid-driven square cavity. Cu–water nanofluid was inside the cavity which its horizontal walls were adiabatic while its sidewalls were sinusoidal heating. Saha et al. [17] have performed the numerical effect of internal heat generation or absorption on MHD mixed convection flow in a lid driven cavity. They significant reduction in the average Nusselt number were produced as the strength of the applied magnetic field was increased. In addition, heat generation predicated to decrease the average Nusselt number whereas heat absorption increases it. Saha et al. [18], numerically investigated the hydro-magnetic mixed convection flow in a lid driven cavity with wavy bottom surface. They observed that the wavy lid-driven cavity can be considered as an effective heat transfer mechanism in presence of magnetic field at large wavy surface amplitude and low Richardson numbers. Guanghong et al. [19], have performed the numerical investigated the mixed convection in rectangular cavities at various aspect ratios with moving isothermal sidewalls and constant flux heat source on the bottom wall.



(a) Case (I)



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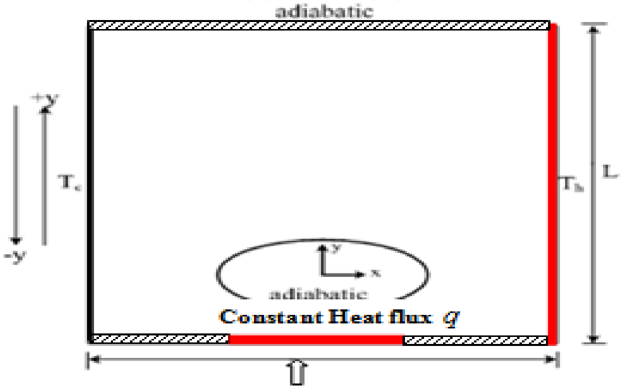

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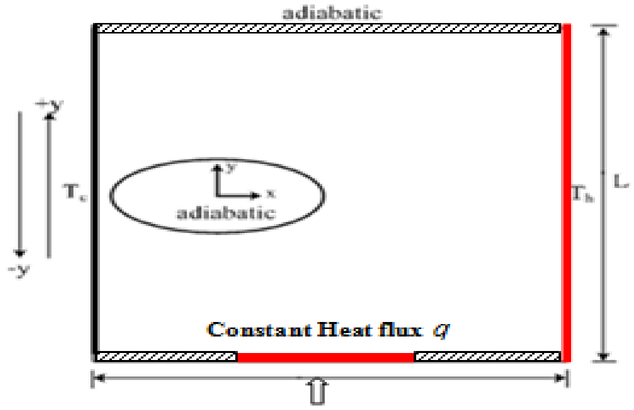


(d) Case (IV)

Fig.1: The Sketch of the Physical Model for Studied Cases

II. MATHEMATICAL FORMULATION

The governing equations describing the problem under consideration are based on the laws of mass, linear momentum and energy with buoyancy forces. The energy equation is written using the Boussinesq approximation. This means that all thermo-physical properties of the fluid at a reference temperature are taken constant except in the buoyancy term of the momentum equation. In addition, the radiation heat exchange is negligible in this study. The non-dimensional governing equations can be written as: (see [24]):

The equations are non-dimensionalzed by using the following dimensionless quantities

$$X=\frac{x}{L}, Y=\frac{y}{L}, U=\frac{u}{u_0}, V=\frac{v}{u_0}, P=\frac{p}{\rho u_0^2}, T=\frac{T-T_c}{T_h-T_c}$$

where $\upsilon = \frac{\mu}{\rho}$ is the reference kinematic viscosity and T is the non-dimensional temperature. After substitution of dimensionless variable we get the non- dimensional governing equations are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RiT$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right)$$

sThe upper wall: $\frac{\partial T}{\partial Y} = 0,\ U = V = 0$

Bottom wall: $\frac{\partial T}{\partial Y} = \begin{cases} 0,\ for\ 0 < X < (1-\varepsilon)/2 \\ -1\ for (1-\varepsilon)/2 \times (1+\varepsilon)/2 \\ 0\ for (1+\varepsilon)/2 < X < 1 \end{cases}$

Right wall: T = 0, U = 0, V = -1
Left wall: T = 0, U = 0, V = ± 1

We define the local heat transfer coefficient $h_x = \frac{q^n}{[T_s - T_c]}$ at a given point on the heat source surface where $T_s(x)$ is the local temperature on the surface. Accordingly the local Nusselt number and the average or over all Nusselt number can be obtained respectively as

$N_u = \frac{h_x W}{k} = \frac{1}{\theta_s(X)}$ and

$\overline{Nu} = \frac{\bar{h}W}{k} = \frac{1}{\varepsilon}\int_0^{\varepsilon} \frac{1}{\theta_s(X)} dX$

where $\theta_s(X)$ is the local dimensionless temperature. The Prandtl number $Pr = \frac{\gamma}{\alpha}$, the Reynolds number $Re = \frac{U_0 W}{\gamma}$, and the Grashof number $Gr = \frac{g\beta\Delta T W^3}{\gamma^2}$, where γ is the kinematic viscosity of the fluid, α is the thermal diffusivity of the fluid, the Richardson number *Ri*, which represents the relative magnitude of the free convection to the forced β is the thermal expansion coefficient of the fluid, and g is the gravitational acceleration. The ratio $\frac{Gr}{Re^2}$ is called convection and playes an important role in designating the convection flow.

III. NUMERICAL TECHNIQUE

The nonlinear governing partial differential equations, i.e., mass, momentum and energy equations are transferred into a system of integral equations by using the Galerkin weighted residual finite-element method. The integration involved in each term of these equations is performed with the aid Gauss quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations with the aid of Newton's method. Lastly, Triangular factorization method is applied for solving those linear equations. For numerical computation and post-processing, the software COMSOL Multiphyics is used.

4.1. Program Validation and Comparison with Previous Work

The computer code and Guanghong et al. [24] was modified and used for the computations in the study. The working fluid is chosen Prandtl number *Pr* = 0.733. The left and right wall is kept heated T_h and upper wall is kept at cold T_c. By performing simulation for natural convection in the lower wall is adiabatic. Streamlines and isotherms are plotted in Fig. 2. showing good agreement.

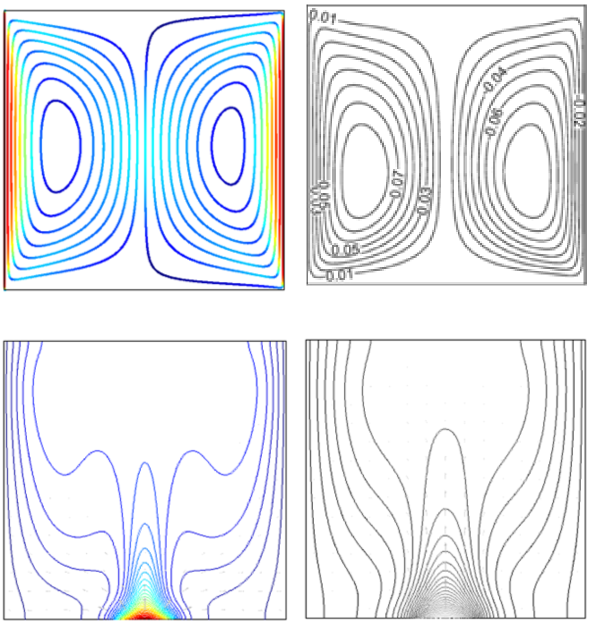


Fig. 2: (a) Present (b) Guanghong et al. obtained streamlines and Isotherms for *Re* = 10, *Gr* = 10 *Pr* = 0.733 and *Ri* = 0.01.

IV. RESULTS AND DISCUSSIONS

In this study, the eccentric elliptic body is located near the top, right, bottom and left wall of the square cavity where the left wall is moving in two opposite direction upward + y and downward – y directions. This work is performed for four values of Richardson number *Ri* = 0.01, 0.1, 1, 10. When the left lid moves in the (+ y) direction, flow and heat transfer results for four locations of the eccentric elliptic body is shown in figure (3)at the value of Richardson number (*Ri* = 0.01, 0.1, 1, 10). When Richardson number is small i. e., (*Ri* = 0.01, 0.1) which represents the forced convection case, the bouncy force effect is small, so for this case the natural convection contribution is small also. From noticed that both streamlines it can be seen that two counter rotating vortices are formed, one small at the left wall and the another is right wall. When the Richardson number (*Ri* = 1, 10) which represents the mixed convection case, it can be seen that three or more counter rotating vortices are formed and velocity increasing.

Fig. (4) Shown isotherms only for four different values of the mixed convection parameter, *Ri* = 0.01, 0.1, 1, 10. The solution is symmetric about the vertical midline due to the symmetry of the problem geometry and boundary conditions.

In each case the flow descends downwards along the moving sidewalls and turns horizontally to the central region after hitting the bottom wall.

Fig. 5 shows that the local Nusselt number when the lid moving upward i.e., + y direction. When *Ri* small the value of local Nusselt number increases until it reaches the lower down again when *Ri* increasing the value start at middle point until it reaches to this value is lower down.

Fig 6: shows that the variation of vertical velocity when the lid moving upward i.e. + y direction. The curves middle point of vertical direction, it can be seen the absolute value of maximum and minimum value of velocity increases with increasing the Richardson number.

Fig 7: Variation of the dimensionless temperature along the bottom wall of the square cavity with lid moving in the + y direction. When Ri small the value of local Nusselt number increases until it reaches the lower down again when Ri increasing the value start at middle point to the left wall until it reaches to the middle point right hot wall.

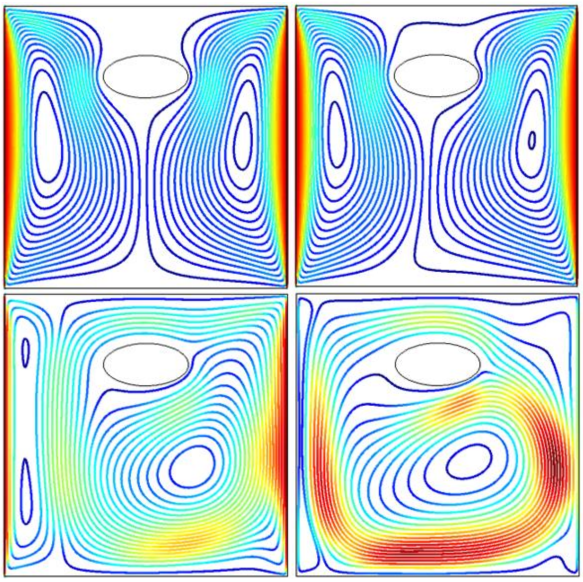
Again the work is performed for four values of Richardson number *Ri* = 0.01, 0.1, 1, 10. When the left lid moves in the (-y) direction, flow and heat transfer results for four locations of the eccentric elliptic body is shown in figure (8) at the value of Richardson number (*Ri* = 0.01, 0.1, 1, 10). When Richardson number is small i. e., (*Ri* = 0.01, 0.1) which represents the forced convection case, the bouncy force effect is small, so for this case the natural convection contribution is small also. From noticed that both streamlines it can be seen that two counter rotating vortices are formed, one small at the left wall and the another is right wall. When the Richardson number (*Ri* = 1, 10) which represents the mixed convection case, it can be seen that three or more counter rotating vortices are formed and velocity increasing.

Fig. (9) Shown isotherms only for four different values of the mixed convection parameter, *Ri* = 0.01, 0.1, 1, 10. The solution is symmetric about the vertical midline due to the symmetry of the problem geometry and boundary conditions. In each case the flow descends downwards along the moving sidewalls and turns horizontally to the central region after hitting the bottom wall.

Fig. 10 shows that the local Nusselt number when the lid moving downward i.e., - y direction. When *Ri* small the value of local Nusselt number increases until it reaches the lower down again when *Ri* increasing the value start at middle point until it reaches to this value is lower down.

Fig. 11 shows that the variation of vertical velocity when the lid moving downward i.e. - y direction. The curves middle point of vertical direction, it can be seen the absolute value of maximum and minimum value of velocity increases with increasing the Richardson number.

Fig. 12 Variation of the dimensionless temperature along the bottom wall of the square cavity with lid moving in the - y direction. When *Ri* small the value of local Nusselt number increases until it reaches the lower down again when *Ri* increasing the value start at middle point to the left wall until it reaches to the middle point right hot wall.



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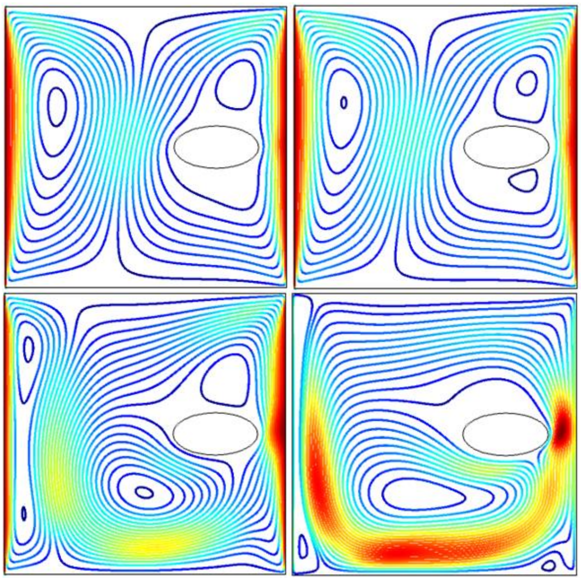

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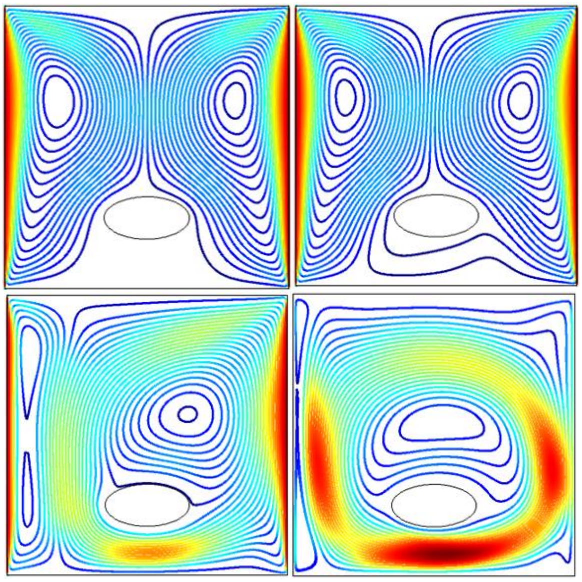

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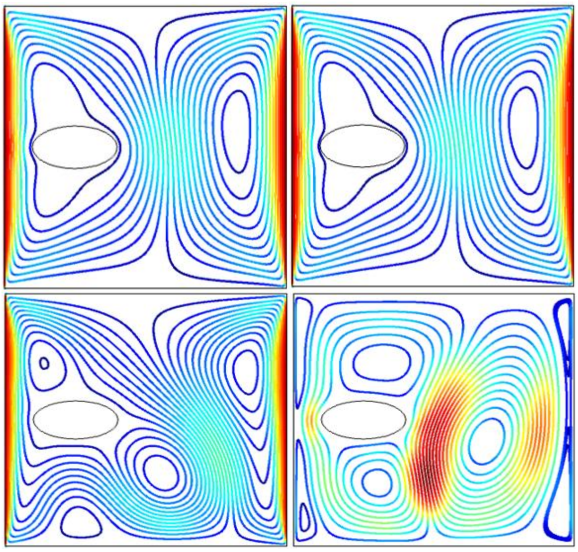
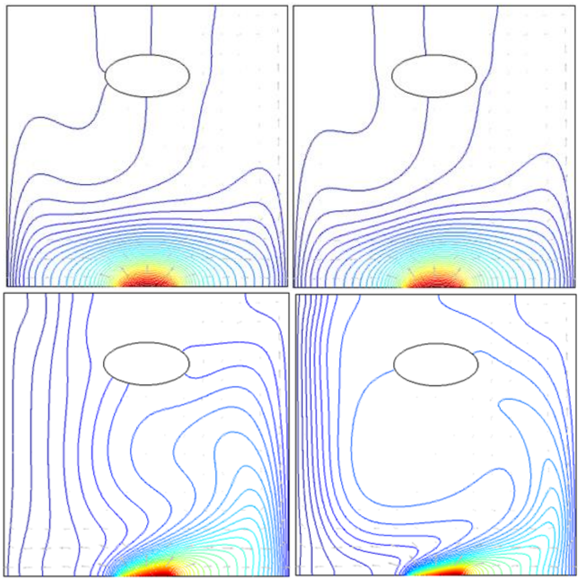
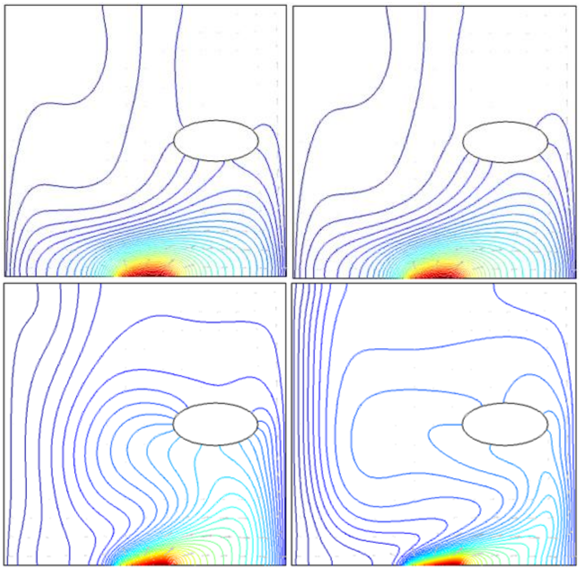


(d) Case (IV)

Fig. 3: Streamlines for case studies with lid moving in the + y direction (upward) at the value of Richardson number *Ri* = 0.01, 0.1, 1 10



(a) Case (I)



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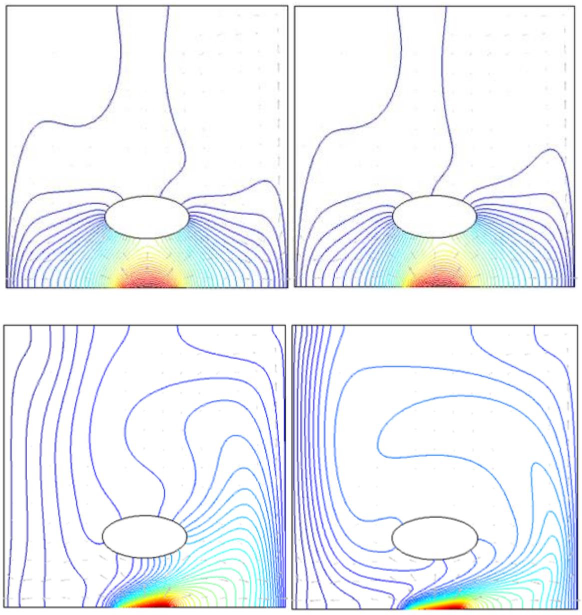

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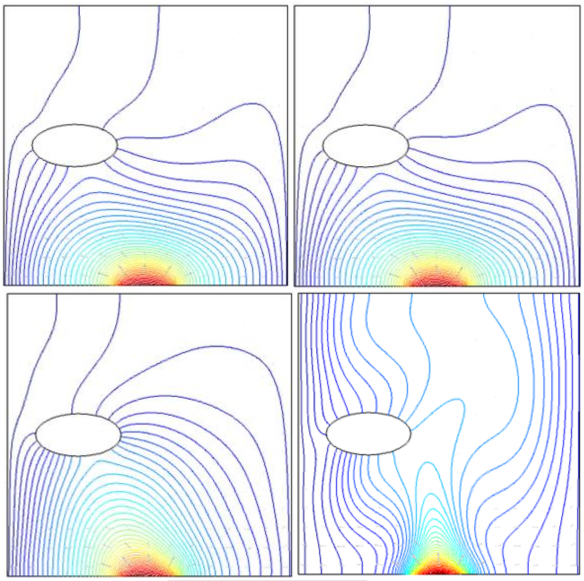


(d) Case (IV)

Fig. 4: Isotherms for case studies with lid moving in the + y direction (upward) at the value of Richardson number Ri = 0.01, 0.1, 1 10

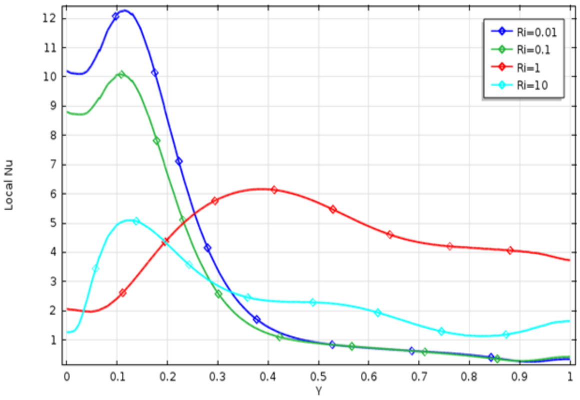


Fig 5: Variation of local Nusselt number along the bottom wall of the square cavity with lid moving in the + y direction (upward) various Richardson number and Pr = 0.733.

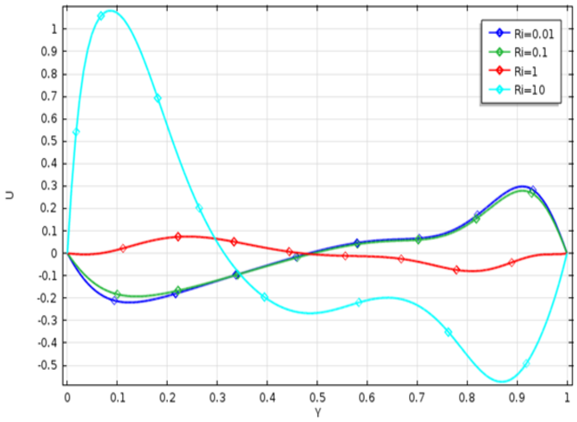


Fig 6: Variation of vertical velocity component along the bottom wall of the square cavity with lid moving in the + y direction (upward) various Richardson number and Pr = 0.733.

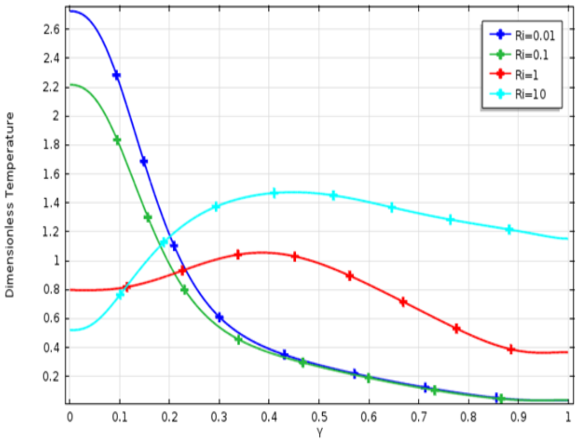
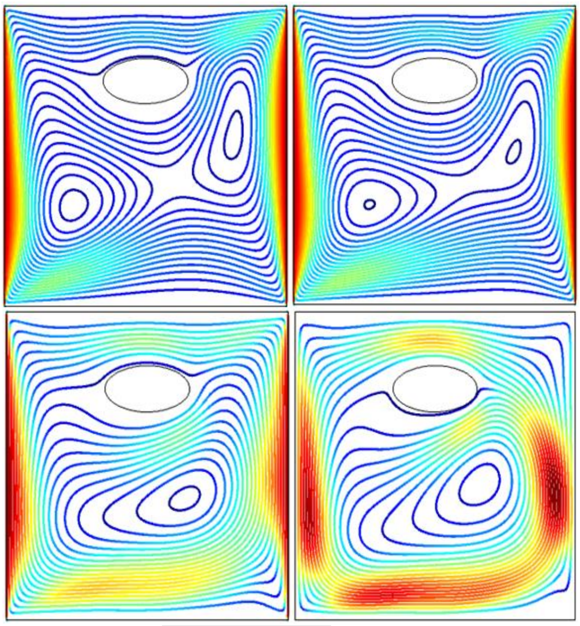
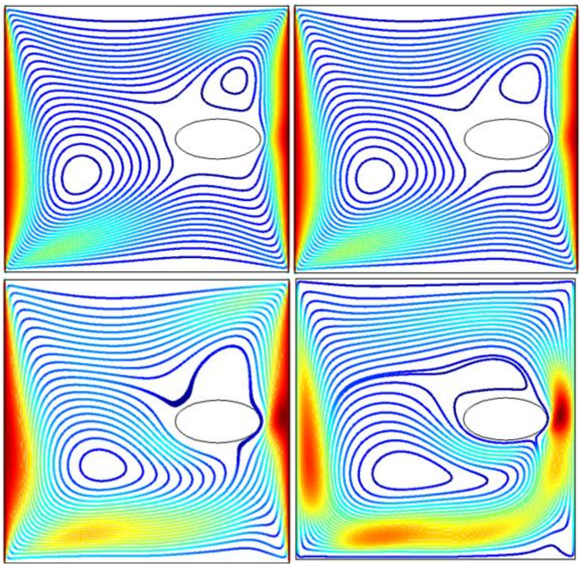


Fig 7: Variation of the dimensionless temperature along the bottom wall of the square cavity with lid moving in the + y direction (upward) h various Richardson number and Pr = 0.733.



(a) Case (I)



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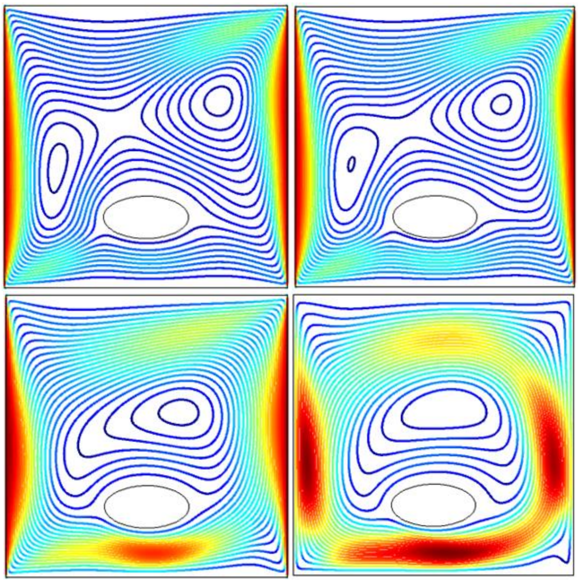

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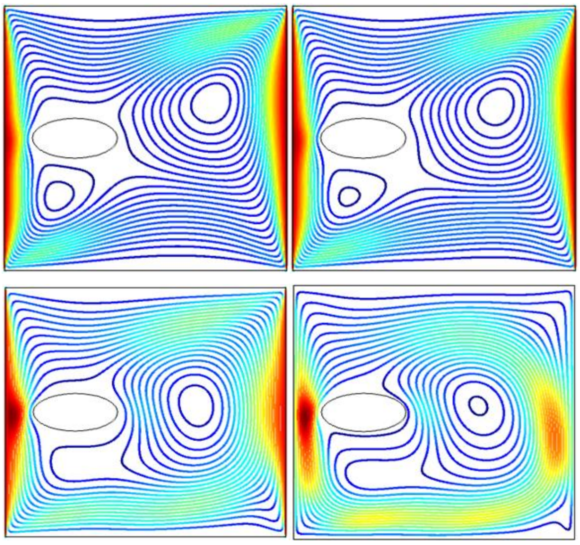
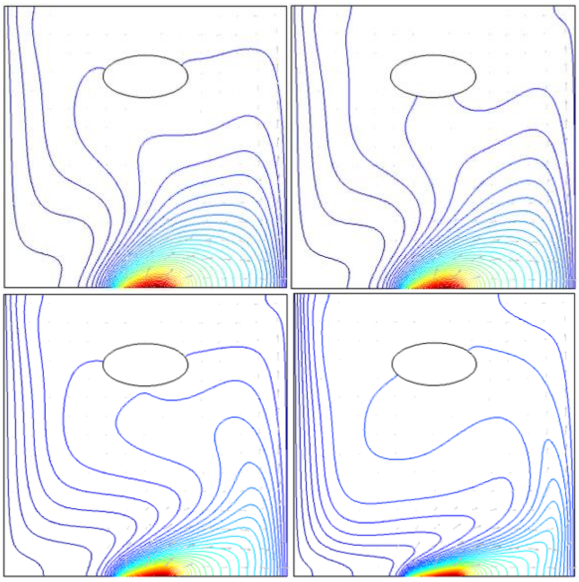
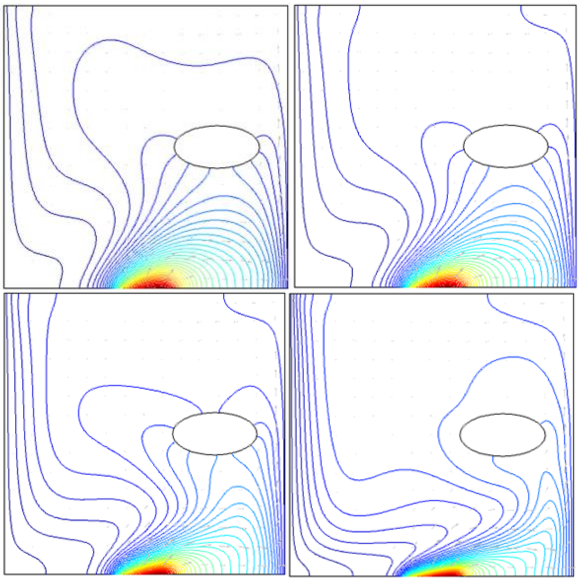


(d) Case (IV)

Fig. 8: Streamlines for case studies with lid moving in the - y direction (downward) at the value of Richardson number Ri = 0.01, 0.1, 1 10



(a) Case (I)



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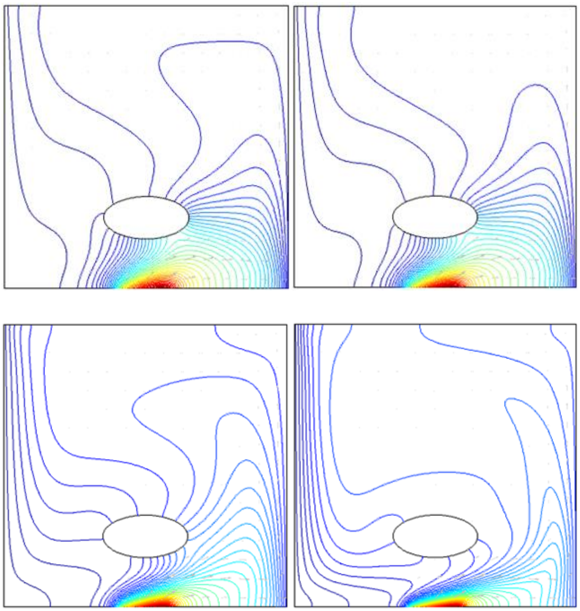

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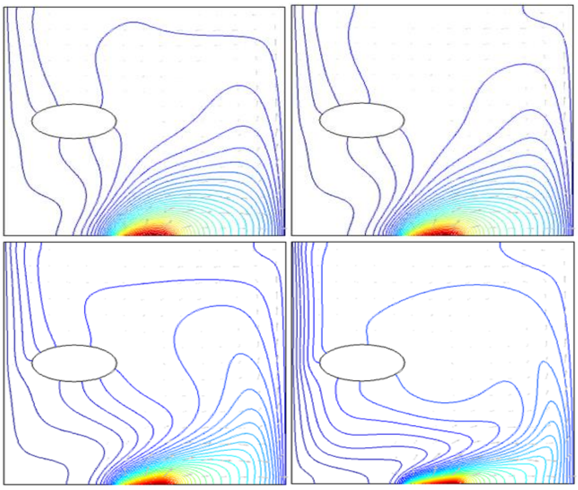

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Fig. 9: Isotherms for case studies with lid moving in the - y direction (downward) at the value of Richardson number Ri = 0.01, 0.1, 1 10

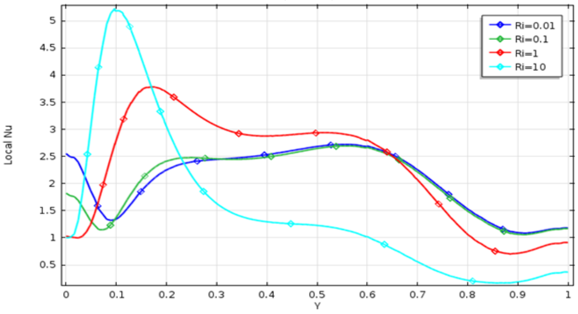

Fig 10: Variation of local Nusselt number along the bottom wall of the square cavity with lid moving in the - y direction (downward) various Richardson number and Pr = 0.733.

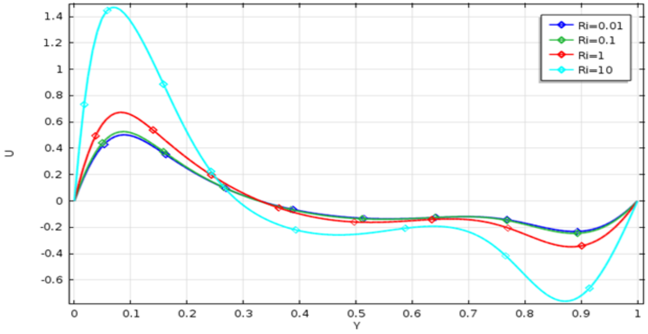

Fig. 11: Variation of vertical velocity component along the bottom wall of the square cavity with lid moving in the - y direction (downward) various Richardson number and Pr = 0.733.

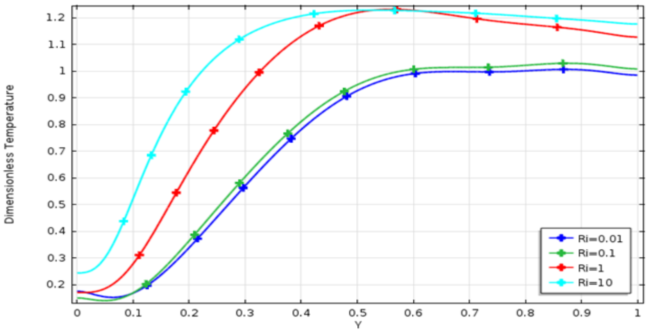

Fig 12: Variation of the dimensionless temperature along the bottom wall of the square cavity with lid moving in the - y direction (downward) h various Richardson number and Pr = 0.733.

V. CONCLUSIONS:

The following conclusions can be drawn from the results of the present work:

A mathematical model to simulate mixed convection heat transfer in a two- dimensional square enclosure and the associated computer coding has been developed. The model is applied to analyze mixed convection in a square cavity where the cold isothermal vertical left wall are moving with constant velocity and a constant flux heat source is placed at the bottom. The direction of lid makes important effect on heat transfer. The behavior of temperature body contours and streamlines refers that the heat transfer process is sensitive to the location of the elliptic body. The results show that the local Nusselt number values increases as the distance along the hot right wall increase for most cases with selected upward and downward lid moving. Higher heat transfer is observed for the case of downward moving wall. The values of local Nusselt number when the lid moving downward have higher values than that when the lid moving upward. It is also noticed that the heat- affected region becomes larger with the increasing heat source length. For a symmetric placement of the heat source, it is observed that the maximum temperature decreases and the average Nusselt number increases as the source is moved more and more towards the sidewall.

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