Application of the resolution of the characteristic-free resolution of Weyl module to Lascoux resolution in case (6,6,3)

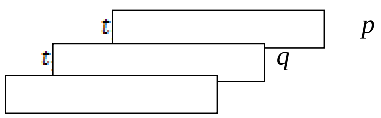
Haytham R. Hassan, Mays M. Mohammed

***Abstract*— In this paper we study the relation between the resolution of Weyl module $K_{(6,6,3)}F$ in characteristic-free mode and in the Lascoux mode (characteristic zero), more precisely we obtain the Lascoux resolution of $K_{(6,6,3)}F$ in characteristic zero as an application of the resolution of $K_{(6,6,3)}F$ in characteristic-free.**

***Index Terms*— Resolution, weyl module, Lascoux module, divided power, characteristic-free.**

I. INTRODUCTION

Let R be commutative ring with 1 and F be free R-module by D_nF we mean the divided power of degree n. we used the resolution of the three-rowed skew-shape $(p+t_1+t_2, q+t_2, r)/(t_1+t_2, t_2, 0)$,and in our case $t_1 = t_2 = 0$, namely , the shape represented by the diagram



In [7], the description of the characteristic zero skeleton by Lascoux in the resolution of skew-shapes. Practically the terms of Lascoux resolution can be recovered with in the formula offered in [3] and [8]. Furthermore in [1], by using letter-place methods and place polarization in a symmetric way we get the application of the results mentioned above. For the corresponding Weyl module to the partition $\lambda = (2,2,2)$ the relation between resolution of $K_{(2,2,2)}(F)$ in the characteristic-free module and in the Lascoux mode (characteristic zero) are studied. By this comparison, the characteristic-free boundary maps are modified to obtain the obvious maps of the Lascoux case. One of the generalization of the techniques used in [2] for the partition λ=(3,3,3) by Hatham R. Hassan.

In section two, we review the terms of characteristic-free resolution of Weyl module in the case of the partition (6,6,3).

In section three we apply this resolution to the Lascoux resolution in the same case by using the way in [1] and [2] with capelli identities [3].

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II. CHARACTERISTIC-FREE RESOLUTION OF THE PARTITION (6,6,3)

We will use the terms of the resolution for three –rowed partition (*p*,*q*,*r*) to discuss our research.

The terms of the resolution are:

Res ([*p*,*q*;*0*])$\otimes D_r \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$ **Res** ([*p*,*q*+*l*+*1*;*l*+*1*])

Res $\otimes D_{r-l-1} \oplus \sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$

([*p*+l_1+1,*q*+l_2+1,$l_2 - l_1$])$\otimes D_{r-(l_1+l_2+2)}$

In particular, if we consider the case when *p*=*q*=6, *r*=2 from above we get

Res([6,6,0])$\otimes D_2 \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$

Res([6,6+*l*+1;*l*+1])$\otimes D_{2-l-1} \oplus \sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$

Res([6+l_1+1,6+l_2+1,$l_2 - l_1$])$\otimes D_{2-(l_1+l_2+2)}$

(3.1.1)

So

Res([6,6+*l*+1;*l*+1])$\otimes D_{2-l-1}$ $\sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$

=$\underline{Z}_{32} y$ **Res**([6,7;1])$\otimes D_2 \oplus \underline{Z}_{32}^{(2)} y$ **Res**([6,8;2])$\otimes D_1 \oplus \underline{Z}_{32}^{(2)} y$ **Res**([6,9;3])$\otimes D_0$

and

$\sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$

Res([6+l_1+1,6+l_2+1;$l_2 - l_1$])$\otimes D_{2-(l_1+l_2+2)}$

= $\underline{Z}_{32} y \underline{Z}_{31} z$ **Res**([7,7;0])$\otimes D_1 \oplus \underline{Z}_{32}^{(2)} y \underline{Z}_{31} z$ **Res**([7,8;1])$\otimes D_0$

Where $\underline{Z}_{32} y$ is the bar complex

$$0 \longrightarrow Z_{32}y \xrightarrow{\partial_y} Z_{32} \longrightarrow 0$$

is the bar complex $\underline{Z}_{32}^{(2)} y$

$$0 \longrightarrow Z_{32}yZ_{32}y \xrightarrow{\partial_y} Z_{32}^{(2)}y \xrightarrow{\partial_y} Z_{32}^{(2)} \longrightarrow 0$$

is the bar complex $\underline{Z}_{32}^{(3)} y$

$$0 \longrightarrow Z_{32}yZ_{32}yZ_{32}y \xrightarrow{\partial_y} \begin{matrix} Z_{32}^{(2)}yZ_{32}y \\ \oplus \\ Z_{32}yZ_{32}^{(2)}y \end{matrix} \xrightarrow{\partial_y} Z_{32}^{(3)}y \xrightarrow{\partial_y} Z_{32}^{(3)} \longrightarrow o$$

and $\underline{Z}_{31} z$ is the bar complex

$$0 \longrightarrow Z_{31}z \xrightarrow{\partial_z} Z_{31} \longrightarrow 0$$

Where *x*, *y* and *z* stand for the separator variables, and the boundary map is $\partial_x + \partial_y + \partial_z$.

Let again Bar(*M*,*A*;*S*) be the free bar module on the set *S*={*x*,*y*,*z*} consisting of three separators *x*, *y* and *z*, where *A* is the free associative (non-commutative) algebra generated by Z_{21},Z_{32} and Z_{31} and their divided powers with the following relation:

and $Z_{21}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{21}^{(a)} Z_{32}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{32}^{(a)}$

and the module *M* is the direct sum of tensor products of divided power module $D_{P_1} \otimes D_{P_2} \otimes D_{P_3}$ for suitable P_1,P_2 and P_3 with the action of Z_{21},Z_{32} and Z_{31} and their divided powers .

Now, from all of the above, we can explicitly describe the terms of the characteristic-free resolution (3.1.1), which are as follows:

○ In dimension zero (M_0) we have $D_6 \otimes D_6 \otimes D_3$
○ In dimension one (M_1) we have

● $Z_{21}^{(b)}xD_{6+b} \otimes D_{6-b} \otimes D_3$ with b=1,2,3,4,5,6 and $Z_{32}^{(b)}yD_6 \otimes D_{6+b} \otimes D_{3-b}$.

○ In dimension two (M_2) we have the sum of the following terms:

● $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_3$; with $|b| = b_1 + b_2 = 2,3,4,5,6$.
● $Z_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{7-b} \otimes D_2$; with b=2,3,4,5,6,7.
● $Z_{32}^{(2)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{8-b} \otimes D_1$; with b=3,4,5,6,7,8.
● $Z_{32}^{(b_1)}yZ_{32}^{(b_2)}yD_6 \otimes D_{6+|b|} \otimes D_{3-|b|}$; with b=2,3.
● $Z_{32}^{(3)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{9-b} \otimes D_0$; with b=4,5,6,7,8,9.
● $Z_{32}^{(b)}yZ_{31}zD_7 \otimes D_{6+b} \otimes D_{2-b}$; with b=1,2.

○ In dimension three (M_3) we have the sum of the following terms:

● $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_3$; with $|b| = b_1 + b_2 + b_3$=3,4,5,6.
● $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_2$; with $|b| = b_1 + b_2$=3,4,5,6,7.
● $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2$=4,5,6,7,8.
● $Z_{32}yZ_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{8-b} \otimes D_1$; with b=3,4,5,6,7,8.
● $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2$ =5,6,7,8,9.
● $Z_{32}yZ_{32}yZ_{32}yD_6 \otimes D_9 \otimes D_0$
● $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{9-b} \otimes D_0$; with b=4,5,6,7,8,9.
● $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{9-b} \otimes D_0$; with b=4,5,6,7,8,9.
● $Z_{32}yZ_{31}zZ_{21}^{(b)}xD_{7+b} \otimes D_{7-b} \otimes D_1$; with b=1,2,3,4,5,6,7.
● $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b)}xD_{7+b} \otimes D_{8-b} \otimes D_0$; with b=2,3,4,5,6,7,8.
● $Z_{32}yZ_{32}yZ_{31}zD_7 \otimes D_8 \otimes D_0$

○ In dimension four (M_4) we have the sum of the following terms:

● $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_3$; with $|b| = b_1 + b_2 + b_3 + b_4$=4,5,6.
● $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_2$; with $|b| = b_1 + b_2 + b_3$=4,5,6,7 and $b_1 \geq 2$.
● $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3$ and $b_1 \geq 3$.
● $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2$=4,5,6,7,8 and $b_1 \geq 3$.
● $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3$=6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{9-b} \otimes D_0$; with b=4,5,6,7,8,9.
● $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2$=5,6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2$=5,6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|} \otimes D_{7-|b|} \otimes D_1$; with $|b| = b_1 + b_2$=2,3,4,5,6,7.
● $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$; with $|b| = b_1 + b_2$=3,4,5,6,7,8 and $b_1 \geq 2$.
● $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b)}xD_{7+b} \otimes D_{8-b} \otimes D_0$; with b=2,3,4,5,6,7,8.

○ In dimension five (M_5) we have the sum of the following terms:

● $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_3$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=5,6.
● $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_2$; with $|b| = b_1 + b_2 + b_3 + b_4$=5,6,7 and $b_1 \geq 2$.
● $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3 + b_4$=6,7,8 and $b_1 \geq 3$.
● $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3$=5,6,7,8 and $b_1 \geq 3$.
● $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2$=5,6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3$=6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3$=6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|} \otimes D_{7-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3$=3,4,5,6,7.
● $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3$=4,5,6,7,8 and $b_1 \geq 2$.
● $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$; with $|b| = b_1 + b_2$=4,5,6,7,8 and $b_1 \geq 2$.

○ In dimension six (M_6) we have the sum of the following terms:

● $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_2$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=6,7 and $b_1 \geq 2$.
● $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=7,8 and $b_1 \geq 3$.
● $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3 + b_4$=6,7,8 and $b_1 \geq 3$.
● $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3$=6,7,8,9 and $b_1 \geq 4$.
● $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=7,8,9 and $b_1 \geq 4$.
● $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{7+|b|} \otimes D_{7-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3 + b_4$=4,5,67.
● $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=5,67,8 and $b_1 \geq 2$.
● $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=4,5,67,8 and $b_1 \geq 2$.

○ In dimension seven (M_7) we have the sum of the following terms:

● $Z_{32}yZ_{21}^{(2)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{13} \otimes D_0 \otimes D_2$
● $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14} \otimes D_0 \otimes D_1$
● $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{8-|b|} \otimes D_1$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=7,8 and $b_1 \geq 3$.
● $Z_{32}^{(3)}yZ_{21}^{(4)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$
● $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4$=7,8,9 and $b_1 \geq 4$.
● $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=8,9 and $b_1 \geq 4$.

- $Z_{32}yZ^{(2)}_{32}yZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=8,9 and $b_1 \geq 4$.

- $Z_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xD_{7+|b|} \otimes D_{7-|b|} \otimes D_1$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=5,6,7.

- $Z^{(2)}_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=6,7,8 and $b_1 \geq 2$.

- $Z_{32}yZ_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4$=5,6,7,8 and $b_1 \geq 2$.

○ In dimension eight (M_8) we have the sum of the following terms:

- $Z_{32}yZ_{32}yZ^{(3)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14} \otimes D_0 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{32}yZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xD_{6+|b|} \otimes D_{9-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=8,9 and $b_1 \geq 4$.

- $Z^{(2)}_{32}yZ_{32}yZ^{(4)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ^{(2)}_{32}yZ^{(4)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xZ^{(b_6)}_{21}xD_{7+|b|} \otimes D_{7-|b|} \otimes D_1$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$+$b_6$=6,7.

- $Z^{(2)}_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xZ^{(b_6)}_{21}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$+$b_6$=7,8 and $b_1 \geq 2$.

- $Z_{32}yZ_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5$=6,7,8 and $b_1 \geq 2$.

○ In dimension nine (M_9) we have the sum of the following terms:

- $Z_{32}yZ_{32}yZ_{32}yZ^{(4)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$
- $Z^{(2)}_{32}yZ_{31}zZ^{(2)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{32}yZ_{31}zZ^{(b_1)}_{21}xZ^{(b_2)}_{21}xZ^{(b_3)}_{21}xZ^{(b_4)}_{21}xZ^{(b_5)}_{21}xZ^{(b_6)}_{21}xD_{7+|b|} \otimes D_{8-|b|} \otimes D_0$

; with $|b| = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$=7,8 and $b_1 \geq 2$.

○ In dimension ten (M_{10}) we have the sum of the following terms:

- $Z_{32}yZ_{32}yZ_{31}zZ^{(2)}_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{15} \otimes D_0 \otimes D_0$

III. LASCOUX RESOLUTION OF THE PARTITION (6,6,3)

The Lascoux resolution of the Weyl module associated to the partition (6,6,3) looks like this

$$\begin{array}{ccccccc} & & D_1 \otimes D_3 \otimes D_2 & & D_7 \otimes D_3 \otimes D_1 & & \\ \oplus & \to & \oplus & \to D_6 \otimes D_6 \otimes D_3 \to 0\ 0 \to & D_1 \otimes D_6 \otimes D_1 & \to & \\ & & D_7 \otimes D_7 \otimes D_1 & & D_6 \otimes D_7 \otimes D_2 & & \end{array}$$

where the position of the terms of the complex determined by the length of the permutations to which they corresponds. The correspondence between the terms of the resolution above and permutations is as follows

$D_6F \otimes D_6F \otimes D_3F \leftrightarrow identity$
$D_5F \otimes D_7F \otimes D_3F \leftrightarrow (12)$
$D_6F \otimes D_2F \otimes D_7F \leftrightarrow (23)$
$D_5F \otimes D_2F \otimes D_8F \leftrightarrow (123)$
$D_1F \otimes D_7F \otimes D_7F \leftrightarrow (132)$

Now, the terms can be presented as below, following Buchsbaum method [1].

$M_0 = A_0$
$M_1 = A_1 \oplus B_1$
$M_2 = A_2 \oplus B_2$
$M_3 = A_3 \oplus B_3$
; for j=4,5,6,7,8,9,10.$M_j = B_j$

Where the A_s are the sums of the lascoux terms, and the B_s are the sums of the others.

Now, we define the map σ_1 from B_1 to A_1 as follows

- $Z^{(2)}_{21}x(v) \mapsto \frac{1}{2}Z_{21}x\partial_{21}(v)$; where $v \in D_8 \otimes D_4 \otimes D_3$
- $Z^{(3)}_{21}x(v) \mapsto \frac{1}{3}Z_{21}x\partial^{(2)}_{21}(v)$; where $v \in D_9 \otimes D_3 \otimes D_3$
- $Z^{(4)}_{21}x(v) \mapsto \frac{1}{4}Z_{21}x\partial^{(3)}_{21}(v)$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z^{(5)}_{21}x(v) \mapsto \frac{1}{5}Z_{21}x\partial^{(4)}_{21}(v)$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z^{(6)}_{21}x(v) \mapsto \frac{1}{6}Z_{21}x\partial^{(5)}_{21}(v)$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z^{(2)}_{32}y(v) \mapsto \frac{1}{2}Z_{32}y\partial_{32}(v)$; where $v \in D_6 \otimes D_8 \otimes D_1$
- $Z^{(3)}_{32}y(v) \mapsto \frac{1}{3}Z_{32}y\partial^{(2)}_{32}(v)$; where $v \in D_6 \otimes D_9 \otimes D_0$

We should point out that the map σ_1 satisfies the identity:

$$\delta_{A_1A_0}\sigma_1 = \delta_{B_1B_0} \tag{3.1}$$

$$\begin{tikzcd} A_1 \arrow[r, "\delta_{A_1A_0}"] & A_0 = B_0 \\ & B_1 \arrow[ul, "\sigma_1"] \arrow[u] \end{tikzcd}$$

Where by $\delta_{A_1A_0}$ we mean the component of the boundary of the fat complex which conveys A_1 to A_0.

We will use notation $\delta_{A_{i+1}A_i}$, $\delta_{A_{i+1}B_i}$ etc. Then we can define $\partial_1 : A_1 \to A_0$ as $\partial_1 = \delta_{A_1A_0}$.

It is easy to show that ∂_1 which we defined above satisfies the condition (3.1), for example:

$$(\delta_{A_1A_0} \circ \sigma_1)\left(Z^{(3)}_{21}x(v)\right) = \delta_{A_1A_0}\left(\frac{1}{3}Z_{21}x\partial^{(2)}_{21}(v)\right) = \frac{1}{3}\left(\partial_{21}\partial^{(2)}_{21}(v)\right) = \partial^{(3)}_{21}(v) = \delta_{B_1B_0}\left(Z^{(3)}_{21}x(v)\right)$$

At this point we are in position to define $\partial_2 : A_2 \to A_1$ by $\partial_2 = \delta_{A_2A_1} + \sigma_1\delta_{A_2B_1}$.

Proposition(3.1): The composition $\partial_1 \circ \partial_2 = 0$

Proof:[1],[2]

$$\partial_1 \circ \partial_2(m) = \delta_{A_1A_0} \circ (\delta_{A_2A_1}(m) + \sigma_1 \circ \delta_{A_2B_1}(m))$$
$$= \delta_{A_1A_0} \circ \delta_{A_2A_1}(m) + \delta_{A_1A_0} \circ \sigma_1 \circ \delta_{A_2B_1}(m)$$

But $\delta_{A_1A_0} \circ \sigma_1 = \delta_{B_1B_0}$. Then we get

$$\partial_1 \circ \partial_2(m) = \delta_{A_1A_0} \circ \delta_{A_2A_1}(m) + \delta_{B_1B_0} \circ \delta_{A_2B_1}(m)$$

Which equal to zero, because of the properties of the boundary map δ [1] , so we get that $\partial_1\partial_2 = 0$.□

Now, we have to define a map $\sigma_2 : B_2 \to A_2$

Such that

$$\delta_{B_2A_1} + \sigma_1 \circ \delta_{B_2B_1} = (\delta_{A_2A_1} + \sigma_1 \circ \delta_{A_2B_1}) \circ \sigma_2 \tag{3.2}$$

We define this map as follows:

- $Z^{(2)}_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_9 \otimes D_3 \otimes D_3$
- $Z_{21}xZ^{(2)}_{21}x(v) \mapsto 0$; where $v \in D_9 \otimes D_3 \otimes D_3$
- $Z^{(3)}_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z^{(2)}_{21}xZ^{(2)}_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}xZ^{(3)}_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z^{(4)}_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z^{(3)}_{21}xZ^{(2)}_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z^{(2)}_{21}xZ^{(3)}_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}xZ^{(4)}_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z^{(5)}_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z^{(4)}_{21}xZ^{(2)}_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z^{(3)}_{21}xZ^{(3)}_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$

- $Z_{21}^{(2)}xZ_{21}^{(4)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(5)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(3)}x(v) \mapsto \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}(v)$; where $v \in D_9 \otimes D_4 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}x(v) \mapsto \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}(v)$; where $v \in D_{10} \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}x(v) \mapsto \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}(v)$; where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(6)}x(v) \mapsto \frac{1}{15}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}$; where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(7)}x(v) \mapsto \frac{1}{21}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}(v)$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}yZ_{21}^{(3)}x(v) \mapsto \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{31}(v)$; where $v \in D_9 \otimes D_5 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(4)}x(v) \mapsto \frac{1}{12}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{31}(v)$; where $v \in D_{10} \otimes D_4 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(5)}x(v) \mapsto \frac{1}{30}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{1}{5}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}(v)$; where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(6)}x(v) \mapsto \frac{1}{60}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v)$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(7)}x(v) \mapsto \frac{1}{105}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{7}Z_{32}yZ_{31}z\partial_{21}^{(6)}(v)$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(8)}x(v) \mapsto \frac{1}{42}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}(v)$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(3)}yZ_{21}^{(4)}x(v) \mapsto \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}(v)$

$-\frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{1}{3}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in D_{11} \otimes D_4 \otimes D_0$

- $Z_{32}^{(3)}yZ_{21}^{(5)}x(v) \mapsto \frac{1}{30}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v) - \frac{1}{6}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{31}(v)$; where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(6)}x(v) \mapsto \frac{1}{90}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v)$

$-\frac{1}{15}Z_{32}yZ_{21}^{(2)}x\partial_{32}^{(2)}\partial_{21}^{(4)}(v) - \frac{2}{9}Z_{32}yZ_{31}z\partial_{32}\partial_{21}^{(5)}(v)$; where $v \in D_{12} \otimes D_3 \otimes D_0$

- $Z_{32}^{(3)}yZ_{21}^{(7)}x(v) \mapsto \frac{1}{210}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}\partial_{31}(v)$

$+\frac{1}{70}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}^{(2)}(v) - \frac{1}{21}Z_{32}yZ_{31}z\partial_{32}\partial_{21}^{(6)}(v)$; where $v \in D_{13} \otimes D_2 \otimes D_0$

- $Z_{32}^{(3)}yZ_{21}^{(8)}x(v) \mapsto \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{21}^{(2)}(v) - \frac{1}{9}Z_{32}yZ_{31}z\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in D_{14} \otimes D_1 \otimes D_0$

; where $v \in D_{15} \otimes D_0 \otimes D_0$

- $Z_{32}^{(3)}yZ_{21}^{(9)}x(v) \mapsto \frac{1}{63}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}^{(2)}(v)$
- $Z_{32}yZ_{32}y(v) \mapsto 0$; where $v \in D_6 \otimes D_8 \otimes D_1$
- $Z_{32}yZ_{32}^{(2)}y(v) \mapsto 0$; where $v \in D_6 \otimes D_9 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}y(v) \mapsto 0$; where $v \in D_6 \otimes D_9 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}z(v) \mapsto \frac{1}{3}Z_{32}yZ_{31}z\partial_{32}(v)$; where $v \in D_7 \otimes D_8 \otimes D_0$

It easy to show that σ_2 which is defined above satisfies the condition (3.2), for example we chose one of them

- $(\delta_{B_2A_1} - \sigma_1\delta_{B_2B_1})\left(Z_{32}yZ_{21}^{(3)}x(v)\right)$;where $v \in D_9 \otimes D_4 \otimes D_2$

$= \sigma_1\left(Z_{21}^{(3)}x\partial_{32}(v)\right) + \sigma_1\left(Z_{21}^{(2)}x\partial_{31}(v)\right) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{3}Z_{21}x\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{2}Z_{21}x\partial_{21}\,\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{3}Z_{21}x\partial_{32}\partial_{21}^{(2)}(v) - \frac{1}{3}Z_{21}x\partial_{21}\partial_{31}(v) + \frac{1}{2}Z_{21}x\partial_{21}\,\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{3}Z_{21}x\partial_{32}\partial_{21}^{(2)}(v) + \frac{1}{6}Z_{21}x\partial_{21}\,\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

and

$(\delta_{A_2A_1} - \sigma_1\delta_{A_2B_1})\left(\frac{1}{3}Z_{32}y\,Z_{21}^{(2)}x\partial_{21}(v)\right)$

$= \sigma_1(\frac{1}{3}Z_{21}^{(2)}x\partial_{32}\partial_{21}(v) + \frac{1}{3}Z_{21}x\partial_{32}\partial_{21}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{6}Z_{21}x\partial_{21}\partial_{32}\partial_{21}(v) + \frac{1}{3}Z_{21}x\partial_{21}\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{6}Z_{21}x\partial_{32}\partial_{21}\partial_{21}(v) - \frac{1}{6}Z_{21}x\partial_{21}\partial_{31}(v) + \frac{1}{3}Z_{21}x\partial_{21}\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

$= \frac{1}{3}Z_{21}x\partial_{32}\partial_{21}^{(2)}(v) + \frac{1}{6}Z_{21}x\partial_{21}\partial_{31}(v) - Z_{32}y\partial_{21}^{(3)}(v)$

Proposition(3.2): we have exactness at A_i

Proof: see[1] and [2].

Now by using σ_2 we can also define

$\partial_3: A_3 \longrightarrow A_2$ by $\partial_3 = \delta_{A_3A_2} + \sigma_2 \circ \delta_{A_3B_2}$

Proposition(3.3): $\partial_2 \circ \partial_3 = 0$

Proof: The same way used in proposition (3.1). □

We need to define $\sigma_3: B_3 \longrightarrow A_3$ which satisfying

$$\delta_{B_3A_2} + \sigma_2 \circ \delta_{B_3B_2} = (\delta_{A_3A_2} + \sigma_2 \circ \delta_{A_3B_2}) \circ \sigma_2 \quad (3.3)$$

As follows

- $Z_{21}yZ_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_9 \otimes D_3 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}xZ_{21}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}xZ_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}^{(2)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}xZ_{21}^{(3)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}xZ_{21}xZ_{21}^{(3)}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(4)}xZ_{21}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}xZ_{21}^{(2)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}xZ_{21}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}^{(3)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}^{(2)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}^{(3)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(4)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}xZ_{21}^{(4)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(3)}xZ_{21}^{(2)}(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}^{(3)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}x(v) \mapsto 0$; where $v \in D_9 \otimes D_4 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{10} \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(3)}x(v) \mapsto 0$; where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(3)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(4)}x(v) \mapsto 0$; where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(6)}xZ_{21}x(v) \mapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}xZ_{21}^{(2)}x(v) \mapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$

- $Z_{32}yZ_{21}^{(4)}xZ_{21}^{(3)}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(4)}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(5)}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}x(v) \longmapsto \frac{1}{4}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v)\right)$; where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}^{(2)}x(v) \longmapsto \frac{1}{2}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v)\right)$; where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}x(v) \longmapsto 0$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \longmapsto \frac{1}{2}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)\right)$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}^{(3)}x(v) \longmapsto \frac{2}{3}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)\right)$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(6)}xZ_{21}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}^{(2)}x(v) \longmapsto 0$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(3)}x(v) \longmapsto \frac{5}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}^{(4)}x(v) \longmapsto \frac{5}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}(v)$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(7)}xZ_{21}x(v) \longmapsto -\frac{1}{7}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}(v)$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(6)}xZ_{21}^{(2)}x(v) \longmapsto -\frac{1}{2}Z_{32}yZ_{31}zZ_{21}xZ_{21}^{(6)}(v)$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}^{(3)}x(v) \longmapsto Z_{32}yZ_{31}zZ_{21}xZ_{21}^{(6)}(v)$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(4)}x(v) \longmapsto 0$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}^{(5)}x(v) \longmapsto 0$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(3)}yZ_{21}^{(4)}xZ_{21}x \longmapsto \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)\right)$; where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}x(v) \longmapsto \frac{4}{15}Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(4)}(v) - \frac{1}{10}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)$; where $v \in D_{12} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \longmapsto \frac{1}{2}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)\right)$; where $v \in D_{12} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(6)}xZ_{21}x(v) \longmapsto -\frac{1}{20}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)\right) - \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(5)}(v)\right)$; where $v \in D_{13} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}^{(2)}x(v) \longmapsto \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(v)\right) - \frac{2}{15}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)$; where $v \in D_{13} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \longmapsto -\frac{1}{2}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}\right) - \frac{5}{6}(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32})$; where $v \in D_{13} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(7)}xZ_{21}x(v) \longmapsto \frac{5}{63}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)\right) - \frac{1}{18}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)$; where $v \in D_{14} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(6)}xZ_{21}^{(2)}x(v) \longmapsto -\frac{2}{9}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v) - \frac{7}{9}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)$; where $v \in D_{14} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(5)}xZ_{21}^{(2)}x(v) \longmapsto \frac{5}{9}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v) - \frac{5}{9}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)$; where $v \in D_{14} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(4)}x(v) \longmapsto -\frac{5}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)\right) - \frac{5}{9}(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(6)})$; where $v \in D_{14} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(8)}xZ_{21}x(v) \longmapsto -\frac{1}{63}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{31}(v)\right)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(7)}xZ_{21}^{(2)}x(v) \longmapsto -\frac{4}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{31}(v)\right)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(6)}xZ_{21}^{(3)}x(v) \longmapsto -\frac{16}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{31}(v)\right)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(5)}xZ_{21}^{(4)}x(v) \longmapsto -\frac{5}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{31}(v)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}yZ_{21}^{(4)}xZ_{21}^{(5)}x(v) \longmapsto -\frac{5}{3}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(7)}(v)\right)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{32}yZ_{21}^{(3)}x(v) \longmapsto 0$; where $v \in D_9 \otimes D_5 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(4)}x(v) \longmapsto 0$; where $v \in D_{10} \otimes D_4 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(5)}x(v) \longmapsto -\frac{1}{10}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v)\right)$; where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(6)}x(v) \longmapsto -\frac{1}{15}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)\right)$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(7)}x(v) \longmapsto -\frac{1}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}(v)\right)$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(8)}x(v) \longmapsto 0$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{32}y(v) \longmapsto 0$; where $v \in D_6 \otimes D_9 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(4)}x(v) \longmapsto -\frac{1}{3}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$; where $v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(5)}x(v) \longmapsto \frac{1}{20}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v)\right) - \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)\right)$; where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(6)}x(v) \longmapsto -\frac{7}{60}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(v)\right) - \frac{1}{10}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)\right)$; where $v \in D_{12} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(7)}x(v) \longmapsto \frac{1}{210}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)\right)$; where $v \in D_{13} \otimes D_2 \otimes D_0$

- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(8)}x(v) \mapsto \frac{1}{42}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)\right) - \frac{1}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)\right)$

; where $v \in D_{14} \otimes D_1 \otimes D_0$

- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(9)}x(v) \mapsto 0$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(4)}x(v) \mapsto -\frac{1}{3}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$; where $v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(5)}x(v) \mapsto -\frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)\right)$; where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(6)}x(v) \mapsto -\frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(v)\right) - \frac{2}{15}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)$

; where $v \in D_{12} \otimes D_3 \otimes D_0$

- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(7)}x(v) \mapsto -\frac{1}{35}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)\right) - \frac{1}{42}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(v)\right)$

; where $v \in D_{13} \otimes D_2 \otimes D_0$

- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(8)}x(v) \mapsto -\frac{1}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)\right)$; where $v \in D_{14} \otimes D_1 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(9)}x(v) \mapsto 0$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{31}zZ_{21}^{(2)}x(v) \mapsto \frac{1}{2}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}(v)\right)$; where $v \in D_9 \otimes D_5 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(3)}x(v) \mapsto \frac{1}{3}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}(v)\right)$; where $v \in D_{10} \otimes D_4 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(4)}x(v) \mapsto \frac{1}{10}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v)\right)$; where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(5)}x(v) \mapsto \frac{1}{15}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)\right)$; where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(6)}x(v) \mapsto \frac{1}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}(v)\right)$; where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(7)}x(v) \mapsto \frac{1}{7}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}(v)\right)$; where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(2)}x(v) \mapsto \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}(v)\right) + \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{31}(v)\right)$

; where $v \in D_9 \otimes D_6 \otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(3)}x(v) \mapsto \frac{1}{6}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}\partial_{31}(v)\right) + \frac{1}{9}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$

; where $v \in D_{10} \otimes D_5 \otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(4)}x(v) \mapsto \frac{1}{30}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v)\right)$; where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(5)}x(v) \mapsto -\frac{1}{60}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)\right) - \frac{1}{12}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{32}\partial_{21}^{(2)}(v)\right)$

; where $v \in D_{12} \otimes D_3 \otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(6)}x(v) \mapsto \frac{1}{35}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)\right)$; where $v \in D_{13} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(7)}x(v) \mapsto \frac{1}{18}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)\right) - \frac{2}{63}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{32}(v)\right)$

; where $v \in D_{14} \otimes D_1 \otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(8)}x(v) \mapsto \frac{1}{21}\left(Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(6)}\partial_{31}(v)\right)$; where $v \in D_{15} \otimes D_0 \otimes D_0$
- $\left(Z_{32}yZ_{32}yZ_{31}x(v)\right) \mapsto 0$; where $v \in D_7 \otimes D_8 \otimes D_0$

Again we can show that σ_2 which defined above satisfies the condition (3.3), and here we chose one of them as an example

- $\left(\delta_{B_2A_2} + \sigma_2\delta_{B_2B_2}\right)\left(Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}^{(2)}\right)$; where $v \in D_{12} \otimes D_2 \otimes D_1$

$= \sigma_2\left(Z_{21}^{(4)}xZ_{21}^{(2)}x\partial_{32}^{(2)}(v)\right) + \sigma_2\left(Z_{21}^{(4)}xZ_{21}x\partial_{32}\partial_{31}(v)\right) + \sigma_2\left(Z_{21}^{(2)}xZ_{21}^{(2)}x\partial_{32}\partial_{31}(v)\right) + \sigma_2\left(Z_{21}^{(2)}xZ_{21}x\partial_{32}\partial_{31}(v)\right) + \sigma_2\left(Z_{21}^{(2)}xZ_{21}^{(2)}x\partial_{31}^{(2)}(v)\right) - \sigma_2\left(15Z_{32}^{(2)}yZ_{21}^{(6)}x(v)\right) + \sigma_2\left(Z_{32}^{(2)}yZ_{21}^{(4)}x\partial_{21}^{(2)}(v)\right)$

$= \frac{-15}{60}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) + \frac{15}{60}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v) + \frac{1}{12}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}\partial_{21}^{(2)}(v) + \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{31}\partial_{21}^{(2)}\partial_{31}(v)$

$= \frac{-1}{4}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) + \frac{5}{2}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) + \frac{1}{4}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)$

$= \frac{5}{2}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)$

and

$\left(\delta_{A_2A_2} + \sigma_2\delta_{A_2B_2}\right)\left(\frac{1}{2}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}(v)\right)$

$= \sigma_2\left(-\frac{1}{2}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(4)}(v)\right) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \sigma_2\left(\frac{1}{2}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(4)}(v)\right) + \frac{5}{2}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v)$

$= \frac{5}{2}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}(v) + \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)$

So from all we have done above we the complex

$$0 \to A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \qquad (3.4)$$

Where ∂_i defined as followes:

- $\partial_2\left(Z_{21}x(v)\right) = \partial_{21}(v)$; with $v \in D_7 \otimes D_5 \otimes D_3$
- $\partial_2\left(Z_{32}y(v)\right) = \partial_{32}(v)$; with $v \in D_6 \otimes D_7 \otimes D_2$
- $\partial_2\left(Z_{32}yZ_{21}^{(2)}x(v)\right) = \frac{1}{2}Z_{21}x\partial_{21}\partial_{32}(v) + Z_{21}x\partial_{31}(v) - Z_{32}y\partial_{21}^{(2)}(v)$; with $v \in D_8 \otimes D_5 \otimes D_2$
- $\partial_2\left(Z_{32}yZ_{31}z(v)\right) = \frac{1}{2}Z_{32}y\partial_{32}\partial_{21}(v) + Z_{21}x\partial_{32}^{(2)}(v) - Z_{32}y\partial_{32}^{(2)}(v)$; with $v \in D_7 \otimes D_7 \otimes D_1$

and the map ∂_3 defined as :

- $\partial_2\left(Z_{32}yZ_{31}zZ_{21}x(v)\right) = Z_{32}yZ_{21}^{(2)}x\partial_{32}(v) + Z_{32}yZ_{31}z\partial_{21}(v)$; with $v \in D_8 \otimes D_6 \otimes D_1$

Proposition (3.4).

The complex $0 \longrightarrow A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \longrightarrow K_{(6,6,3)}$ is exact

Proof: see [1] and [2].□

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