A Novel Enrolments Forecasting Model Based On Automatic Clustering Techniques and Time-Variant Fuzzy Logical Relationship Groups

Nghiem Van Tinh, Nguyen Cong Dieu

***Abstract* – Most fuzzy forecasting approaches are based on model fuzzy logical relationships according to the past data. In this paper, a hybrid forecasting model based on two computational methods, time-variant fuzzy logical relationship groups and clustering technique, is presented for academic enrolments. Firstly, we use the automatic clustering algorithm to divide the historical data into clusters and adjust them into intervals with unequal lengths. Then, based on the new intervals, we fuzzify all the historical data of the enrolments of the University of Alabama and calculate the forecasted output by the proposed method. Compared to the other methods existing in literature, particularly to the first-order fuzzy time series, our method gets a higher average forecasting accuracy rate than the existing methods.**

***Index Terms*—Fuzzy time series, forecasting, Time-variant fuzzy logical relationship groups, automatic clustering, enrolments.**

I. INTRODUCTION

It is obvious that forecasting activities play an important role in our daily life. Therefore, many more forecasting models have been developed to deal with various problems in order to help people to make decisions, such as crop forecast [7], [8] academic enrolments [2], [11], the temperature prediction [14], stock markets[15], etc. There is the matter of fact that the traditional forecasting methods cannot deal with the forecasting problems in which the historical data are represented by linguistic values. Ref. [2,3] proposed the time-invariant fuzzy time and the time-variant time series model which use the max–min operations to forecast the enrolments of the University of Alabama. However, the main drawback of these methods is huge computation burden. Then, Ref. [4] proposed the first-order fuzzy time series model by introducing a more efficient arithmetic method. After that, fuzzy time series has been widely studied to improve the accuracy of forecasting in many applications. Ref. [5] considered the trend of the enrolment in the past years and presented another forecasting model based on the first-order fuzzy time series. Ref. [13] pointed out that the effective length of the intervals in the universe of discourse can affect the forecasting accuracy rate. In other words, the choice of the length of intervals can improve the forecasting results. Ref. [6] presented a heuristic model for fuzzy forecasting by integrating Chen's fuzzy forecasting method [4]. At the same time, Ref. [9],[12] proposed several forecast models based on the high-order fuzzy time series to deal with the enrolments forecasting problem. In [10], the length of intervals for the fuzzy time series model was adjusted to get a better forecasted accuracy.

Recently, Ref.[17] presented a new hybrid forecasting model which combined particle swarm optimization with fuzzy time series to find proper length of each interval. Ref. [19] presented a method to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) based on fuzzy time series and clustering techniques. Additionally, Ref.[18] proposed a new method to forecast enrolments based on automatic clustering techniques and fuzzy logical relationships.

In this paper, we proposed a new forecasting model combining the time-variant fuzzy relationship groups and automatic clustering technique in **Error! Reference source not found.**. The method is different from the approach in [4] and [17] in the way where the fuzzy relationships are created. Based on the model proposed in [10], we have developed a new weighted fuzzy time series model by combining the automatic clustering technique and time-variant fuzzy relationship groups with the aim to increase the accuracy of the forecasting model.

In case study, we applied the proposed method to forecast the enrolments of the University of Alabama. Computational results show that the proposed model outperforms other existing methods.

The rest of this paper is organized as follows. In Section 2, we provide a brief review of fuzzy time series and algorithms. In Section 3, we using an automatic clustering algorithm in [20] combining time-variant fuzzy logical relationship groups for forecasting the enrolments of the University of Alabama. Then, the computational results are shown and analyzed in Section 4. Conclusions are presented in Section 5

II. FUZZY TIME SERIES AND ALGORITHMS

In this section, we briefly review the basic concepts of fuzzy time series(FTS), forecasting method based on time – variant fuzzy logical relationship groups(TV-FLRGs) and an automatic clustering algorithm.

A. FTS Definitions

This section briefly summarizes the basic fuzzy and fuzzy time series concepts. The main difference between the fuzzy time series and traditional time series is that the values of the fuzzy time series are represented by fuzzy sets rather than real value. Let $U = \{u_1, u_2, \ldots, u_n\}$ be an universal set; a fuzzy set A of U is defined as $A = \{f_A(u_1)/u_1 + \cdots + f_A(u_n)/u_n\}$, where f_A is a membership function of a given set A, $f_A : U \rightarrow [0,1]$, $f_A(u_i)$ indicates the grade of membership of u_i in the fuzzy set A, $f_A(u_i) \in [0, 1]$, and $1 \leq i \leq n$. General definitions of fuzzy time series are given as follows:

Definition 1: Fuzzy time series

Let $Y(t)$ $(t = \ldots 0, 1, 2 \ldots)$, a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2…) are defined and

Nghiem Van Tinh, Electronics Faculty, Thai Nguyen University of Technology, Thai Nguyen University, Thainguyen, Vietnam.
Nguyen Cong Dieu, Institute of Information Technology, Vietnam Academy of Science and Technology, Hanoi, Vietnam.

if *F(t)* be a collection of $f_i(t)$ $(i = 1, 2 \ldots)$. Then, *F(t)* is called a fuzzy time series on $Y(t)$ $(t = \ldots 0, 1, 2 \ldots)$.

Definition 2: Fuzzy logical relationship

Let $F(t-1) = A_i$ and $F(t) = A_j$. The relationship between two consecutive observations, $F(t)$ and $F(t-1)$, is referred to as a fuzzy logical relationship (FLR) and denoted by $A_i \rightarrow A_j$, where *Ai* is called the left-hand side (LHS) and *Aj* the right-hand side (RHS) of the FLR.

Definition 3: λ- Order Fuzzy Relations

Let *F(t)* be a fuzzy time series. If F(t) is caused by F(t-1), $F(t-2), \ldots, F(t-\lambda+1)\ F(t-\lambda)$ then this fuzzy relationship is represented by $F(t-\lambda), \ldots, F(t-2),\ F(t-1) \rightarrow F(t)$ and is called an λ-order fuzzy time series.

Definition 4: Fuzzy Relationship Group (FLRG)

Relationships with the same fuzzy set on the left hand side can be further grouped into a relationship group. Relationship groups are also referred to as fuzzy logical relationship groups. Suppose there are relationships such that:

$A_i \rightarrow A_j$

$A_i \rightarrow A_k$

…….

So, based on [4], these fuzzy logical relationship can be grouped into the same FLRG as : $A_i \rightarrow A_j, A_k \ldots$

Definition 5: Time-variant fuzzy relationship groups

The fuzzy relationship is determined by the relationship of F(t-1) $\rightarrow F(t)$. If, let $F(t) = A_i(t)$ and $F(t-1) = A_j(t-1)$, we will have the relationship $A_j(t-1) \rightarrow A_i(t)$. At the time t, we have the following fuzzy relationship:

$A_j(t-1) \rightarrow A_i(t),\ A_j(t1-1) \rightarrow A_{i1}(t1), \ldots, A_j(tp-1) \rightarrow A_{ip}(tp)$

with $t1, t2, \ldots, tp \leq t$. It means that if the fuzzy relationship took place before $A_j(t-1) \rightarrow A_i(t)$, we can group the fuzzy logic relationship to be $A_j(t-1) \rightarrow A_{i1}(t1), A_{i2}(t2), A_{ip}(tp), A_i(t)$. It is called time-variant fuzzy logical relationship groups(TV-FLRGs).

B. Forecasting Method Based On TV-Flrgs

The main steps for the FTS forecasting algorithm based on TV-FLRGs is shown in the following algorithm

Step 1: Partition the universe of discourse into equally lengthy intervals.

Step 2: Define fuzzy sets on the universe of discourse.

Step 3: Fuzzify historical data

Step 4: Identify the time – variant fuzzy relationships

Step 5: Establish the time – variant fuzzy relationship groups according to Definition 5.

Step 6: Defuzzify and calculate the forecasted output.

C. An Automatic Clustering Algorithm for Generating Intervals from Numerical Data.

In this section, we briefly summarize an automatic clustering algorithm to cluster numerical data into intervals. The algorithm is introduced in [20]. The algorithm is composed of the main following steps.

Step 1: Sort the numerical data in an ascending sequence having n different numerical data.

$d_1, d_2, d_3, \ldots, d_i, \ldots, d_n$.

where d_1 is the smallest datum among the n numerical data, d_n is the largest datum among the n numerical data, and $1 \leq i \leq n$.

Step 2: Put each numerical datum into a cluster, show as follows: $\{d_1\}, \{d_2\}, \{d_3\}, \ldots, \{d_i\}, \ldots, \{d_n\}$.

Where the symbol "{ }" denotes a cluster, d_1 is the smallest datum among the n numerical data, d_n is the largest datum among the n numerical data and $1 \leq i \leq n$.

Step 3: Based on the clustering results obtained in Step 2, adjust these clusters into contiguous intervals.

III. A Novel Method For Forecasting The Enrolments Based On Tv-Flrgs And Clustering Techniques.

In this section, we present a hybrid method for forecasting enrolments based on the automatic clustering algorithm and TV-FLRGs. The historical data of enrolments of the University of Alabama are listed in Table I. The proposed method is now presented as follows:

Table I. Historical data of enrolments

Year	Actual	Year	Actual
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

Step 1: Partition the universe of discourse into n intervals.

In this step, we apply the automatic clustering algorithm [20] to cluster the historical enrolments in Table 1 into clusters and adjust the clusters into 21 intervals according to the rules in [20]. Then, calculate the midpoint of each interval as shown in Table II.

Table II. The midpoint of each intervals u_i $(1 \leq i \leq 21)$

No	U_i= Intervals	M_i = Midpoint
1	[13055, 13354]	13204.5
2	[13354, 13862]	13608
3	[13862, 14166]	14014
4	[14166, 14397]	14281.5
5	[14397, 14995]	14696
6	[14995, 15145]	15070
7	[15145, 15163]	15154
8	[15163, 15331]	15247
9	[15331, 15603]	15467
10	[15603, 15861]	15732
11	[15861, 15984]	15922.5
12	[15984, 16089]	16036.5
13	[16089, 16687]	16388
14	[16687, 16807]	16747
15	[16807, 16919]	16863
16	[16919, 17851]	17385
17	[17851, 18449]	18150
18	[18449, 18876]	18662.5
19	[18876, 18970]	18923
20	[18970, 19328]	19149
21	[19328, 19337]	19332.5

Source: *In [2, 3]*

Step 2: Define fuzzy sets A_i, where $(1 \leq i \leq n)$

Each interval in Step 1 represents a linguistic variable of enrolment. For 21 intervals, there are 21 linguistic variables. Each linguistic variable represents a fuzzy set A_i $(1 \leq i \leq 21)$ and its definition is described in (1)

$$A_i = \sum_{j=1}^{21} \frac{a_{ij}}{u_j} = \begin{cases} 1 & \text{if } j == i \\ 0.5 & \text{if } j == i-1 \text{ or } j == i+1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $a_{ij} \in [0,1]$, $1 \leq i \leq 21$, and $1 \leq j \leq 21$. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set A_i.

Step 3: Fuzzify variations of the historical enrolment data.

In order to fuzzify all historical data, it's necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set that each interval belongs to with the highest membership degree. For example, the historical enrolment of year 1975 is 15460 which falls within u_9 = (15331, 15603], so it belongs to interval u_9 Based on Eq. (1), Since the highest membership degree of u_9 occurs at A_9, the historical time variable F(1975) is fuzzified as A_9. A complete overview of fuzzified enrolments is shown Table III.

Table III. Fuzzified enrolments of the University of Alabama

Year	Actual	Fuzzy set	Year	Actual	Fuzzy set
1971	13055	A1	1982	15433	A9
1972	13563	A2	1983	15497	A9
1973	13867	A3	1984	15145	A7
1974	14696	A5	1985	15163	A8
1975	15460	A9	1986	15984	A12
1976	15311	A8	1987	16859	A15
1977	15603	A10	1988	18150	A17
1978	15861	A11	1989	18970	A20
1979	16807	A15	1990	19328	A21
1980	16919	A16	1991	19337	A21
1981	16388	A13	1992	18876	A19

Step 4: Identify the time – variant fuzzy relationships

Relationships are identified from the fuzzified historical data. So, based on Table III and according to Definition 2, we get first – order fuzzy logical relationships are shown in Table 4. where the fuzzy logical relationship $A_i \to A_k$ means "If the enrolment of year i is A_i, then that of year i + 1 is A_k", where A_i is called the current state of the enrolment, and A_k is called the next state of the enrolment (*Note that even though the same relationships can appear more than once*).

Table IV: The first-order fuzzy logical relationships

A1 -> A2; A2 -> A3; A3 -> A5; A5 -> A9; A9 -> A8; A8 -> A10; A10 -> A11; A11 -> A15; A15 -> A16; A16 -> A13; A13- > A9; A9 -> A9; A9 -> A7; A7 -> A8 A8 -> A12; A12 -> A15; A15 -> A17; A17 -> A20; A20 -> A21; A21 -> A21; A21 -> A19

Step 5: Create all time-variant FLRGs

In [4], all the fuzzy relationship having the same fuzzy set on the left-hand side or the same current state can be put together into one fuzzy relationship group. But, according to the Definition 5, we need to consider the appearance history of the fuzzy sets on the right-hand side too. Therefore, only the element on the right hand side appearing before the left-hand side of the relationship group is taken into the same fuzzy logic relationship group. Thus, from Table IV and based on Definition 5, we can obtain 21 fuzzy logical relationship groups shown in Table V.

Table V: Fuzzy time – variant logical relationship groups

No	Actual	Time	Fuzzy set	TV-FLRGs
	13055	t=1	A1	#
1	13563	t=2	A2	A2
2	13867	t=3	A3	A3
3	14696	t=4	A5	A5
4	15460	t=5	A9	A9
5	15311	t=6	A8	A8
6	15603	t=7	A10	A10
7	15861	t=8	A11	A11
8	16807	t=9	A15	A15
9	16919	t=10	A16	A16
10	16388	t=11	A13	A13
11	15433	t=12	A9	A9
12	15497	t=13	A9	A8, A9
13	15145	t=14	A7	A8, A9, A7
14	15163	t=15	A8	A8
15	15984	t=16	A12	A10, A12
16	16859	t=17	A15	A15
17	18150	t=18	A17	A16, A17
18	18970	t=19	A20	A20
19	19328	t=20	A21	A21
20	19337	t=21	A21	A21
21	18876	t=22	A19	A21, A19

Step 6: Defuzzify and calculate the forecasting output.

Calculate the forecasted output at time t by using the following rules:

Rule 1: If the fuzzified enrolment of year t-1 is A_j and there is only one fuzzy logical relationship in the fuzzy logical relationship group whose current state is A_j, shown as follows: $A_j(t-1) \to A_k(t)$, then the forecasted enrolment of year t is m_k, where m_k is the midpoint of the interval u_k and the maximum membership value of the fuzzy set A_k occurs at the interval u_k.

Rule 2: If the fuzzified enrolment of year t -1 is A_j and there are the following fuzzy logical relationship group whose current state is A_j, shown as follows:

$A_j(t-1) \to A_{i1}(t1), A_{i2}(t2), A_{ip}(tk)$

then the forecasted enrolment of year t is calculated as follows: $forecasted = \frac{1*m_{i1}+2*m_{i2}+3*m_{i3}+\cdots+p*m_{ip}}{1+2+\cdots+p}$;

where m_{i1}, m_{i2}, m_{ik} are the middle values of the intervals u_{i1}, u_{i2} and u_{ip} respectively, and the maximum membership values of $A_{i1}, A_{i2}, \ldots, A_{ip}$ occur at intervals $u_{i1}, u_{i2}, \ldots, u_{ip}$, respectively.

From Tables III and V and based on the Principles in Step 5, we can forecast the enrolments of the University of Alabama from 1971s to 1992s by the proposed method. For example, assume that we want to forecast the enrolment of years 1975 and 1983 are calculated as follows:

[F(t)=F(1975)]. From Table III, we can see that the fuzzified enrolments of years F(t-1)= F(1974) is A_5 . From Table V, we can see that there is a fuzzy logical relationship $A_5(t-1) \to A_9(t)$, in Group 4 and the maximum membership value of the fuzzy set A_9 occurs at the interval u_9. Based on **rule 1**, the forecasted enrolment of year 1975 can be calculated as follows:

$$forecasted = m_9 = \frac{15331+15603}{2} = 15467$$

[F(t)=F(1983)]. From Table III, we can see that the fuzzified enrolments of years F(t-1)= F(1982) is A_9 . From Table V, we can see that there is a fuzzy logical relationship $A_9(t-1) \to A_8(t1), A_9(t)$; $(t1 < t)$ in Group 13 and the maximum membership value of the fuzzy set A_8 and A_9 occurs at the intervals u_8 and u_9, respectively. Based on **rule 2**, the forecasted enrolment of year 1983 can be calculated as follows:

$$forecasted = \frac{1*m_8+2*m_9}{1+2} = \frac{15247+2*15467}{3} = 1539.6$$

Where, $m_8 = \frac{15163+15331}{2} = 15247$ and $m_9 = \frac{15331+15603}{2} = 15467$

In the same way, we can get the forecasted enrolments of the other years of the University of Alabama from 1971s to 1992s based on the first-order fuzzy time series, as listed in Table VI.

Table VI**:** Forecasted enrolments of the proposed method using the first-order FTS.

Year	**Actual**	**Fuzzified**	**Results**
1971	13055	A1	Not forecasted
1972	13563	A2	13608
1973	13867	A3	14014
1974	14696	A5	14696
1975	15460	A9	15467
1976	15311	A8	15247
1977	15603	A10	15732
1978	15861	A11	15922.5
1979	16807	A15	16863
1980	16919	A16	17385
1981	16388	A13	16388
1982	15433	A9	15467
1983	15497	A9	15393.67
1984	15145	A7	15273.83
1985	15163	A8	15247
1986	15984	A12	15935
1987	16859	A15	16863
1988	18150	A17	17895
1989	18970	A20	19149
1990	19328	A21	19332.5
1991	19337	A21	19332.5
1992	18876	A19	19059.5
MSE			**20818**

❖ To measure the forecasted performance of proposed method in the fuzzy time series, the mean square error (MSE) is employed as an evaluation criterion to represent the forecasted accuracy. The MSE value is computed as follows:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\mathrm{Fo_i} - \mathrm{Ac_i})^2 \qquad (2)$$

Where, Ac_i notes actual data on date i, Fo_i forecasted value on date i, n is number of the forecasted data

IV. Computational Results

The performance of the proposed method will be compared with the existing methods, such as the **SCI** model [2], the **C96** model [4], the **H01** model [5], **CC06F** model [11] and **HPSO** model [17] by using the enrolment of Alabama University from 1971 to 1992. It can be shown in Table VII

Table II**:** A comparison of the forecasted results of our proposed model with the existing models with first-order of the FTS series under different number of intervals.

Year	**Actual data**	**SCI**	**C96**	**H01**	**CC06F**	**HPSO**	**Our proposed**
1971	13055	Not forecasted	Not forecasted	Not forecasted	Not forecasted	Not forecasted	Not forecasted
1972	13563	14000	14000	14000	13714	13555	13608
1973	13867	14000	14000	14000	13714	13994	14014
1974	14696	14000	14000	14000	14880	14711	14696
1975	15460	15500	15500	15500	15467	15344	15467
1976	15311	16000	16000	15500	15172	15411	15247
1977	15603	16000	16000	16000	15467	15411	15732
1978	15861	16000	16000	16000	15861	15411	15922.5
1979	16807	16000	16000	16000	16831	16816	16863
1980	16919	16813	16833	17500	17106	17140	17385
1981	16388	16813	16833	16000	16380	16464	16388
1982	15433	16789	16833	16000	15464	15505	15467
1983	15497	16000	16000	16000	15172	15411	15393.67
1984	15145	16000	16000	15500	15172	15411	15273.83
1985	15163	16000	16000	16000	15467	15344	15247
1986	15984	16000	16000	16000	15467	16018	15935
1987	16859	16000	16000	16000	16831	16816	16863
1988	18150	16813	16833	17500	18055	18060	17895
1989	18970	19000	19000	19000	18998	19014	19149
1990	19328	19000	19000	19000	19300	19340	19332.5
1991	19337	19000	19000	19500	19149	19340	19332.5
1992	18876	19000	19000	19149	19014	19014	19059.5
MSE		**423027**	**407507**	**226611**	**35324**	**22965**	**20818**

Table VII shows a comparison of MSE according to (2) of the proposed method using the first-order fuzzy time series with different number of intervals. The the forecasted accuracy is computed by (3) as follows.

$$MSE = \frac{\sum_{i=1}^{n}(Foi-Aci)^2}{N} = \frac{(13608-13563)^2+(14014-13867)^2...+(19059.5-18876)^2}{21} = 20818 \qquad (3)$$

From Table VII, we can see that the proposed method has a smaller MSE value than SCI model [2] the C96 model [4], the H01 model [5], the CC06F model [11], the HPSO model [17] for forecasting enrolments of the University of Alabama.

To be clearly visualized, Fig.1 displays the forecasting results of the H01 model, the CC06F model, the HPSO model and the our proposed method. The trend of the enrolment forecasting by first-order of the fuzzy time series in comparison to the actual enrollment are shown.

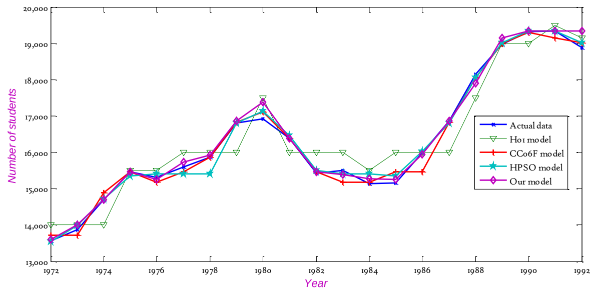

Fig.1**:** The curves of the actual data and the H01, CC06F, HPSO models and our model for forecasting enrolments of University of Alabama from 1972s to 1992s based on the first-order FTS

The trend of the curves in Fig.1 indicates the our model is still stable and is close to the actual enrolment of students each year, from 1972s to 1992s for the first-order FTS model.

V. Conclusion

The lengths of intervals and fuzzy logical relationships are two critical factors that affect forecasting accuracy of time series data. In this paper, we have proposed a new forecasting method in the first-order fuzzy time series model based on the time-variant fuzzy logical relationship groups and the automatic clustering techniques. In this method, we tried to classify the historical data of Alabama University into clusters by clustering techniques and then, adjust the clusters into intervals with different lengths. In case study, we have applied the proposed method to forecast the number of students enrolling in the University of Alabama from 1972s to 1992s. The simulation result showed that the proposed method is able to obtain the forecasted value with better accuracy compared to other methods existing in literature. The detail of comparison was presented in Table VII and Fig.1.
Although this study shows the superior forecasting capability compared with existing forecasting models; but the proposed model is a new forecasting model and only tested by the enrolment data. To assess the effectiveness of the forecasting model, there are two suggestions for future research: The first, we can apply proposed model to deal with more complicated real-world problems for decision-making such as weather forecast, crop production, stock markets, and etc. The second, we use more intelligent methods (e.g., particle swarm optimization, ant colony or a neural network) to deal with forecasting problems. That will be the future work of this research.

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Nghiem Van Tinh is PhD student at Institute of Information Technology, Vietnam Academy of Science and Technology, Hanoi, Vietnam. He is working at Electronics Faculty, Thai Nguyen University of Technology, Vietnam. His research interests include forecasting, clustering, Fuzzy logics, and Fuzzy time series.


Nguyen Cong Dieu is a Doctor of computer science. He is working at Institute of Information Technology, Vietnam Academy of Science and Technology, Hanoi, Vietnam. His research interests include forecasting, Fuzzy logics, and Fuzzy time series, machine learning.