Stabilization of Uncertain Non-Bilinear Descriptor Dynamical System Via Logarithmic Norm Approach

Radhi Ali Zaboon, Ghazwa Faisal Abd

***Abstract*— In this paper a feedback controller stabilization of non-bilinear descriptor system have been developed via logarithmic norm approach. The sufficient conditions on parametric uncertainty where the system is regular or impulse free have been given. Some theoretical results supported have been adopted with suitable illustrations example for designing a stabilizing controller for non-bilinear uncertain descriptor systems based on the theoretical result have also been developed.**

***Index Terms*— Bilinear system, Descriptor system, Dini derivative, Logarithmic Norm ,Parametric matrix uncertainty.**

I. INTRODUCTION

Logarithmic norms were often used to estimate stability and perturbation bounds in linear ordinary differential equation [7],[10]. Extensions of other classes of problems such as nonlinear dynamics descriptor system need a careful modification of logarithmic norm.. One important problem for stability of bilinear systems is given an integral equation such that any solution of bilinear systems satisfies this equation, this is require to solving the singular linear part and studying the consistent initial conditions that was already given in [2],[3] .the robust stabilization of bilinear systems with parametric uncertainty by nonlinear state feedback was considered in [13]. We present and discuses possibilities for stability of non-bilinear descriptor system with bounded perturbation using logarithmic norm concept.

II. PROBLEM FORMULATION

Consider the descriptor non- bilinear system

$$Ex^{\alpha}(t) = (A+\delta A)x(t) + (B+\delta B)u(x)x(t) + g(x(t)) \quad \ldots(1)$$

with

1- x be n-dimentional vector space.
2- E be $n \times n$ singular matrix with index k and $rank\,(E) = p$.
3- A, B are $n \times n$ constant matrices .
4- $\delta A, \delta B$ are constant perturbations matrices with $\|\delta A\| \leq a, \|\delta B\| \leq b$, and a, b, are positive integers.
5- $u(t)$be nonlinear input control
6-$g: \mathbb{R}^n \to \mathbb{R}^n$is a vector of nonlinear functions which may represent a known non-linearity.

III. BASIC CONCEPT

3.1. Definition: [6]

Let f be a function defined on $I = [\,a\,,b\,]$ then

$$D^+f(x) = \lim_{h\to 0^+} sup\,\frac{f(x+h)-f(x)}{h}$$

$$D_+f(x) = \lim_{h\to 0^+} inf\,\frac{f(x+h)-f(x)}{h}$$

Be right upper and lower Dini derivative respectively

3.2. Remark

The properties of the Dini derivative can be found in [5],[12].

3.3.Definition: [8]

The Logarithmic norm is a real valued functional on operators and is derived from either an inner product or vector norm or it's induced operator norm .

3.4.Definition: [4] , [11] , [14]

Let A be square matrix and $\|.\|$ be an induced matrix norm then associated logarithmic norm μ of A is defined

$$\mu(A) = \lim_{h\to 0^+} sup\,\frac{\|I+hA\|-1}{h}$$

Where I is identity matrix of the same dimension of , h is real, positive number.

3.5.Remark

The properties of the logarithmic norm can be found in [3], [11], [7].

3.6.Lemma (Generalization of Gronwell's lemma) [1]

Let $a, b, n, k \in \mathbb{R}$, $a < b$, $n > 1$ and $K > 0$,

$f:\ [a,b]\ \to \mathbb{R}^+$an integral function such that

$\forall\ \alpha, \beta \in\ [a,\ b]\ (\alpha < b): \int_{\alpha}^{\beta} f(s)\ ds\ > 0$ and

$x: [a,b]\ \to \mathbb{R}^+$

If

$x(t) \leq K + \int_0^t f(s)\,[x\,(s)]^n$ ds

and$1-\ (n-1)\,K^{n-1}\int_{\alpha}^{t} f(s)\ ds > 0$

Then $x(t) \leq \dfrac{K}{\left[1-(n-1)K^{n-1}\int_{\alpha}^{t} f(s)\,ds\right]^{\frac{1}{n-1}}}$

IV. STABILIZATION OF UNCERTAIN NON-BILINEAR DESCRIPTOR SYSTEM VIA LOGARTHMIC NORM APPROACH

4.1. Lemma

Consider the non-bilinear descriptor system (1)

If there exist a non singular matrices P,Q such that:

1- $W = p^{-1}X = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix},\ \ w_1 \in \mathbb{R}^{n_1}, w_2 \in \mathbb{R}^{n_2},$

$n_1 = p\,, n_2 = n-p$

2- $QEP = diag\left(I_{n_1}, 0\right),\ QAP = diag\left(A_1, I_{n_2}\right),$

$QBP = \begin{bmatrix} B_1 & B_3 \\ B_2 & B_4 \end{bmatrix}, Q\delta BP = \begin{bmatrix} \delta B_1 & \delta B_3 \\ \delta B_2 & \delta B_4 \end{bmatrix},$

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$Q\delta AP = \begin{bmatrix} \delta A_1 & \delta A_3 \\ \delta A_2 & \delta A_4 \end{bmatrix}$, with appropriate dimension.

3- $Qg(x(t)) = Qg(Pw(t)) = Qg(w_1(t), w_2(t)) = \begin{bmatrix} h_1(w_1(t), w_2(t)) \\ h_2(w_1(t), w_2(t)) \end{bmatrix}$

Then the decomposite system has been as:

$$\left.\begin{aligned} w_1'(t) = (A_1 + \delta A_1)w_1(t) + \delta A_3 w_2(t) + \\ (B_1 + \delta B_1)u(pw(t))w_1(t) + (B_3 + \delta B_3) \\ u(pw(t))w_2(t) + h_1(w_1(t), w_2(t)) \end{aligned}\right\} \quad \ldots(2)$$

$$\left.\begin{aligned} 0 = (\delta A_2)w_1(t) + (I_{n2} + \delta A_4)w_2(t) + (B_2 + \delta B_2) \\ u(pw(t))w_1(t) + (B_4 + \delta B_4)u(pw(t))w_2(t) \\ + h_2(w_1(t), w_2(t)) \end{aligned}\right\} \ldots(3)$$

Proof:

Consider the system "(1)" with E singular of index k and rank p if the nominal system is regular then there exist two non-singular matrices Q and P such that

$W = p^{-1}X = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$, $w_1 \in \mathbb{R}^p$, $w_2 \in \mathbb{R}^{n-p}$ and

$Ex^{\sigma}(t) = (A + \delta A)x(t) + (B + \delta B)u(x)x(t) + g(x(t))$

$$\left.\begin{aligned} QEPw^{\sigma}(t) = Q(A + \delta A)Pw(t) + Q(B + \delta B) \\ Pu(pw(t))w(t) + Qg(Pw(t)) \end{aligned}\right\}$$

$$\left.\begin{aligned} \begin{bmatrix} I_{n1} & 0 \\ 0 & 0 \end{bmatrix} w^{\sigma}(t) = \begin{bmatrix} A_1 + \delta A_1 & \delta A_3 \\ \delta A_2 & I_{n2} + \delta A_4 \end{bmatrix} w(t) + \\ \begin{bmatrix} B_1 + \delta B_1 & B_3 + \delta B_3 \\ B_2 + \delta B_2 & B_4 + \delta B_4 \end{bmatrix} u(pw(t))w(t) \\ + \begin{bmatrix} h_1(w_1(t), w_2(t)) \\ h_2(w_1(t), w_2(t)) \end{bmatrix} \end{aligned}\right\}$$

$$\left.\begin{aligned} w_1'(t) = (A_1 + \delta A_1)w_1(t) + \delta A_3 w_2(t) + \\ (B_1 + \delta B_1)u(pw(t))w_1(t) + \\ (B_3 + \delta B_3)u(pw(t))w_2(t) + h_1(w_1(t), w_2(t)) \end{aligned}\right\}$$

$$\left.\begin{aligned} 0 = (\delta A_2)w_1(t) + (I_{n2} + \delta A_4)w_2(t) \\ +(B_2 + \delta B_2)u(pw(t))w_1(t) + (B_4 + \delta B_4) \\ u(pw(t))w_2(t) + h_2(w_1(t), w_2(t)) \end{aligned}\right\}$$

4.2. Remark

1-If the nominal system is not regular one can transform the system to regular one see [3].

2- as special case if one take

$QBP = \begin{bmatrix} B_1 & 0 \\ B_2 & 0 \end{bmatrix}$, $Q\delta BP = \begin{bmatrix} \delta B_1 & 0 \\ 0 & 0 \end{bmatrix}$, $Q\delta AP = \begin{bmatrix} \delta A_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\|\delta A_1\| \le a_1$ and $\|\delta B_1\| \le b_1$ for some positive constant

a, b and $Qg(w_1(t), w_2(t)) = \begin{bmatrix} h_1(w_1(t), w_2(t)) \\ 0 \end{bmatrix}$ for uncertain nonlinear $Qg(w_1(t), w_2(t))$ then "(2)" and "(3)" In the decomposite system be as following

$$\left.\begin{aligned} w_1' = (A_1 + \delta A_1)w_1 + (B_1 + \delta B_1)u(pw(t))w_1(t) \\ + h_1(w_1(t), w_2(t)) \end{aligned}\right\} \ldots(4)$$

$$0 = w_2 + B_2 u(pw(t))w_1(t) \quad \ldots(5)$$

On using the result of lemma (4.1) and logarithmic norm approach with a necessary conditions, the following stabilizing theorem is developed.

Theorem (4.1) : Consider the system "(1)" with the composite non-bilinear descriptor system "(4)" and "(5)" if the following condition satisfied

1-The nominal system is regular

2- δA is chosento be constant perturbation such that $e^{\mu[A+\delta A]t} \le \gamma e^{-\alpha t} \forall t \ge 0$, α, γ positive integers.

3-

$u(pw(t)) = \dfrac{f(w_1(t))}{\|w_1(0)\|^2}$

$f(w_1(t))$ is a vector function satisfy

$\|f(w_1(t))\| \le f(t)\|w_1(t)\|^2$

The nonlinear control satisfies

$\|u(w(t))\| \le \dfrac{f(t)\,\|(w_1(t))\|^2}{\|w_1(0)\|^2}$

4- the vector of non-linear functions

$\|h_1(w_1(t), w_2(t))\| \le \dfrac{e^{-kt}\|(w_1(t))\|^3}{\|w_1(0)\|^2}$

For some positive constant k.

Then the system "(1)" is an exponentially stable.

Proof: Substitute the above third condition in the algebraic equation "(5)" one can construct the space of consistent initial condition as follows

$$w_k = \{(w_1(0), w_2(0)) | w_2(0) = -B_2 \frac{f(w_1(0))}{\|w_1(0)\|^2} w_1(0),$$

$$w_1(0) \in \mathbb{R}^p \text{ and } w_1(0) \ne 0\}$$

From "(4)"

$$\left.\begin{aligned} w_1' = (A_1 + \delta A_1)w_1 + (B_1 + \delta B_1)u(pw(t))w_1(t) \\ + h_1(w_1(t), w_2(t)) \end{aligned}\right\}$$

Using Dini derivative

$$D_t^+\|w_1(t)\| \le \lim_{h \to 0^+} \sup \frac{\|w_1(t+h)\| - \|w_1(t)\|}{h}$$

$$= \lim_{h \to 0^+} \frac{\|w_1(t) + hw_1'(t)\| - \|w_1(t)\|}{h}$$

$$\left.\begin{aligned} \le \lim_{h \to 0^+} \frac{\|w_1(t) + h(A_1 + \delta A_1)w_1(t)\| - \|w_1(t)\|}{h} \\ + \|(B_1 + \delta B_1)u(pw(t))w_1(t) + h_1(w)\| \end{aligned}\right\}$$

$$\left.\begin{aligned} \le \lim_{h \to 0^+} \frac{\|I + h(A_1 + \delta A_1)\| - 1}{h} \|w_1(t)\| \\ + \|(B_1 + \delta B_1)u(pw(t))w_1(t) + h_1(w)\| \end{aligned}\right\}$$

$$\left.\begin{array}{c} = \mu[A_1+\delta A_1]\|w_1(t)\| \\ +\|(B_1+\delta B_1)u(pw(t))w_1(t)+h_1(w)\| \end{array}\right\}$$

Where

$$\mu[A_1+\delta A_1] = \lim_{h\to 0^+}\frac{\|I+h(A_1+\delta A_1)\|-1}{h} \triangleq \bar{\mu}$$

Now

$$\left.\begin{array}{c} D_t^+\|w_1(t)\| \le \bar{\mu}\|w_1(t)\| + \\ \|(B_1+\delta B_1)u(pw(t))w_1(t)+h_1(w)\| \end{array}\right\} \quad \ldots(6)$$

Multiply both sides of "(6)" by $e^{-\bar{\mu}t}$ to get

$$\left.\begin{array}{c} D_t^+\|w_1(t)\|e^{-\bar{\mu}t}-\bar{\mu}e^{-\bar{\mu}t}\|w_1(t)\| \\ \le \|(B_1+\delta B_1)u(pw(t))w_1(t)+h_1(w)\|e^{-\bar{\mu}t} \end{array}\right\} \quad \ldots(7)$$

And by integrate both sides of "(7) "one can get

$$\left.\begin{array}{c} \|w_1(t)\|e^{-\bar{\mu}t}-\|w_1(0)\| \\ \le \int_0^t\|(B_1+\delta B_1)u(pw(t))w_1(t)+h_1(w)\|e^{-\bar{\mu}s} \end{array}\right\} \quad \ldots(8)$$

Substitute $u(w_1(t))$ in (8) to have

$$\left.\begin{array}{c} \|w_1(t)\| \le e^{\bar{\mu}t}\|w_1(0)\| \\ +\int_0^t e^{\bar{\mu}(t-s)}\left(\|B_1+\delta B_1\|\frac{\|f(s)\|\|w_1(s)\|^2}{\|w_1(0)\|^2}\|w_1(s)\|+h_1(w)\right)ds \end{array}\right\}$$

$$\left.\begin{array}{c} \|w_1(t)\| \le e^{\bar{\mu}t}\|w_1(0)\| \\ \left[1+\int_0^t e^{-\bar{\mu}s}\|B_1+\delta B_1\|\|f(s)\|\|w_1(s)\|^3\|w_1(0)\|^{-3}+\frac{h_1(w)}{\|w_{10}\|}ds\right] \end{array}\right\}$$

By divide both side of the last inequality by $e^{\bar{\mu}t}\|w_1(0)\|$

$$\left.\begin{array}{c} \frac{\|w_1(t)\|}{e^{\bar{\mu}t}\|w_1(0)\|} \le 1+ \\ \int_0^t e^{2\bar{\mu}s}\|B_1+\delta B_1\|\,\|f(s)\|\left[\frac{\|w_1(s)\|}{e^{\mu s}\|w_1(0)\|}\right]^3 \\ +\frac{e^{-ks}\|(w_1(s))\|^3}{e^{\bar{\mu}s}\|w_1(0)\|^3}ds \end{array}\right\}$$

$$\left.\begin{array}{c} \frac{\|w_1(t)\|}{e^{\bar{\mu}t}\|w_1(0)\|} \le 1+ \\ \int_0^t (e^{2\bar{\mu}s}\|B_1+\delta B_1\|\,\|f(s)\|+e^{-ks})\left[\frac{\|w_1(s)\|}{e^{\bar{\mu}s}\|w_1(0)\|}\right]^3 ds \end{array}\right\}$$

From the first condition we have that

$$e^{\bar{\mu}[A+\delta A]t} \le \gamma e^{-\alpha t}\forall t \ge 0$$

On applying lemma (3.6)

$$\left.\begin{array}{c} \frac{\|w_1(t)\|}{e^{\mu t}\|w_1(0)\|} \\ \le \frac{1}{\sqrt{1-2\int_0^t(\gamma^2(\|B_1\|+b_1)e^{-2\alpha s}\|f(s)\|+e^{-ks})ds}} \end{array}\right\}$$

$$\left.\begin{array}{c} \|w_1(t)\| \\ \le \frac{\gamma e^{-\alpha t}\|w_1(0)\|}{\sqrt{1-2\int_0^t(\gamma^2(\|B_1\|+b_1)e^{-2\alpha s}\|f(s)\|+e^{-ks})ds}} \end{array}\right\}$$

Hence $\lim_{t\to\infty}\|w_1(t)\| \to 0$

From "(5)"

$$w_2(t) = -B_2u(pw(t))w_1(t) = -B_2\frac{f(w_1(t))}{\|w_1(0)\|^2}w_1(t)$$

$$\|w_2(t)\| \le -\|B_2\|\frac{\|f(t)\|\|(w_1(t))\|^3}{\|w_1(0)\|^2}$$

Since $w_1(t)$ is exponentially stable , then

$\|w_2(t)\| \to 0 \quad as \ t \to \infty$

And by the linear transformation $w = p^{-1}x$

Thus the original system "(1)" is exponentially stable.

4.3.Example

Consider the descriptor non- bilinear system "(1)"

$$Ex^{\sigma}(t) = (A+\delta A)x(t)+(B+\delta B)u(x)x(t)+g(x(t))$$

Where

$$E=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, A=\begin{bmatrix}0&1&0&0\\1&0&0&0\\-1&0&0&1\\0&1&1&1\end{bmatrix},$$

$$B=\begin{bmatrix}-2&0&0&0\\0&0&2&0\\1&0&-2&0\\0&0&-2&0\end{bmatrix}, \delta A=\begin{bmatrix}-0.02&0&0.1&0\\0&0&-0.1&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$\delta B=\begin{bmatrix}0.2&0&0.01&0\\0&0&-0.1&0\\0&0&0&0\\0&0&0&0\end{bmatrix},, g(x(t))=\begin{bmatrix}e^{-2t}\\e^{-2t}\\0\\0\end{bmatrix}.$$

Solution:

Step(1): $Rank(E) = p = 2$.

Step(2): The nominal system is regular since both E, A sequre matrix and for $\lambda = 1 \notin \sigma(E,A)${the set of all finite spectrum eigenvalue}, $|\lambda E - A| \ne 0$.

Step(3): There is two nonsingular matrices Q, P such that $W = p^{-1}X = \begin{bmatrix}W_1\\w_2\end{bmatrix}$, $w_1 \in \mathbb{R}^2$, $w_2 \in \mathbb{R}^2$, $w_1 = \begin{bmatrix}z_1\\z_2\end{bmatrix}$,$w_2 = \begin{bmatrix}z_3\\z_4\end{bmatrix}$

$$\text{Where } Q=\begin{bmatrix}1&0&1&-1\\0&1&0&0\\0&0&-1&1\\0&0&1&0\end{bmatrix}, P=\begin{bmatrix}1&0&0&0\\-1&-1&1&0\\0&1&0&0\\1&0&0&1\end{bmatrix}$$

$$\text{and } QEP=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, QAP=\begin{bmatrix}-1&-1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},$$

$$QBP=\begin{bmatrix}-1&0&0&0\\0&2&0&0\\-1&0&0&0\\1&2&0&0\end{bmatrix}, Q\delta AP=\begin{bmatrix}-0.02&0.1&0&0\\0&-0.1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$,Q\delta BP=\begin{bmatrix}0.2&0.01&0&0\\0&-0.1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, Qg=\begin{bmatrix}e^{-2t}\\e^{-2t}\\0\\0\end{bmatrix}$$

$$u(w)=\begin{bmatrix}z_1\sin(z_1)&0&0&0\\z_2\cos(z_1)&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

Step(4): To determine the space of consistent initial condition W_k

$$= \left\{ (w_1(0), w_2(0)) | w_2(0) = -B_2 \frac{f(w_1(0))}{\|w_1(0)\|^2} w_1(0),\ w_1(0) \neq 0, \quad where\ w_1 = \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}, \quad w_2 = \begin{bmatrix} w_{21} \\ w_{22} \end{bmatrix} \right\}$$

One can use $w_1(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$\Rightarrow w_2(0) = -\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} / 4$$

$$\Rightarrow w_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$W_k = \{(w_1(0), w_2(0)) | (w_1(0), w_2(0)) = (0, 2, 0, 0)\}.$$

Step(5): Using the Dini derivative to get logarithmic norm as :

$$\mu[A_1 + \delta A_1] = \lim_{h \to 0^+} \frac{\left\| I + h \begin{bmatrix} -1.02 & -1.1 \\ 1 & -0.1 \end{bmatrix} \right\| - 1}{h} \triangleq \bar{\mu}$$

Step(6): On setting $\gamma = 1, \propto = 2$,

$$\|h_1\| = \left\| \begin{bmatrix} e^{-2t} \\ e^{-2t} \end{bmatrix} \right\| \leq \frac{e^{-2t}\|(w_1(t))\|^3}{\|w_1(0)\|^2}$$

one can find

$$\|w_1(t)\| \leq \frac{e^{-2t}\|w_1(0)\|}{\sqrt{1 - 2\int_0^t (1^2(2+1)e^{-4s}\|f(s)\| - e^{-2s})ds}}$$

Hence $\lim_{t \to \infty} \|x_1(t)\| \to 0$

And since $w_2(t) = -B_2 u(w(t)) w_1(t)$

$$\leq -\|B_2\| \frac{\|f(t)\| \|(w_1(t))\|^3}{\|w_1(0)\|^2} = -1/2 \|(w_1(t))\|^3$$

Since $w_1(t)$ is exponentially stable , then
$\|w_2(t)\| \to 0 \quad as\ t \to \infty$

And by the linear transformation $w = p^{-1}x$

Thus the original system “(1)” is exponentially stable.

V. CONCOLUSION

A stabilization feedback controller for uncertain bilinear descriptor had been designed using logarithmic norm approach, with illustration.

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