International Journal of Engineering and Applied Sciences (IJEAS)
ISSN: 2394-3661, Volume-2, Issue-10, October 2015

Event-Triggered Control and H_∞ Control Co-Design for Platoon Control Systems with Parameter Uncertainties and External Disturbances

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***Abstract*—In this paper, an event-triggered control problem in array-like autonomous platoon control system with network-induced delay, parameter uncertainties and external disturbances is investigated. Firstly, a novel six-order linear kinematic model of each autonomous vehicle and a novel state error model of array-like autonomous platoon control system are constructed. Secondly, the corresponding delay system is modelled. Then, by employing the constructed model and Lyapunov functional approach, a co-design method of both the H_∞ controller and the parameters of event-triggering condition for each following vehicle is proposed. The feedback gain matrix and the event-triggering matrix corresponding to each autonomous vehicle can be obtained by employing LMI technique. Finally, a simulation example is presented to demonstrate the effectiveness of the proposed co-design method.**

***Index Terms*—Autonomous Platoon Control, Event-Triggered Control, H_∞ Control, Linear Matrix Inequality, Uncertainties**

I. Introduction

In an autonomous platoon, the spacing and velocity errors of one vehicle may affect the other following vehicles or even amplify as they propagate upstream along the platoon. Therefore, aside from stabilizing each individual vehicle, one significant aspect of platoon control is to guarantee string stability. So far, many literatures have already contributed to this research topic. In [1], a hybrid platoon model with the effects of actuator delay and sensing range limitation is established. Besides, a framework of guaranteed-cost controller design is presented, which can robustly stabilize the platoon and guarantee zero steady-state spacing error.

In recent years, autonomous platoon control systems are usually implemented by introducing wireless network to inter-vehicle network communication [1-7]. However, the amount of transmitted data between vehicles could be extremely large, which may further result in increasing the inter-vehicle network transmission load and the energy consumption of the sensor nodes and controller nodes in each autonomous vehicle. Therefore, how to reduce network transmission load and energy consumption in a network-based autonomous platoon is still a problem need to be addressed.

Traditionally, the periodic control mechanism is used to design control laws in networked control systems (NCSs). Nevertheless, such control strategy requires too much resource usage (i.e., sampling rate, CPU time) to ensure the desired control performance. To overcome such defect, the idea of event-triggered control [8-12] was proposed. The main idea of event-triggered control, that is, the control signal is kept constant until violation of a triggering condition on certain signal of the plant triggers re-computation of the control signals. Such control mechanism can reduce the number of re-computations of the control signals, the amount of data transmission in NCSs, and the energy consumption of sensor or controller nodes. Meanwhile, the desired level of control performance can be guaranteed under event-triggered control.

In [10], the event-triggered control with triggering condition $\|x(t)-x(t_k)\| \le \bar{e}$ was proposed for linear systems with external disturbances. In [11], event-triggered controller design approaches for both linear and nonlinear systems were proposed. However, event detectors in [10-11] need to continuously supervise the plant's state. Under such circumstance, self-triggered scheme, in which the event-triggering time instants are determined by a predictive approach, was proposed in [12]. The self-triggered scheme in [12] can save more energy for sensor nodes and implement the NCSs with less complexity. Unfortunately, as shown in [8] and [12], the average inter-event time based on self-triggered scheme is often smaller than that based on event-triggered scheme. In [13-14], a novel dynamic output feedback based event-triggered scheme for nonlinear NCSs was proposed, which can avoid zero inter-event time phenomenon.

In [15-16], the proposed event detector only needs a supervision of the plant's state in discrete time. Besides, the method proposed in [15-16] can provide the co-design of both the H_∞ controller and the parameters of event-triggering condition. In [17], a co-design method of both the event-triggered H_∞ controller and triggering condition with the effects of data packet dropout was proposed.

In this paper, we introduce the idea of event-triggered control to array-like autonomous platoon control system with the effects of network-induced delay from event detector to controller existing in each vehicle, parameter uncertainties in each vehicle's kinematic model, and external disturbances caused by wind gust and road surface condition. Based on the third-order linear model in [1-2], a novel six-order linear kinematic model of each vehicle and a novel state error model of platoon control system are established, respectively. Furthermore, we model the delay system, which corresponds

This work is supported by National Nature Science Foundation of China (61473153), Natural Science Foundation of Jiangsu Province (BK20131352), A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), and Six Talent Peaks Project in Jiangsu Province (2015-XXRJ-006).

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to the novel state error model we have constructed. Then, a co-design method of both the H_∞ controller and the parameters of event-triggering condition in platoon control systems is proposed. Eventually, the event-triggered based simulation result is presented.

The rest of this paper is organized as follows. In Section Ⅱ, after a novel six-order linear kinematic model of each vehicle and a novel state error model of the platoon control system are constructed, the delay system corresponding to the platoon control system is given. In Section Ⅲ, we present the co-design method of both the H_∞ controller and the parameters of event-triggering condition in platoon control system. Simulation results are presented in Section Ⅳ, showing the advantages of the proposed approach in Section Ⅲ. Finally, the conclusions are given in Section Ⅴ.

II. PROBLEM FORMULATION

Consider an array-like platoon which consists of $(M+1)\times(N+1)$ vehicles. In the platoon, leading vehicle is driven by human driver, and it is numbered as $(0,0)$. Other following vehicles are unmanned vehicles, and their numbers are shown in Figure 1.

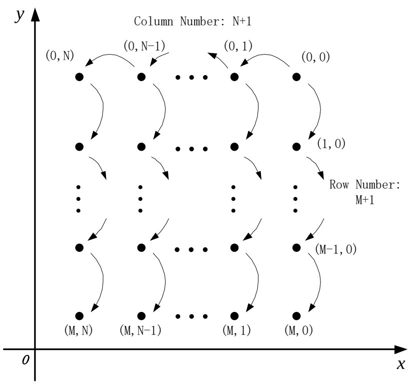

Figure 1. Structure of the array-like platoon and structure of the communication link

In this paper, let the following assumptions be satisfied (see Figure 1):

(1) Each vehicle is assumed to be a mass point. We abstract the platoon control system as a decoupled model without considering vehicle's complex steering system;

(2) As to vehicles in first row, each vehicle (excluding leading vehicle) updates the following information periodically: the longitudinal and lateral displacement, velocity, and acceleration of the vehicle itself and its right neighboring vehicle;

(3) As to vehicles which are not in first row, each vehicle updates the following information periodically: the longitudinal and lateral displacement, velocity, and acceleration of the vehicle itself and its forward neighboring vehicle;

(4) As to vehicles in first row, each vehicle (excluding leading vehicle) can simultaneously (delay-free) receive the update of its right neighboring vehicle's desired longitudinal and lateral acceleration when the event-triggering condition of itself is satisfied;

(5) As to vehicles which are not in first row, each vehicle can simultaneously (delay-free) receive the update of its forward neighboring vehicle's desired longitudinal and lateral acceleration when the event-triggering condition of itself is satisfied.

A. Array-Like Platoon Modelling

In this section, based on the third-order linear model in [1-2], a novel six-order linear kinematic model of each vehicle is established. Then, the state error model of the platoon with the effects of parameter uncertainties and external disturbances is given. In the following part of this paper, the subscription (i,j) in variables and equations represents the vehicle (i,j).

In the first row of the platoon, define vehicle's lateral and longitudinal spacing error as

$$\begin{aligned}\tilde{\delta}_{x,(0,j)}(t)&=\delta_{x,(0,j-1)}(t)-\delta_{x,(0,j)}(t)-L_{x,(0,j)},\\ \tilde{\delta}_{y,(0,j)}(t)&=\delta_{y,(0,j-1)}(t)-\delta_{y,(0,j)}(t),\end{aligned}\tag{1}$$

where $L_{x,(0,j)}$ is the desired lateral spacing error of the vehicle. $\delta_{x,(0,j)}$ and $\delta_{y,(0,j)}$ are the lateral and longitudinal displacement of the vehicle, respectively.

Define the lateral and longitudinal velocity error as

$$\begin{aligned}\tilde{v}_{x,(0,j)}(t)&=v_{x,(0,j-1)}(t)-v_{x,(0,j)}(t),\\ \tilde{v}_{y,(0,j)}(t)&=v_{y,(0,j-1)}(t)-v_{y,(0,j)}(t),\end{aligned}\tag{2}$$

where $v_{x,(0,j)}$ and $v_{y,(0,j)}$ are the lateral and longitudinal velocity of the vehicle, respectively.

Define the lateral and longitudinal acceleration error as

$$\begin{aligned}\tilde{a}_{x,(0,j)}(t)&=a_{x,(0,j-1)}(t)-a_{x,(0,j)}(t),\\ \tilde{a}_{y,(0,j)}(t)&=a_{y,(0,j-1)}(t)-a_{y,(0,j)}(t),\end{aligned}\tag{3}$$

where

$$\begin{aligned}\dot{a}_{x,(0,j)}(t)&=-a_{x,(0,j)}(t)/\varsigma_{x,(0,j)}+a^c_{x,(0,j)}(t)/\varsigma_{x,(0,j)},\\ \dot{a}_{y,(0,j)}(t)&=-a_{y,(0,j)}(t)/\varsigma_{y,(0,j)}+a^c_{y,(0,j)}(t)/\varsigma_{y,(0,j)},\end{aligned}$$

$a_{x,(0,j)}$ and $a_{y,(0,j)}$ are the lateral and longitudinal acceleration of the vehicle, respectively. $a^c_{x,(0,j)}$ and $a^c_{y,(0,j)}$ are the desired lateral and longitudinal acceleration of the vehicle, respectively. $\varsigma_{x,(0,j)}$ and $\varsigma_{y,(0,j)}$ are the time constant of the lag in tracking any desired lateral and longitudinal acceleration of the vehicle, respectively.

Define the lateral and longitudinal desired acceleration error as

$$\begin{aligned}\tilde{u}_{x,(0,j)}(t)&=a^c_{x,(0,j-1)}(t)-a^c_{x,(0,j)}(t),\\ \tilde{u}_{y,(0,j)}(t)&=a^c_{y,(0,j-1)}(t)-a^c_{y,(0,j)}(t).\end{aligned}\tag{4}$$

Let $y_{(0,j)}=\left(\delta_{x,(0,j)},\delta_{y,(0,j)},v_{x,(0,j)},v_{y,(0,j)},a_{x,(0,j)},a_{y,(0,j)}\right)^T$, and define the state vector of the vehicle $(0,j)$ for the state error model between the vehicle $(0,j-1)$ and the vehicle $(0,j)$

$$x_{(0,j)}=\left(\tilde{\delta}_{x,(0,j)},\tilde{\delta}_{x,(0,j)},\tilde{v}_{x,(0,j)},\tilde{v}_{y,(0,j)},\tilde{a}_{x,(0,j)},\tilde{a}_{y,(0,j)}\right)^T.\tag{5}$$

Define the control input vector of the vehicle $(0,j)$ for the state error model between the vehicle $(0,j-1)$ and the vehicle $(0,j)$

$$\tilde{u}_{(0,j)}(t)=\left(\tilde{u}_{x,(0,j)}(t)\quad \tilde{u}_{y,(0,j)}(t)\right)^T. \tag{6}$$

Let $q_{(0,j)}(t)=\left(L_{x,(0,j)}\quad 0\quad 0\quad 0\quad 0\quad 0\right)^T$. Therefore, $x_{(0,j)}(t)=y_{(0,j-1)}(t)-y_{(0,j)}(t)-q_{(0,j)}(t)$. So the state space expression corresponding to the state error model is

$$\dot{x}_{(0,j)}(t)=A_{(0,j)}x_{(0,j)}(t)+B_{(0,j)}\tilde{u}_{(0,j)}(t),\ j=1,2,...,N, \tag{7}$$

where

$$A_{(0,j)}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & -\dfrac{1}{\varsigma_{x,(0,j)}} & 0\\ 0 & 0 & 0 & 0 & 0 & -\dfrac{1}{\varsigma_{y,(0,j)}} \end{bmatrix},$$

$$B_{(0,j)}=\begin{bmatrix} 0 & 0 & 0 & 0 & \dfrac{1}{\varsigma_{x,(0,j)}} & 0\\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{\varsigma_{y,(0,j)}} \end{bmatrix}^T.$$

Similar to modelling the vehicles which are in first row, we can easily obtain the state error model of the vehicles which are not in first row. For vehicles which are numbered as (i,j), $i=1,2,...,M$, $j=0,1,2,...,N$, define vehicle's lateral and longitudinal spacing error as

$$\begin{aligned}\tilde{\delta}_{x,(i,j)}(t)&=\delta_{x,(i-1,j)}(t)-\delta_{x,(i,j)}(t),\\ \tilde{\delta}_{y,(i,j)}(t)&=\delta_{y,(i-1,j)}(t)-\delta_{y,(i,j)}(t)-L_{y,(i,j)},\end{aligned} \tag{8}$$

where $L_{y,(i,j)}$ is the desired longitudinal spacing error of the vehicle. $\delta_{x,(i,j)}$ and $\delta_{y,(i,j)}$ are the lateral and longitudinal displacement of the vehicle, respectively.

Concisely speaking, the definitions of the lateral and longitudinal velocity error, the lateral and longitudinal acceleration error as well as the lateral and longitudinal desired acceleration error between the vehicle $(i-1,j)$ and the vehicle (i,j) are similar to the case of the vehicles which are in first row, except for the treatment of subscripts of the variables.

Let $y_{(i,j)}=\left(\delta_{x,(i,j)},\delta_{y,(i,j)},v_{x,(i,j)},v_{y,(i,j)},a_{x,(i,j)},a_{y,(i,j)}\right)^T$, and define the state vector of the vehicle (i,j) for the state error model between the vehicle $(i-1,j)$ and the vehicle (i,j)

$$x_{(i,j)}=\left(\tilde{\delta}_{x,(i,j)},\tilde{\delta}_{x,(i,j)},\tilde{v}_{x,(i,j)},\tilde{v}_{y,(i,j)},\tilde{a}_{x,(i,j)},\tilde{a}_{y,(i,j)}\right)^T. \tag{9}$$

Define the control input vector of the vehicle (i,j) for the state error model between the vehicle $(i-1,j)$ and the vehicle (i,j)

$$\tilde{u}_{(i,j)}(t)=\left(\tilde{u}_{x,(i,j)}(t)\quad \tilde{u}_{y,(i,j)}(t)\right)^T. \tag{10}$$

Let $q_{(i,j)}(t)=\left(0\quad L_{y,(i,j)}\quad 0\quad 0\quad 0\quad 0\right)^T$. Therefore, $x_{(i,j)}(t)=y_{(i-1,j)}(t)-y_{(i,j)}(t)-q_{(i,j)}(t)$. So the state space expression corresponding to the state error model is

$$\begin{aligned}&\dot{x}_{(i,j)}(t)=A_{(i,j)}x_{(i,j)}(t)+B_{(i,j)}\tilde{u}_{(i,j)}(t),\\ &i=1,2,...,M,\ j=0,1,2,...,N,\end{aligned} \tag{11}$$

where $A_{(i,j)}$ and $B_{(i,j)}$ are obtained from $A_{(0,j)}$ and $B_{(0,j)}$ by replacing $\varsigma_{x,(0,j)}$ and $\varsigma_{y,(0,j)}$ with $\varsigma_{x,(i,j)}$ and $\varsigma_{y,(i,j)}$, respectively.

Then, we also consider parameter uncertainties of parameters $\varsigma_{x,(i,j)}$ and $\varsigma_{y,(i,j)}$ in the state error model, where $i=0,1,2,...,M$, $j=0,1,2,...,N$, and i,j are not equal to zero simultaneously. The matrices $A_{(i,j)}$ and $B_{(i,j)}$ satisfy the following assumption

$$\begin{aligned}&A_{(i,j)}=A_{0,(i,j)}+\Delta A_{(i,j)}(t),B_{(i,j)}=B_{0,(i,j)}+\Delta B_{(i,j)}(t),\\ &\left[\Delta A_{(i,j)}(t)\quad \Delta B_{(i,j)}(t)\right]=H_{(i,j)}F_{(i,j)}(t)\left[E_{1,(i,j)}\quad E_{2,(i,j)}\right],\end{aligned} \tag{12}$$

where $A_{0,(i,j)},B_{0,(i,j)},H_{(i,j)},E_{1,(i,j)}$, and $E_{2,(i,j)}$ are known constant matrices, and $F_{(i,j)}^T(t)F_{(i,j)}(t)\le I$.

We assume that $H_{(i,j)},E_{1,(i,j)}$, and $E_{2,(i,j)}$ have the following form

$$H_{(i,j)}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & h_{1,(i,j)} & 0\\ 0 & 0 & 0 & 0 & 0 & h_{2,(i,j)} \end{bmatrix}, \tag{13}$$

$$F_{(i,j)}(t)=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & f_{1,(i,j)} & 0\\ 0 & 0 & 0 & 0 & 0 & f_{2,(i,j)} \end{bmatrix}\cdot\sin(t), \tag{14}$$

$$E_{1,(i,j)}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -\varepsilon_{1,(i,j)} & 0\\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2,(i,j)} \end{bmatrix}, \tag{15}$$

$$E_{2,(i,j)}=\begin{bmatrix} 0 & 0 & 0 & 0 & \varepsilon_{1,(i,j)} & 0\\ 0 & 0 & 0 & 0 & 0 & \varepsilon_{2,(i,j)} \end{bmatrix}^T. \tag{16}$$

Next, we also consider the external disturbances influencing each vehicle in the platoon. These external disturbances are caused by wind gust and road surface condition. So the state space expression corresponding to the state error model of each following vehicle can be rewritten as

$$\dot{x}_{(i,j)}(t)=A_{(i,j)}x_{(i,j)}(t)+B_{(i,j)}\tilde{u}_{(i,j)}(t)+B_{\omega,(i,j)}\omega_{(i,j)}(t), \tag{17}$$

$$z_{(i,j)}(t)=C_{(i,j)}x_{(i,j)}(t)+D_{(i,j)}\tilde{u}_{(i,j)}(t), \tag{18}$$

where $x_{(i,j)}(t)\in\mathbb{R}^6$, $\tilde{u}_{(i,j)}(t)\in\mathbb{R}^2$, $\omega_{(i,j)}(t)\in\mathbb{R}^p$, and $z_{(i,j)}(t)\in\mathbb{R}^6$ are the state vector, control input vector, external disturbance input vector, and output vector of the state error model, respectively. Each element of $B_{\omega,(i,j)}\omega_{(i,j)}(t)$ denotes external disturbances influencing $\delta_{x,(i,j)}$, $\delta_{y,(i,j)}$, $v_{x,(i,j)}$, $v_{y,(i,j)}$, $a_{x,(i,j)}$, and $a_{y,(i,j)}$, respectively. It is noted that the units of the corresponding elements in $B_{\omega,(i,j)}\omega_{(i,j)}(t)$ are m, m, m/s, m/s, m/s^2 and m/s^2, respectively. $\omega_{(i,j)}(t)\in L_2[0,\infty)$.

B. Delay System Modelling

In this paper, we assume that the array-like platoon control system formulated by (17)-(18) is controlled through network, and the control structure of the system is shown in Figure 2 and Figure 3.

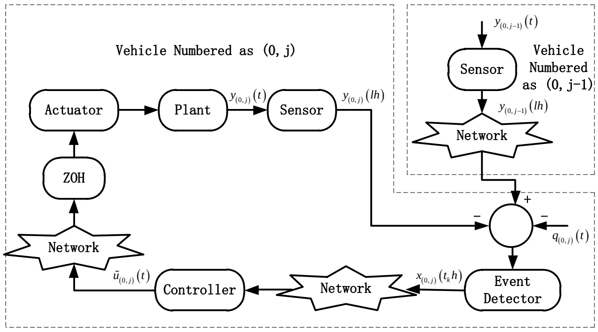

Figure 2. Control structure of each following vehicle in first row ($j=1,2,...,N$)

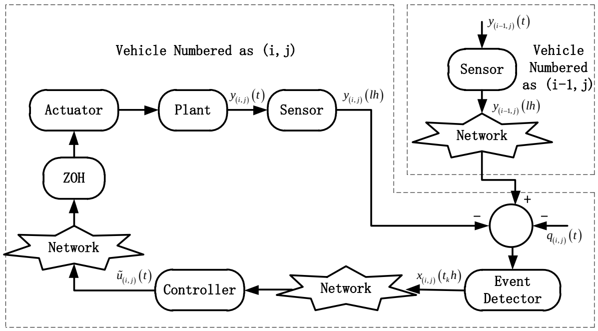

Figure 3. Control structure of each following vehicle which are not in first row ($i=1,2,...,M,\ j=0,1,2,...,N$)

The purpose of this paper is to design an appropriate state feedback controller $\tilde{u}_{(i,j)}(t)=K_{(i,j)}x_{(i,j)}(t)$ for each following vehicle, such that the resulting closed-loop system satisfies the required performance. In this paper, we choose the H_∞ performance as the required performance. Additionally, in order to significantly reduce data transmission in network, we add an event detector in each vehicle, as shown in Figure 2 and Figure 3.

Assume that the sampling time instants of sensors in each vehicle are $t=lh, l=0,1,2,...$, where $h>0$ is a fixed sampling period. Moreover, assume that the event-triggering time instants of each following vehicle are $t=t_{k,(i,j)}h$, $k=0,1,2,...$, where $t_{0,(i,j)}h=0$ is the initial time, and $s_{k,(i,j)}h=t_{k+1,(i,j)}h-t_{k,(i,j)}h$ denotes the inter-event time. For the purpose of concisely presenting the analysis process in the following part of this paper, we use $t_k h$ to denote each following vehicle's event-triggering time.

We design the following event-triggering condition

$$\eta^T_{(i,j)}(i_k h)\Omega_{(i,j)}\eta_{(i,j)}(i_k h)\le \sigma x^T_{(i,j)}(t_k h+lh)\Omega_{(i,j)}x_{(i,j)}(t_k h+lh), \tag{19}$$

where $\eta_{(i,j)}(i_k h)=x_{(i,j)}(i_k h)-x_{(i,j)}(t_k h)$, $i_k h=t_k h+lh$, $\Omega_{(i,j)}$ are positive definite matrices, $l=1,2,...$, and $\sigma\in[0,1)$.

Remark 1: According to (19), the state vector $x_{(i,j)}(t_k h+lh)$ which satisfies the inequality (19) will not be sent to the controller of the vehicle (i,j). Only the one which violates the inequality (19) will be sent to the controller through network.

For each following vehicle, we assume that there exists network-induced delay τ_k in network communication channel from event detector to controller, where $\tau_k\in[0,\bar{\tau})$ and $0<\bar{\tau}<h$.

Under the event-triggering condition (19) and the state feedback controller $\tilde{u}_{(i,j)}(t)=K_{(i,j)}x_{(i,j)}(t_k h)$, $t\in[t_k h+\tau_k, t_{k+1}h+\tau_{k+1})$, the state error model (17)-(18) can be rewritten as

$$\dot{x}_{(i,j)}(t)=A_{(i,j)}x_{(i,j)}(t)+B_{(i,j)}K_{(i,j)}x_{(i,j)}(t_k h)+B_{\omega,(i,j)}\omega_{(i,j)}(t), \tag{20}$$

$$z_{(i,j)}(t)=C_{(i,j)}x_{(i,j)}(t)+D_{(i,j)}K_{(i,j)}x_{(i,j)}(t_k h). \tag{21}$$

By employing the delay system analysis method in [15-17], we can rewrite (19)-(21) in another form. The event-triggering condition (19) can be rewritten as

$$e^T_{k,(i,j)}(t)\Omega_{(i,j)}e_{k,(i,j)}(t)\le \sigma x^T_{(i,j)}(t-\tau(t))\Omega_{(i,j)}x_{(i,j)}(t-\tau(t)), \tag{22}$$

where $0\le\tau_k\le\tau(t)\le h+\bar{\tau}\triangleq\tau_M$. The definitions of $\tau(t)$ and $e_{k,(i,j)}(t)$ are similar to the definitions in [15-16].

For $t\in[t_k h+\tau_k, t_{k+1}h+\tau_{k+1})$, (20)-(21) can be rewritten as

$$\begin{aligned}\dot{x}_{(i,j)}(t)&=A_{(i,j)}x_{(i,j)}(t)+B_{(i,j)}K_{(i,j)}x_{(i,j)}(t-\tau(t))\\&+B_{(i,j)}K_{(i,j)}e_{k,(i,j)}(t)+B_{\omega,(i,j)}\omega_{(i,j)}(t),\end{aligned} \tag{23}$$

$$\begin{aligned}z_{(i,j)}(t)&=C_{(i,j)}x_{(i,j)}(t)+D_{(i,j)}K_{(i,j)}x_{(i,j)}(t-\tau(t))\\&+D_{(i,j)}K_{(i,j)}e_{k,(i,j)}(t).\end{aligned} \tag{24}$$

Here, we give the initial function of $x_{(i,j)}(t)$ as

$$x_{(i,j)}(t)=\phi_{(i,j)}(t), t\in[-\tau_M,0], \tag{25}$$

where $\phi_{(i,j)}(t)$ is a continuous function defined on the time interval $[-\tau_M,0]$.

III. THE CO-DESIGN OF EVENT DETECTOR AND EVENT-TRIGGERED H∞ CONTROLLER

In this section, for given a disturbance attenuation level γ, under the event-triggering condition (22), considering the system described by (23)-(24), the co-design method of the event-triggering condition and linear state feedback event-triggered H_∞ controller is proposed, such that the platoon control system (23)-(24) is robustly asymptotically stable.

Before presenting the main results, we give the definition of robustly asymptotically stable and a lemma.

Definition 1: The closed-loop system (23)-(24) is said to be asymptotically stable with an H_∞ disturbance attenuation level γ if it satisfies the following two requirements:

(1) When $\omega_{(i,j)}(t)\equiv 0$, the closed-loop system (23)-(24) is asymptotically stable;

(2) Under zero initial condition, for any nonzero $\omega_{(i,j)}(t)\in L_2[0,\infty)$, the output vector $z_{(i,j)}(t)$ of the closed-loop system (23)-(24) satisfies $\left\|z_{(i,j)}(t)\right\|_2 \le \gamma\left\|\omega_{(i,j)}(t)\right\|_2$.

Lemma 1 [15]: For matrices $R>0$ and $X^T=X$, we have $-XR^{-1}X\le\rho^2R-2\rho X$, where ρ is an arbitrarily selected constant.

In this section, we will extend the main results in [15] to a distributed control system formulated by (23)-(24). Furthermore, we will apply the extended theoretical results in this paper to the array-like platoon control system constructed in the previous section of this paper.

By using the Lyapunov functional approach, we first provide a robustly asymptotic stability criterion for the closed-loop system (23)-(24).

Theorem 1: For some given parameters γ, σ and matrix $K_{(i,j)}$, under the event-triggering condition (22), the system (23)-(24) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $P>0$, $Q>0$, $R>0$, and $\Omega_{(i,j)}>0$, matrices N and M with appropriate dimensions, such that for $r=1,2$

$$\begin{bmatrix} W & * & * & * & * \\ \Phi_{21}(r) & -R & * & * & * \\ \Phi_{31} & 0 & -\gamma^2 I & * & * \\ \Phi_{41} & 0 & \sqrt{\tau_M}RB_{\omega,(i,j)} & -R & * \\ \Phi_{51} & 0 & 0 & 0 & -I \end{bmatrix}<0, \quad (26)$$

where

$$W=\Phi_{11}+\Gamma+\Gamma^T, \Gamma=\begin{bmatrix}-N & N-M & M & 0\end{bmatrix},$$

$$\Phi_{11}=\begin{bmatrix} PA_{(i,j)}+A_{(i,j)}^TP+Q & * & * & * \\ K_{(i,j)}^TB_{(i,j)}^TP & \sigma\Omega_{(i,j)} & * & * \\ 0 & 0 & -Q & * \\ K_{(i,j)}^TB_{(i,j)}^TP & 0 & 0 & -\Omega_{(i,j)} \end{bmatrix},$$

$$\Phi_{21}(1)=\sqrt{\tau_M}N^T, \Phi_{21}(2)=\sqrt{\tau_M}M^T,$$

$$\Phi_{31}=\begin{bmatrix}B_{\omega,(i,j)}^TP & 0 & 0 & 0\end{bmatrix},$$

$$\Phi_{41}=\begin{bmatrix}\sqrt{\tau_M}RA_{(i,j)}, \sqrt{\tau_M}RB_{(i,j)}K_{(i,j)}, 0, \sqrt{\tau_M}RB_{(i,j)}K_{(i,j)}\end{bmatrix},$$

$$\Phi_{51}=\begin{bmatrix}C_{(i,j)} & D_{(i,j)}K_{(i,j)} & 0 & D_{(i,j)}K_{(i,j)}\end{bmatrix}.$$

Proof: Construct the following Lyapunov function

$$V_{(i,j)}(t)=x_{(i,j)}^T(t)Px_{(i,j)}(t)+\int_{t-\tau_M}^{t}x_{(i,j)}^T(s)Qx_{(i,j)}(s)ds +\int_{t-\tau_M}^{t}\int_{s}^{t}\dot{x}_{(i,j)}^T(v)R\dot{x}_{(i,j)}(v)dvds, \quad (27)$$

where P, Q, and R are positive definite matrices with appropriate dimensions. By using a similar method to the proof in [18] and recalling (22), we can conclude that if (26) is satisfied, then the system (23)-(24) is asymptotically stable with an H_∞ disturbance attenuation level γ. Subject to the page limitation, detailed proof process is omitted here.

It should be noted that $\Delta A_{(i,j)}(t)$ and $\Delta B_{(i,j)}(t)$ presenting the parameter uncertainties of the state error model are contained in (26). Therefore, Theorem 1 cannot be directly used to determine the event-triggering matrix $\Omega_{(i,j)}$.

Then, we will provide a sufficient condition for guaranteeing the feasibility of (26). Such robustly asymptotic stability criterion can be directly used to determine the event-triggering matrix $\Omega_{(i,j)}$. By using Shur complement and combining (12) and (26), the proof of Theorem 2 can easily be concluded.

Theorem 2: For some given parameters γ, σ and matrix $K_{(i,j)}$, under the event-triggering condition (22), the system (23)-(24) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $P>0$, $Q>0$, $R>0$, and $\Omega_{(i,j)}>0$, matrices N and M with appropriate dimensions and a scalar $\varepsilon>0$, such that for $r=1,2$

$$\begin{bmatrix} W' & * & * & * & * & * \\ \Phi_{21}(r) & -R & * & * & * & * \\ \Phi_{31} & 0 & -\gamma^2 I & * & * & * \\ \Phi_{41}' & 0 & \sqrt{\tau_M}RB_{\omega,(i,j)} & -R & * & * \\ \Phi_{51} & 0 & 0 & 0 & -I & * \\ \Phi_{61} & 0 & 0 & \Phi_{64} & 0 & \Phi_{66} \end{bmatrix}<0, \quad (28)$$

where $W'=\Phi_{11}'+\Gamma+\Gamma^T$, Φ_{11}' and Φ_{41}' are obtained from Φ_{11} and Φ_{41} by replacing $A_{(i,j)}$ and $B_{(i,j)}$ with $A_{0,(i,j)}$ and $B_{0,(i,j)}$, respectively, and

$$\Phi_{61}=\begin{bmatrix} H_{(i,j)}^TP & 0 & 0 & 0 \\ \varepsilon E_{1,(i,j)} & \varepsilon E_{2,(i,j)}K_{(i,j)} & 0 & \varepsilon E_{2,(i,j)}K_{(i,j)} \end{bmatrix},$$

$$\Phi_{64}=\begin{bmatrix}\sqrt{\tau_M}H_{(i,j)}^TR \\ 0\end{bmatrix}, \Phi_{66}=diag\{-\varepsilon I,-\varepsilon I\}.$$

Next, we will provide a robustly asymptotic stability criterion which can be directly used to co-design the feedback gain matrix $K_{(i,j)}$ and the event-triggering matrix $\Omega_{(i,j)}$.

Theorem 3: For some given parameters γ, σ, and ρ, under the event-triggering condition (22) and the feedback gain matrix $K_{(i,j)}=YX^{-1}$, the system (23)-(24) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $X>0$, $\tilde{Q}>0$, $\tilde{R}>0$, and $\tilde{\Omega}_{(i,j)}>0$, matrices $\tilde{N}$, $\tilde{M}$, and Y with appropriate dimensions and a scalar $\mu>0$, such that for $r=1,2$

$$\begin{bmatrix} \hat{W} & * & * & * & * & * \\ \Sigma_{21}(r) & -\tilde{R} & * & * & * & * \\ \Sigma_{31} & 0 & -\gamma^2 I & * & * & * \\ \Sigma_{41} & 0 & \sqrt{\tau_M} B_{\omega,(i,j)} & \hat{W}_{44} & * & * \\ \Sigma_{51} & 0 & 0 & 0 & -I & * \\ \Sigma_{61} & 0 & 0 & \Sigma_{64} & 0 & \Sigma_{66} \end{bmatrix} < 0, \quad (29)$$

where

$$\hat{W} = \Sigma_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T, \hat{W}_{44} = \rho^2 \tilde{R} - 2\rho X, \tilde{\Gamma} = \begin{bmatrix} -\tilde{N} & \tilde{N} - \tilde{M} & \tilde{M} & 0 \end{bmatrix},$$

$$\Sigma_{11} = \begin{bmatrix} A_{0,(i,j)} X + X A_{0,(i,j)}^T + \tilde{Q} & * & * & * \\ Y^T B_{0,(i,j)}^T & \sigma \tilde{\Omega}_{(i,j)} & * & * \\ 0 & 0 & -\tilde{Q} & * \\ Y^T B_{0,(i,j)}^T & 0 & 0 & -\tilde{\Omega}_{(i,j)} \end{bmatrix},$$

$$\Sigma_{21}(1) = \sqrt{\tau_M} \tilde{N}^T, \Sigma_{21}(2) = \sqrt{\tau_M} \tilde{M}^T, \Sigma_{31} = \begin{bmatrix} B_{\omega,(i,j)}^T & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_{41} = \begin{bmatrix} \sqrt{\tau_M} A_{0,(i,j)} X & \sqrt{\tau_M} B_{0,(i,j)} Y & 0 & \sqrt{\tau_M} B_{0,(i,j)} Y \end{bmatrix},$$

$$\Sigma_{51} = \begin{bmatrix} C_{(i,j)} X & D_{(i,j)} Y & 0 & D_{(i,j)} Y \end{bmatrix},$$

$$\Sigma_{61} = \begin{bmatrix} \mu H_{(i,j)}^T & 0 & 0 & 0 \\ E_{1,(i,j)} X & E_{2,(i,j)} Y & 0 & E_{2,(i,j)} Y \end{bmatrix},$$

$$\Sigma_{64} = \begin{bmatrix} \sqrt{\tau_M} \mu H_{(i,j)}^T \\ 0 \end{bmatrix}, \Sigma_{66} = diag\{-\mu I, -\mu I\}.$$

Proof: Defining $X = P^{-1}$, pre-multiplying and post-multiplying (28) with $diag\{X, X, X, X, X, I, R^{-1}, I, \varepsilon^{-1} I, \varepsilon^{-1} I\}$. Defining new matrix variables $\tilde{Q} = XQX$, $\tilde{R} = XRX$, $\tilde{\Omega}_{(i,j)} = X\Omega_{(i,j)} X$, $\tilde{N} = diag\{X, X, X, X\} NX$, $\tilde{M} = diag\{X, X, X, X\} MX$, $Y = K_{(i,j)} X$, $\mu = \varepsilon^{-1}$. According to Lemma 1, we can obtain that $-R^{-1} = -X\tilde{R}^{-1} X \le \rho^2 \tilde{R} - 2\rho X$. Then, (29) can be concluded from (28), which completes the proof.

Remark 2: If given parameters γ, σ, and ρ, then we can co-design $K_{(i,j)}$ and $\Omega_{(i,j)}$ by solving a set of LMIs in (29).

Remark 3: When the parameters γ, σ, and ρ are fixed, we can obtain the upper bound of τ_M by using Theorem 3. Since $\tau_M = h + \overline{\tau}$, so if we know $\overline{\tau}$, then the allowable maximum sampling period of sensors in each vehicle is $h = \tau_M - \overline{\tau}$. By employing such method to choose sampling period, we can further reduce data transmissions between neighboring vehicles which have network communication link. Meanwhile, sensors installed on each vehicle will become far more energy-saving.

IV. SIMULATION RESULT

We consider an array-like platoon which consists of 3×3 vehicles. The desired lateral and longitudinal spacing error between neighboring vehicles are $4m$ and $5m$, respectively. The initial state of the leading vehicle is $y_{(0,0)}(0) = (10 \quad 0 \quad 0 \quad 12 \quad 0 \quad 0)^T$. The units of the corresponding elements in both $y_{(i,j)}(t)$ and $x_{(i,j)}(t)$ are m, m, m/s, m/s, m/s^2 and m/s^2, respectively. Moreover, the leading vehicle keeps moving toward the positive direction of the y axis with constant velocity 12 m/s. The initial state errors of each following vehicle for the state error model are given as follows

$$x_{(0,1)}(0) = (2 \quad 1.5 \quad 3 \quad -2 \quad 1 \quad 0.5)^T,$$
$$x_{(0,2)}(0) = (-1 \quad 0.5 \quad 2 \quad -1 \quad 1.2 \quad -0.8)^T,$$
$$x_{(1,0)}(0) = (-1.8 \quad 1 \quad -2.5 \quad -0.5 \quad -0.8 \quad 0.4)^T,$$
$$x_{(2,0)}(0) = (1.5 \quad -0.5 \quad -3 \quad 2 \quad -1 \quad -0.5)^T,$$
$$x_{(1,1)}(0) = (-1 \quad -0.3 \quad 2 \quad 0.8 \quad -1 \quad 0.4)^T,$$
$$x_{(2,1)}(0) = (2 \quad 1 \quad -2.5 \quad -1 \quad -1.2 \quad 0.5)^T,$$
$$x_{(1,2)}(0) = (1.8 \quad -0.8 \quad 3 \quad 2 \quad -1 \quad -0.5)^T,$$
$$x_{(2,2)}(0) = (-2 \quad -1 \quad -1 \quad 0.5 \quad 0.8 \quad 0.2)^T.$$

Each vehicle's time constant of the lag in tracking any desired lateral and longitudinal acceleration are given as follows

$$\varsigma_{x,(0,1)} = 0.35, \varsigma_{y,(0,1)} = 0.20, \varsigma_{x,(0,2)} = 0.42, \varsigma_{y,(0,2)} = 0.25,$$
$$\varsigma_{x,(1,0)} = 0.45, \varsigma_{y,(1,0)} = 0.30, \varsigma_{x,(2,0)} = 0.38, \varsigma_{y,(2,0)} = 0.35,$$
$$\varsigma_{x,(1,1)} = 0.35, \varsigma_{y,(1,1)} = 0.40, \varsigma_{x,(2,1)} = 0.37, \varsigma_{y,(2,1)} = 0.35,$$
$$\varsigma_{x,(1,2)} = 0.45, \varsigma_{y,(1,2)} = 0.30, \varsigma_{x,(2,2)} = 0.43, \varsigma_{y,(2,2)} = 0.25.$$

We suppose that parameters which are related to parameter uncertainties in $\varsigma_{x,(i,j)}$ and $\varsigma_{y,(i,j)}$ are chosen as follows

$$h_{1,(i,j)} = 0.5, h_{2,(i,j)} = 0.6, f_{1,(i,j)} = 1.0,$$
$$f_{2,(i,j)} = 1.0, \varepsilon_{1,(i,j)} = 0.6, \varepsilon_{2,(i,j)} = 1.0,$$

where $i = 0,1,2$, $j = 0,1,2$, and i, j are not equal to zero simultaneously. As to the external disturbances, we suppose that

$$B_{\omega,(i,j)} = (1 \quad 1 \quad 1 \quad 1)^T, \omega_{(i,j)}(t) = \begin{cases} 0.02\sin(2\pi t), t \in [0,15), \\ 0, otherwise, \end{cases}$$

By choosing $\rho = 0.5$, $\gamma = 50$, $\sigma = 0.1$, and applying Theorem 3, we can obtain the maximum allowable value of τ_M corresponding to each following vehicle. Then, we choose the minimum one among these maximum allowable values of τ_M for further simulation, which is $0.34s$. Also, we suppose that $\overline{\tau} = 0.14s$, so $h = 0.2s$. By solving the LMIs (29), we can obtain $K_{(i,j)}$ and $\Omega_{(i,j)}$ corresponding to each following vehicle. Subject to the page limitation, we only present the results of $K_{(0,1)}$ and $\Omega_{(0,1)}$ as follows

$$K_{(0,1)} = \begin{pmatrix} -0.41 & -0.02 & -1.33 & -0.12 & -0.31 & -0.02 \\ -0.01 & -0.15 & -0.01 & -0.91 & -0.01 & -0.11 \end{pmatrix},$$

$$\Omega_{(0,1)} = \begin{pmatrix} 0.61 & -0.01 & -0.66 & 0.01 & 0.53 & -0.01 \\ -0.01 & 0.12 & 0 & -0.09 & 0 & 0.12 \\ -0.66 & 0 & 1.24 & 0 & -1.39 & 0.01 \\ 0.01 & -0.09 & 0 & 0.18 & 0 & -0.26 \\ 0.53 & 0 & -1.39 & 0 & 2.80 & -0.01 \\ -0.01 & 0.12 & 0.01 & -0.26 & -0.01 & 0.44 \end{pmatrix},$$

Under event-triggered control, the state error response of the vehicle (0, 1) is shown in Figure 4. Each following vehicle's event-triggering time and inter-event time are shown in Figure 5 and Figure 6. In each following vehicle, from event detector to controller, the average inter-event time and the percentage of transmitted data in total sampled data are shown in Table 1.

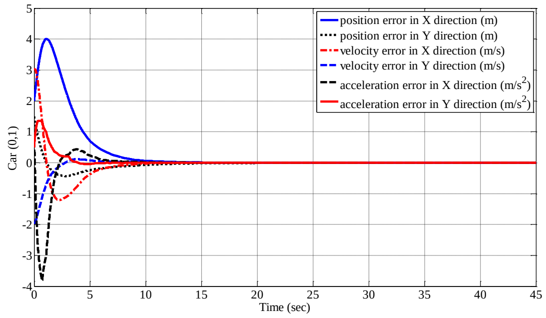

Figure 4. State error response of the vehicle (0, 1)

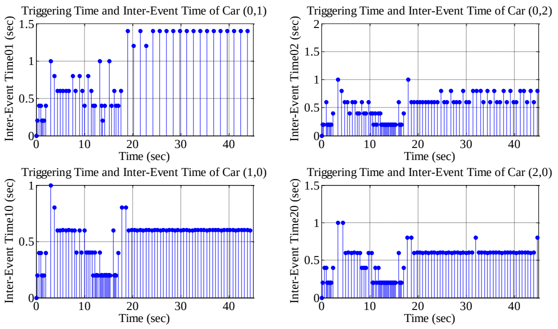

Figure 5. Event-triggering time and inter-event time of vehicles numbered as (0, 1), (0, 2), (1, 0), (2, 0)

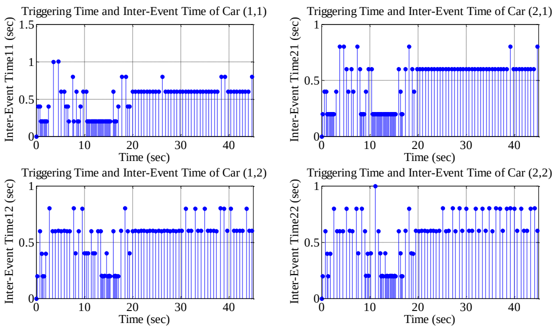

Figure 6. Event-triggering time and inter-event time of vehicles numbered as (1, 1), (2, 1), (1, 2), (2, 2)

Table 1. The average inter-event time and the percentage of transmitted data in total sampled Data

Vehicle Number	Average Inter-Event Time (Second)	Percentage of Transmitted Data (%)
(0, 1)	0.8036	25.22
(0, 2)	0.4891	41.15
(1, 0)	0.5011	39.82
(2, 0)	0.4695	42.48
(1, 1)	0.4619	43.36
(2, 1)	0.4392	45.58
(1, 2)	0.6000	38.05
(2, 2)	0.5349	37.17

From Figure 4, it can be seen that, before $t = 10s$, the state error of the vehicle (0, 1) tends to be zero. The state error responses of other following vehicles are similar to the state error response of this one. From Figure 5, Figure 6, and Table 1, it can be concluded that the proposed event-triggered scheme can reduce the transmitted data between neighboring vehicles and greatly reduce data transmissions from event detector to controllers in each autonomous vehicle.

V. CONCLUSION

In this paper, we have investigated event-triggered control in array-like autonomous platoon control system with network-induced delay, parameter uncertainties and external disturbances. Firstly, a novel six-order linear kinematic model of each autonomous vehicle and a novel state error model of the platoon control system were constructed. Secondly, the corresponding delay system was modelled. Then, a co-design method of both the H_∞ controller and the parameters of event-triggering condition for each following vehicle was proposed. Finally, a simulation example was presented. The simulation result shows that the proposed co-design method can robustly stabilize the platoon longitudinally and laterally, reduce the transmitted data between neighboring vehicles and greatly reduce data transmissions from event detector to controller in each autonomous vehicle.

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