International Journal of Engineering and Applied Sciences (IJEAS)
ISSN: 2394-3661, Volume-2, Issue-6, June 2015

Some Classications of Ditopological Texture Spaces via Cardinal Functions

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***Abstract*— In this paper we study under which conditions equality of some pairs of dicardinal functions such as weight-coweight and densification-codensification, holds. We obtain some useful results on "bounds of S", the set P of all p-sets and the set Q of all q-sets by choosing the class of all ditopological texture spaces or the subclass satisfying axiom T_0.**

***Index Terms*— Texture, Ditopological texture space, Cardinal function, Dicardinal function.**

I. INTRODUCTION

The concept of fuzzy structure introduced first by L. M. Brown in [13, 14] is renamed texture space by being developed in [10, 9]. The structure open the way for investigating mathematical concepts without any complement in consideration of the fact that, in a texture space $(S,\mathcal{S})$, $\mathcal{S}$ doesn't need to be closed under set-theoretical complement. Based on the structure of texture space, it is obvious intuitively that a convenient topology on a texture space doesn't need to hold the existence of the known duality of interior and closure and so not need to hold both axioms of open sets and ones of closed sets.

L. M. Brown et al. present first two papers in 2004 and last one in 2006 [5, 6, 7]. In the first of them subtitled 'Basic concepts', the authors introduce a systematic form of the concepts of direlation, difunction, the category *dfTex* ditopological texture space in a categorical setting. In second paper, the category *dfDitop* of ditopological texture spaces and bicontinuous difunctions is defined. The subject of third paper is on separation axioms in general ditopological texture space. In a ditopological setting , L. M. Brown and M. M. Gohar study compactness in 2009, and strong compactness one year later [11, 21].

There is no doubt that cardinal invariants play a major role in general topology of which set theory forms the basis. They are most useful tools in classifying topological spaces; so they distinguish some important classes of topological spaces, e.g. compact spaces, finally compact spaces, the spaces with a countable basis, and separable spaces. Also, the remarkable feature of them is that they need not have an additional structure on topological spaces. Moreover, cardinal invariants enable us to compare quantitively topological properties, and to generalize the some known results of them.

The theory of cardinal functions contributed by many researchers has been being developed since 1920. In the 1920's, Alexandroff and Urysohn show that every compact, perfectly normal space has cardinality $\leq 2^{\omega}$ [1]. In 1940's, it is shown by Hewitt, Marczewski, and Pondiczery that a product of at most 2^{ω} separable space is separable [22, 27, 29]. In 1965, one of Groot's results which generalizes Alexandroff and Urysohn's result above states that a Hausdorff space in which every subspace is Lindelöf has cardinality $\leq 2^{\omega}$ [17]. In 1969, Arhangel'skii show that every Lindelöf, first countable, Hausdorff space has cardinality $\leq 2^{\omega}$ [2].

In the paper [28], we gave first concept of dicardinal function and then defined (co)weight, (co)net weight, (co)densification, (co)pseudo character. They are ones of most useful tools in classifying ditopological texture spaces. Also, we investigated relationships between the set S ($\mathcal{P}$ or $\mathcal{Q}$) and eight dicardinal functions defined on ditopological texture spaces as seen from above. We represented dicardinal functions and some important theorems for (a particular subclass of) the class of all ditopological texture spaces: for every ditopological texture space $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$, $co-r(\mathfrak{D})\leq w(\mathfrak{D})$ and $r(\mathfrak{D})\leq co-w(\mathfrak{D})$; if $\mathfrak{D}$ is T_0, then $|\mathcal{Q}|\leq \max\{o(\mathfrak{D}),c(\mathfrak{D})\}$ and $|S|\leq 2^{\max\{nw(\mathfrak{D}),co-nw(\mathfrak{D})\}}$. In particular, for every Kolmogorov ditopological coseparated texture space, $|S|\leq \max\{o(\mathfrak{D}),c(\mathfrak{D})\}$. If $\mathfrak{D}$ is $co-T_1$, then $|S|\leq nw(\mathfrak{D})^{\Psi(\mathfrak{D})}$.

In this study, as a continuation of our paper [28], we investigate under which conditions equality of the pair of dicardinal functions, weight-coweight and densification-codensification, holds. Also, in Kolmogorov ditopological texture spaces, we show that the cardinality of S is bounded by the dicardinal functions weight and coweight. In the section 2 titled 'Texture Spaces', we recall the basic definitions of texture space, ditopology on the texture space and then, some definitions and theorems regarding the subjects. The concepts of ordinals, cardinals, cardinality of a set and cardinal functions and some theorems which are related to cardinals are given in the section 3. Finally, in the last section, we give the definition of dicardinal function in ditopological texture spaces. Then, we represent dicardinal functions and some important theorems for (a particular subclass of) the class of all ditopological texture spaces: for every complemented ditopological texture space, $w(\mathfrak{D})=co-w(\mathfrak{D})$ and $r(\mathfrak{D})=co-r(\mathfrak{D})$; for every Kolmogorov ditopological texture space, $|\mathcal{Q}|\leq 2^{\max\{w(\mathfrak{D}),co-w(\mathfrak{D})\}}$.

II. TEXTURE

The following definitions and propositions were introduced in [4, 3, 10, 9, 5, 6, 7, 8, 18, 19, 13, 14, 12, 30].

A *texturing* on a non-empty set S is a set $\mathcal{S}$ containing S, $\emptyset$ of subsets of S with respect to inclusion satisfying the

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conditions: (i) $(S,\subseteq)$ is a complete lattice (ii) $\mathcal{S}$ is completely distributive, (iii) $\mathcal{S}$ separates the points of S, (iv) Meets $\wedge$ and finite joins $\vee$ coincide with intersections $\bigcap$ and unions $\bigcup$, respectively. $(S,\mathcal{S})$ is then called *texture*.

For each $s \in S$, the p-*set* P_s is defined by

$$P_s = \bigcap \{A \in \mathcal{S} \mid s \in A\},$$

and q-*set*

$$Q_s = \vee \{A \in \mathcal{S} \mid s \notin A\} = \vee \{P_r \mid r \in S, \quad s \notin P_r\}.$$

We recall that a texture $(S,\mathcal{S})$ is said *coseparated* if

$$\forall s,t \in S,\ Q_s \subseteq Q_t \Rightarrow P_s \subseteq P_t.$$

We define $\mathcal{P} = \{P_s \mid s \in S\}$ and $\mathcal{Q} = \{Q_s \mid s \in S\}$.

Let $(S,\mathcal{S})$ be a texture and $\sigma : \mathcal{S} \to \mathcal{S}$ a function. σ is said a complementation on $\mathcal{S}$ and $(S,\mathcal{S},\sigma)$ is called complemented texture if the following conditions are satisfied: for every sets $A, B \in \mathcal{S}$

1. $\sigma(\sigma(A)) = A$,
2. $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$.

It is obvius that σ is a bijection. Furthermore, for each $\mathcal{A} \subseteq \mathcal{S}$, the restriction $\sigma_{\mathcal{A}}$ of σ is injective.

Proposition 1 *For every complemented texture,*

1. $\sigma(\bigcap_{i\in I} A_i) = \vee_{i\in I} \sigma(A_i)$,
2. $\sigma(\vee_{i\in I} A_i) = \bigcap_{i\in I} \sigma(A_i)$.

Definition 1 *Let $(S,\mathcal{S})$ be a texture, and τ,κ subsets of $\mathcal{S}$. $\mathfrak{D} = (S,\mathcal{S},\tau,\kappa)$ is called a ditopological texture space, and the pair (τ,κ) be a ditopology on $(S,\mathcal{S})$ if (τ,κ) satisfies []*

1. $(G1) S, \emptyset \in \tau$,
2. $(G2) G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$,
3. $(G3) \mathcal{G} \subseteq \tau \Rightarrow \vee\ \mathcal{G} \in \tau$,
4. $(F1) S, \emptyset \in \kappa$,
5. $(F2) F_1, F_2 \in \kappa \Rightarrow F_1 \cup F_2 \in \kappa$,
6. $(F3) \mathcal{F} \subseteq \kappa \Rightarrow \bigcap\ \mathcal{F} \in \kappa$.

Let $(S,\mathcal{S},\sigma)$ be a complemented texture and (τ,κ) a ditopology on $(S,\mathcal{S})$. (τ,κ) is said a complemented ditopology on $(S,\mathcal{S},\sigma)$ and $(S,\mathcal{S},\tau,\kappa,\sigma)$ is called a complemented ditopological texture space if the following condition is satisfied: for every $A \in \mathcal{S}$,

$$A \in \tau \Leftrightarrow \sigma(A) \in \kappa.$$

Definition 2 *Let $\mathfrak{D} = (S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space, and $\mathcal{B}$ a subset of τ (κ). $\mathcal{B}$ is a base (cobase) for (τ,κ) if, for all $C \in \tau$ (κ), there exists a subset $\mathcal{B}_C$ of $\mathcal{B}$ such that $C = \vee\ \mathcal{B}_C$ $(\bigcap\ \mathcal{B}_C)$.*

Proposition 2 *For every ditopological texture space, the following statements are equivalent:*

1. $\mathcal{B}$ is a base for $\mathfrak{D}$,
2. $\forall G \in \tau\ \exists B \in \mathcal{B},\ G \not\subseteq Q_s \Rightarrow (B \not\subseteq Q_s \wedge B \subseteq G)$,
3. $\forall G \in \tau\ \exists B \in \mathcal{B},\ G \not\subseteq Q_s \Rightarrow P_s \subseteq B \subseteq G$.

Proposition 3 *For every ditopological texture space, the following statements are equivalent:*

1. $\mathcal{F}$ is a cobase for $\mathfrak{D}$,
2. $\forall K \in \kappa\ \exists F \in \mathcal{F},\ P_s \not\subseteq K \Rightarrow (K \subseteq F \wedge P_s \not\subseteq F)$,
3. $\forall K \in \kappa\ \exists F \in \mathcal{F},\ P_s \not\subseteq K \Rightarrow K \subseteq F \subseteq Q_s$.

Definition 3 *Let $\mathfrak{D} = (S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. The interior and the closure of the set $A \in \mathcal{S}$ is defined, respectively:*

$$]A[= \vee \{G \in \tau \mid G \subseteq A\},\ [A] = \bigcap \{K \in \kappa \mid A \subseteq K\}.$$

Definition 4 *A set $A \in \mathcal{S}$ is said dense (codense) in (τ,κ) if $[A] = S$ $(]A[= \emptyset)$.*

Definition 5 *Let $\mathfrak{D} = (S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. $(\tau \cup \kappa)^{\vee}$ denote the set of arbitrary joins of sets in $(\tau \cup \kappa)$ and $(\tau \cup \kappa)^{\cap}$ the set of arbitrary intersections of sets in $\tau \cup \kappa$. $\mathfrak{D}$ is said T_0 if*

$$\forall s,t \in S\ \exists C \in (\tau \cup \kappa)^{\vee},\ Q_s \not\subseteq Q_t \Rightarrow P_s \not\subseteq C \not\subseteq Q_t,$$

or equivalently,

$$\forall s,t \in S\ \exists C \in (\tau \cup \kappa)^{\cap},\ Q_s \not\subseteq Q_t \Rightarrow P_s \not\subseteq C \not\subseteq Q_t.$$

Proposition 4 *The following are characteristic properties of T_0 ditopological texture space:*

1. $\forall s,t \in S\ \exists C \in \tau \cup \kappa,\ Q_s \not\subseteq Q_t \Rightarrow P_s \not\subseteq C \not\subseteq Q_t$,
2. $\forall s,t \in S,\ ([P_s] \subseteq [P_t] \wedge]Q_s[\subseteq]Q_t[) \Rightarrow Q_s \subseteq Q_t$.

III. Cardinal Functions

The following definitions and propositions were introduced in [23, 24, 25, 26].

A set A is called *transitive* if and only if $\forall y \forall z (z \in y \wedge y \in A \Rightarrow z \in A)$. An *ordinal* α is a transitive set such that all $\beta \in \alpha$ are transitive. The least infinite ordinal is denoted by ω. Also, we recall that a *cardinal* is an ordinal that there is no bijection from itself to a smaller ordinal.

Let X be a set. Assuming axiom of choice, there exists an ordinal that can be mapped one-to-one onto X. The smallest one of the ordinals is called the *cardinality* or *cardinal number* of X, written as $|X|$. We recall that the class of cardinals is well-ordered.

The binary operations of addition and multiplication of cardinals are defined by means the operations of set union and Cartesian product as follows: $\kappa + \lambda := |\kappa \times \{0\} \cup \lambda \times \{1\}|$ and $\kappa.\lambda := |\kappa \times \lambda|$, respectively, where κ and λ are cardinals.

The following proposition is useful for our proofs in the paper :

Proposition 5 *If one of cardinal numbers κ and λ is nonzero and the other infinite, then*

$$\kappa + \lambda = \kappa.\lambda = max\{\kappa,\lambda\}.$$

We recall that a cardinal function is a function ϕ from the class of all topological spaces into the class of all infinite cardinals such that, if X and Y are homeomorphic, then $\phi(X) = \phi(Y)$.

The interested reader is referred to [1, 2, 15, 16, 17, 20, 22, 23, 24, 25, 27, 29, 26] for more information about cardinals and cardinal functions.

IV. Main Results

Theorem 6 *[28] For every texture space $(S,\mathcal{S})$, $|S| = |\mathcal{P}|$.*

Theorem 7 *[28] If a texture $(S,\mathcal{S})$ is coseparated, then $|S| = |\mathcal{Q}|$.*

Definition 6 *[28] A function ϕ from the class of all ditopological texture spaces (or a particular subclass) into the class of all infinite cardinal numbers is called a dicardinal function if, for every pair $\mathfrak{D}_i = (S_i, \mathcal{S}_i, \tau_i, \kappa_i)_{i\in\{1,2\}}$ of ditopological texture spaces,*

$$\text{`}\mathfrak{D}_1, \mathfrak{D}_2 \text{ are dihomeomorphic}' \Rightarrow \phi(\mathfrak{D}_1) = \phi(\mathfrak{D}_2)$$

where $(S_1,\mathcal{S}_1), (S_2,\mathcal{S}_2)$ are isomorphic.

Definition 7 *[28] Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. $o(\mathfrak{D})$ and $c(\mathfrak{D})$ are defined as the number of open sets in $\mathfrak{D}$ plus ω and the number of closed sets in $\mathfrak{D}$ plus ω, respectively. $oc(\mathfrak{D})$ is defined as the number of sets in $\tau \cup \kappa$ plus ω.*

[28] Clearly, $\max\{o(\mathfrak{D}), c(\mathfrak{D})\} \leq oc(\mathfrak{D}) \leq |\mathcal{S}| \leq 2^{|S|}$. Also, $o(\mathfrak{D})$ and $c(\mathfrak{D})$ don't need to dominate each other.

Theorem 8 *If $\mathfrak{D}$ is a complemented ditopological texture space, then $o(\mathfrak{D}) = c(\mathfrak{D})$.*

Proof Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa, \sigma)$ be a complemented ditopological texture space. Define a function $\varphi: \tau \to \kappa$ by, for each $G \in \tau$, $\varphi(G) = \sigma(G)$. Since $\tau \subseteq \mathcal{S}$, φ is the restriction of σ. Then, φ is injective and so $|\tau| \leq |\kappa|$. Similar arguments can be used to show that $|\kappa| \leq |\tau|$. This completes the proof.

Theorem 9 *[28] If a ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ is T_0 (Kolmogorov), then $|Q| \leq max\{o(\mathfrak{D}), c(\mathfrak{D})\}$.*

Corollary 10 *[28] For every Kolmogorov ditopological coseparated texture space, $|S| \leq o(\mathfrak{D}) = c(\mathfrak{D})$.*

Definition 8 *[28] Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. The weight and coweight of $\mathfrak{D}$ are defined as follows,*

$w(\mathfrak{D}) = \min\{|\mathcal{B}| \mid \mathcal{B} \text{ a } base \ for \ (\tau, \kappa)\}$,

$co - w(\mathfrak{D}) = \min\{|\mathcal{B}| \mid \mathcal{B} \text{ a } cobase \ for \ (\tau, \kappa)\}$,

respectively.

Theorem 11 *For every ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$, we have*

1. $w(\mathfrak{D}) \leq o(\mathfrak{D})$,
2. $co - w(\mathfrak{D}) \leq c(\mathfrak{D})$.

Proof 1. Let $\mathcal{B}$ be a base for the ditopological texture space $\mathfrak{D}$ such that $|\mathcal{B}| \leq w(\mathfrak{D})$. Then $\mathcal{B} \subseteq \tau$ and so $|\mathcal{B}| \leq |\tau| = o(\mathfrak{D})$.

2. Let $\mathcal{F}$ be a cobase for the ditopological texture space $\mathfrak{D}$ such that $|\mathcal{F}| \leq co - w(\mathfrak{D})$. Then $\mathcal{F} \subseteq \kappa$ and so $|\mathcal{F}| \leq |\kappa| = c(\mathfrak{D})$.

Theorem 12 *If $\mathfrak{D}$ is a complemented ditopological texture space, then $w(\mathfrak{D}) = co - w(\mathfrak{D})$.*

Proof Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa, \sigma)$ be a complemented ditopological texture space and $\mathcal{B}$ a base for $\mathfrak{D}$ such that $|\mathcal{B}| \leq w(\mathfrak{D})$. Let us set $\mathcal{F}_{\mathcal{B}} := \{\sigma(B) \mid B \in \mathcal{B}\}$. Then $\mathcal{F}_{\mathcal{B}} \subseteq \kappa$. Given an arbitrary closed set $K \in \kappa$. Then there exists $G \in \tau$ such that $K = \sigma(G)$. Since $\mathcal{B}$ be a base for $\mathfrak{D}$, there exists $\mathcal{B}_0 \subseteq \mathcal{B}$ such that $G = \bigvee \mathcal{B}_0$. Therefore, $K = \sigma(G) = \sigma(\bigvee \mathcal{B}_0)$. It follows from Proposition 1 that $K = \sigma(\bigvee \mathcal{B}_0) = \bigcap \mathcal{F}_0$ where $\mathcal{F}_0 := \{\sigma(B) \mid B \in \mathcal{B}_0\} \subseteq \mathcal{F}_{\mathcal{B}}$; this shows that $\mathcal{F}_{\mathcal{B}}$ is a cobase for $\mathfrak{D}$. Define a function $\varphi: \mathcal{B} \to \mathcal{F}_{\mathcal{B}}$ such that if $B \in \mathcal{B}$, then $\varphi(B) = \sigma(B)$. It is clear that φ is well-defined. That φ is injective follows from the fact that for each $\mathcal{A} \subseteq \mathcal{S}$, the restriction $\sigma|_{\mathcal{A}}$ of σ is injective. Furthermore, by the definition of $\mathcal{F}_{\mathcal{B}}$, it is easily seen that φ is surjective; thus $|\mathcal{F}_{\mathcal{B}}| = |\mathcal{B}|$ and so $co - w(\mathfrak{D}) \leq w(\mathfrak{D})$.

Let $\mathcal{F}$ be a cobase for the ditopological texture space $\mathfrak{D}$ such that $|\mathcal{F}| \leq co - w(\mathfrak{D})$. Set $\mathcal{B}_{\mathcal{F}} := \{\sigma(F) \mid F \in \mathcal{F}\}$. Then $\mathcal{B}_{\mathcal{F}} \subseteq \kappa$. Given an arbitrary open set $G \in \tau$. Then there exists $K \in \kappa$ such that $G = \sigma(K)$. Since $\mathcal{F}$ be a cobase for $\mathfrak{D}$, $G = \sigma(K) = \sigma(\bigcap \mathcal{F}_0)$. It follows from Proposition 1 that $G = \sigma(\bigcap \mathcal{F}_0) = \bigvee \mathcal{B}_0$ where $\mathcal{B}_0 := \{\sigma(F) \mid F \in \mathcal{F}_0\} \subseteq \mathcal{B}_{\mathcal{F}}$; this shows that $\mathcal{B}_{\mathcal{F}}$ is a base for $\mathfrak{D}$. Define a function $\phi: \mathcal{F} \to \mathcal{B}_{\mathcal{F}}$ such that if $F \in \mathcal{F}$, then $\phi(F) = \sigma(F)$. Then ϕ is well-defined. The fact that ϕ is injective follows from that for each $\mathcal{A} \subseteq \mathcal{S}$, the restriction $\sigma|_{\mathcal{A}}$ of σ is injective. Moreover, by the definition of $\mathcal{B}_{\mathcal{F}}$, it can be shown that ϕ is surjective; thus $|\mathcal{B}_{\mathcal{F}}| = |\mathcal{F}|$ and so $w(\mathfrak{D}) \leq co - w(\mathfrak{D})$.

Corollary 13 *If $\mathfrak{D}$ is a complemented ditopological texture space, then $w(\mathfrak{D}) = co - w(\mathfrak{D}) \leq o(\mathfrak{D}) = c(\mathfrak{D}) \leq |\mathcal{S}|$.*

By Remark 4, Theorem 8 and Theorem 12, that the statement above is valid can be easily seen.

Theorem 14 *If a ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ is T_0 (Kolmogorov), then $|Q| \leq 2^{max\{w(\mathfrak{D}), co-w(\mathfrak{D})\}}$.*

Proof Let a ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be T_0, $\mathcal{B}$ a base for $\mathfrak{D}$ such that $|\mathcal{B}| \leq w(\mathfrak{D})$, and $\mathcal{F}$ a cobase for $\mathfrak{D}$ such that $|\mathcal{F}| \leq co - w(\mathfrak{D})$. Now, let us define a function $\varphi: Q \to \mathcal{P}(\mathcal{B} \times \mathcal{F})$ by, for each $s \in S$,

$$\varphi(Q_s) := \{(B, F) \in \mathcal{B} \times \mathcal{F} \mid P_s \subseteq B \vee F \subseteq Q_s\}.$$

Then φ is clearly well-defined. Given two q-sets Q_s, Q_t with $Q_s \not\subseteq Q_t$. Since $\mathfrak{D}$ is T_0, by Proposition 4, $\exists C \in \tau \cup \kappa$, $P_s \not\subseteq C \not\subseteq Q_t$. In the case $C \in \tau$, since $\mathcal{B}$ is a base for $\mathfrak{D}$, by Proposition 2, $\exists B \in \mathcal{B}$, $P_t \subseteq B \subseteq C$. Moreover, the fact that $P_s \not\subseteq B$ follows from that $B \subseteq C$ and $P_s \not\subseteq C$. Therefore, $P_t \subseteq B$ and $P_s \not\subseteq B$. Thus $\varphi(Q_s) \neq \varphi(Q_t)$. In the case $C \in \kappa$, since $\mathcal{F}$ is a cobase for $\mathfrak{D}$, by Proposition 3, $\exists F \in \mathcal{F}$ $C \subseteq F \subseteq Q_s$. Furthermore, it follows from $C \subseteq F$ and $C \not\subseteq Q_t$ that $F \not\subseteq Q_t$. Hence $F \subseteq Q_s$ and $F \not\subseteq Q_t$. Thus $\varphi(Q_s) \neq \varphi(Q_t)$. This shows that φ is injective. Then $|Q| \leq |\mathcal{P}(\mathcal{B} \times \mathcal{F})|$. Since $|\mathcal{P}(\mathcal{B} \times \mathcal{F})| = 2^{|\mathcal{B} \times \mathcal{F}|}$ and $|\mathcal{B} \times \mathcal{F}| = \max\{|\mathcal{B}|, |\mathcal{F}|\}$, $|Q| \leq 2^{\max\{|\mathcal{B}|, |\mathcal{F}|\}}$ and so $|Q| \leq 2^{\max\{w(\mathfrak{D}), co-w(\mathfrak{D})\}}$.

Corollary 15 *For every Kolmogorov complemented ditopological coseparated texture space, $|S| \leq 2^{w(\mathfrak{D})} = 2^{co-w(\mathfrak{D})}$.*

Proof By Theorem 7, Theorem 12 and Theorem 14, that the statement above is valid can be easily seen.

Definition 9 *[28] Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. A subset $\mathcal{A}$ of $\mathcal{S}$ is said densifier (codensifier) in $\mathfrak{D}$ if $\bigvee \mathcal{A}$ ($\bigcap \mathcal{A}$) is dense (codense) in (τ, κ).*

Definition 10 *[28] The densification and codensification of $\mathfrak{D}$ are defined as follows,*

$r(\mathfrak{D}) = \min\{|\mathcal{A}| \mid \mathcal{A} \ densifier \ in \ (\tau, \kappa)\}$,

$co - r(\mathfrak{D}) = \min\{|\mathcal{A}| \mid \mathcal{A} \ codensifier \ in \ (\tau, \kappa)\}$,

respectively.

Now, we show that, in a ditopological texture space $\mathfrak{D}$, how there are relationships between (co)weight and (co)densification.

Theorem 16 *[28] For every ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$, we have*

1. $co - r(\mathfrak{D}) \leq w(\mathfrak{D})$,
2. $r(\mathfrak{D}) \leq co - w(\mathfrak{D})$.

Corollary 17 *For every ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$, we have*

1. $co - r(\mathfrak{D}) \leq o(\mathfrak{D})$,

2. $r(\mathfrak{D}) \leq c(\mathfrak{D})$.

Proof By Theorem 11 and Theorem 16, that the statement above is valid can be easily seen.

Theorem 18 *If $\mathfrak{D}$ is a complemented ditopological texture space, then $r(\mathfrak{D}) = co - r(\mathfrak{D})$.*

Proof Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa, \sigma)$ be a complemented ditopological texture space and $\mathcal{M}$ a densifier of $\mathfrak{D}$. Let us set $\mathcal{N}_{\mathcal{M}} := \{\sigma(M) \mid M \in \mathcal{M}\}$. By the definition of complementation on a texture, $]\bigcap \mathcal{N}_{\mathcal{M}}[= \sigma(\sigma(]\bigcap \mathcal{N}_{\mathcal{M}}[))$. By the definitions of interior and closure, and Proposition 1, $]\bigcap \mathcal{N}_{\mathcal{M}}[= \sigma([\sigma(\bigcap \mathcal{N}_{\mathcal{M}})])$ and so $]\bigcap \mathcal{N}_{\mathcal{M}}[= \sigma([\bigvee \mathcal{M}])$. Since $\mathcal{M}$ is a densifier of $\mathfrak{D}$, $\sigma([\bigvee \mathcal{M}]) = \sigma(S) = \emptyset$. Hence $]\bigcap \mathcal{N}_{\mathcal{M}}[= \emptyset$; that is, $\mathcal{N}_{\mathcal{M}}$ is a codensifier of $\mathfrak{D}$. Now, let us define a function $\varphi : \mathcal{M} \rightarrow \mathcal{N}_{\mathcal{M}}$ by, for each $M \in \mathcal{M}$, $\varphi(M) = \sigma(M)$. Then, it is clear that φ is well-defined. Moreover, it can be shown that φ is a bijection; that is, $|\mathcal{M}| = |\mathcal{N}_{\mathcal{M}}|$ and so $co - r(\mathfrak{D}) \leq r(\mathfrak{D})$.

Let $\mathcal{N}$ be a codensifier of $\mathfrak{D}$. Let us set $\mathcal{M}_{\mathcal{N}} := \{\sigma(N) \mid N \in \mathcal{N}\}$. By the definition of complementation on a texture, $[\bigvee \mathcal{M}_{\mathcal{N}}] = \sigma(\sigma([\bigvee \mathcal{M}_{\mathcal{N}}]))$. By the definitions of interior and closure, and Proposition 1, we have $[\bigvee \mathcal{M}_{\mathcal{N}}] = \sigma(]\sigma(\bigvee \mathcal{M}_{\mathcal{N}})[)$ and so $[\bigvee \mathcal{M}_{\mathcal{N}}] = \sigma(]\bigcap \mathcal{N}[)$. Since $\mathcal{N}$ is a codensifier of $\mathfrak{D}$, $\sigma(]\bigcap \mathcal{N}[) = \sigma(\emptyset) = \sigma(\sigma(S)) = S$. Hence $[\bigvee \mathcal{M}_{\mathcal{N}}] = S$; that is, $\mathcal{M}_{\mathcal{N}}$ is a densifier of $\mathfrak{D}$. Now, let us define a function $\phi : \mathcal{N} \rightarrow \mathcal{M}_{\mathcal{N}}$ by, for each $N \in \mathcal{N}$, $\phi(N) = \sigma(N)$. Then, it is clear that ϕ is well-defined. Moreover, it can be shown that ϕ is a bijection; that is, $|\mathcal{N}| = |\mathcal{M}_{\mathcal{N}}|$ and so $r(\mathfrak{D}) \leq co - r(\mathfrak{D})$. This completes the proof.

Corollary 19 *If $\mathfrak{D}$ is a complemented ditopological texture space, then $r(\mathfrak{D}) = co - r(\mathfrak{D}) \leq o(\mathfrak{D}) = c(\mathfrak{D}) \leq |\mathcal{S}|$.*

Proof By Corollary 13, Theorem 16 and Theorem 18, that the statement above is valid can be easily seen.

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