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# Homotopy Analysis to Heat and Mass Transfer of MHD Flow of a Viscous Fluid over a Moving Vertical Plate in a Porous Medium with Viscous Dissipation

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***Abstract*— The present paper deals with the MHD boundary layer flow over a linearly moving porous semi infinite vertical plate taking suction and viscous dissipation into account. The fluid considered is viscous, incompressible and electrically conducting. After transferring the governing equations into ordinary differential equations using suitable dimensionless variables, analytical solutions are generated using homotopy analysis method (HAM). The effects of various parameters on dimensionless velocity, temperature and concentration profiles are presented in the form of graphs and tables. HAM results are in good agreement with the results available in the literature.**

***Index Terms*— Porous Media, Viscous Dissipation, MHD Flow, Moving Vertical Plate, Homotopy Analysis Method(HAM).**

## I. INTRODUCTION

In recent years, there are number of studies on convection flow and heat transfer in saturated porous media due to their wide ranging applications in engineering field like heat exchanger devices, geothermal and geophysical engineering, petroleum reservoirs, underground disposal of nuclear waste and others. The vertical free convection boundary layer flow in porous medium owing to combined heat and mass transfer has been studied by Bejan and Khair [1]. Lai and Kulacki [2] studied the coupled heat and mass transfer by natural convection from vertical surface in porous medium. Kim and Vafai [3] have analyzed the buoyancy driven flow about a vertical plate for constant wall temperature and heat flux.

Helmy [4] worked on MHD unsteady free convection flow past a vertical plate embedded in a porous medium. Raptis et al. [5] constructed similarity solutions for boundary layer near a vertical surface in porous medium with constant temperature and concentration. Thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink was investigated by Chamka [6]. Yih [7] observed free convection effect on MHD coupled heat and mass transfer of a moving permeable vertical surface. Soundalgekar [8] has discussed the viscous dissipation effect on unsteady free convection flow past an infinite vertical porous plate with constant suction.

In this study, HAM [10, 11] is adopted to find the analytical solutions for velocity, temperature and concentration profiles.

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Convergence of the obtained solutions is explicitly shown. Velocity, temperature and concentration profiles are drawn for various values of flow parameters like magnetic parameter $M$, buoyancy parameters $Gr$ and $Gc$, suction parameter $F_w$, permeability parameter $K$, Schmidt number $Sc$ and Eckert number $Ec$. The results are discussed in detailed, compared with the available results in literature and are in good agreement.

**Governing equations**

Consider the free convection effects on MHD boundary layer flow of a viscous incompressible fluid over a linearly started porous vertical semi infinite plate embedded in a porous medium with suction and viscous dissipation. The $x$-coordinate is taken along the plate in ascending direction. The $y$-coordinate is taken normal to the plate. The velocity of the fluid far away from the plate surface is assumed to be zero for a quiescent state fluid. The variations of surface temperature and concentration are linear. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field, Hall effects and Joule heating are negligible. Under these assumptions, along with the Boussinesq approximations, the boundary layer equations describing this flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty)-\frac{\nu}{K'}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

The corresponding boundary conditions are

$$\begin{aligned}&u=Bx,\ v=V,\ T=T_w=T_\infty+ax,\ C=C_w=C_\infty+bx \quad at\ y=0,\\ &u\to 0,\ \ T\to T_\infty,\ \ C\to C_\infty \qquad\qquad as\ y\to\infty,\end{aligned} \tag{5}$$

where $u$ and $v$ are the velocity components along the $x$ and $y$ axes respectively. $T$ and $C$ are the temperature and concentration in the boundary layer, $a$ and $b$ denote the stratification rate of the gradient of ambient temperature and concentration, $\nu$ is the kinematics viscosity, $\rho$ is the density, $\sigma$ is the electric conductivity of the fluid, $\beta_T$ is the volumetric coefficient of thermal expansion, $\beta_c$ is the volumetric coefficient of concentration expansion, $C_\infty$ is the free stream concentration, $B_0$ is the magnetic induction, $D_m$ is the mass diffusivity, $g$ is the acceleration due to gravity, $K'$ is the permeability of the porous medium and $\alpha$ is the thermal diffusivity, $c_p$ is the specific heat at constant pressure.

Now using the non-dimensionless parameters and variables reported in [8]

$$\eta = y\sqrt{\frac{B}{\nu}}, \qquad \psi(x,y) = x\sqrt{\nu B}\, f(\eta),$$
$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \tag{6}$$

where $\psi(x,y)$ is the stream function, $f(\eta)$ is a dimensionless stream function, $\theta(\eta)$ is a dimensionless temperature of the fluid in the boundary layer region, $\phi(\eta)$ is a dimensionless species concentration of the fluid in the boundary layer region and $\eta$ is the similarity variable.

Taking the Cauchy-Riemann equations

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}, \tag{7}$$

the conservation of mass, equation (1) is identically satisfied. Substituting equation (7) in equations (2) to (5), we obtain

$$f''' + f f'' - f'^2 - M f' + Gr\,\theta + Gc\,\phi - K f' = 0, \tag{8}$$
$$\theta'' + \Pr f\theta' - \Pr f'\theta + Ec f''^2 = 0, \tag{9}$$
$$\phi'' + Sc\, f\phi' - Sc\, f'\phi = 0. \tag{10}$$

The boundary conditions in non-dimensional form are

$$\begin{array}{lllll} f = -F_w, & f' = 1, & \theta = 1, & \phi = 1 & at \quad \eta = 0, \\ f' = 0, & \theta = 0, & \phi = 0, & & as \quad \eta \to \infty, \end{array} \tag{11}$$

where $M = \frac{\sigma B_0^2}{\rho B}$ is the magnetic parameter, $Gr = \frac{g\,\beta_T (T_w - T_\infty)}{x B^2}$ is the local temperature Grashof number, $Gr = \frac{g\,\beta_c (C_w - C_\infty)}{x B^2}$ is the local concentration Grashof number, $K = \frac{\nu}{K' x B}$ is the Permeability parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Ec = \frac{B^2 x^2}{c_p (T_w - T_\infty)}$ is the Eckert number, $Sc = \frac{\nu}{D_m}$ is the Schmidt number, $F_w = \frac{V}{\sqrt{B\nu}}$ is the suction parameter.

**Solution using HAM**

To solve the dimensionless equations (8) to (10) together with the boundary conditions (11) analytically using HAM, the initial approximations and auxiliary linear operators are chosen as

$$f_0(\eta) = 1 - e^{-\eta},$$
$$\theta_0(\eta) = e^{-\eta},$$
$$\phi_0(\eta) = e^{-\eta},$$
$$L_1(f) = f''' - f',$$
$$L_2(\theta) = \theta'' - \theta,$$
$$L_3(\phi) = \phi'' - \phi,$$

which satisfy

$$L_1\left(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\right) = 0,$$
$$L_2\left(C_4 e^{\eta} + C_5 e^{-\eta}\right) = 0,$$
$$L_3\left(C_6 e^{\eta} + C_7 e^{-\eta}\right) = 0,$$

where $C_i$ $(i = 1\ to\ 7)$ are the arbitrary constants.

If $p \in [0,1]$ is the embedding parameter, $\hbar_1, \hbar_2$ and $\hbar_3$ are the non-zero auxiliary parameters, the following equations are constructed.

**Zeroth order deformation equations**

$$(1-p)L_1\big(f(\eta;p) - f_0(\eta)\big) = p\hbar_1 N_1[f(\eta;p), \theta(\eta;p), \phi(\eta;p)], \tag{12}$$

$$(1-p)L_2\big(\theta(\eta;p) - \theta_0(\eta)\big) = p\hbar_2 N_2[f(\eta;p), \theta(\eta;p)], \tag{13}$$

$$(1-p)L_3\big(\phi(\eta;p) - \phi_0(\eta)\big) = p\hbar_3 N_3[f(\eta;p), \phi(\eta;p)], \tag{14}$$

$$\begin{array}{lll} f(0;p) = -F_w, & f'(0;p) = 1, & f'(\infty;p) = 0, \\ \theta(0;p) = 1, & & \theta(\infty;p) = 0, \\ \phi(0;p) = 1, & & \varphi(\infty;p) = 0, \end{array} \tag{15}$$

$$N_1[f(\eta;p),\theta(\eta;p),\phi(\eta;p)]=\frac{\partial^3 f(\eta;p)}{\partial\eta^3}+f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2}-\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2-M\frac{\partial f(\eta;p)}{\partial\eta}$$
$$+Gr\,\theta(\eta;p)+Gc\,\phi(\eta;p)-K\frac{\partial f(\eta;p)}{\partial\eta},$$
$$N_2[f(\eta;p),\theta(\eta;p)]=\frac{\partial^2\theta(\eta;p)}{\partial\eta^2}+\Pr f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta}-\Pr\frac{\partial f(\eta;p)}{\partial\eta}\theta(\eta;p)+Ec\left(\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right)^2,$$
$$N_3[f(\eta;p),\phi(\eta;p)]=\frac{\partial^2\phi(\eta;p)}{\partial\eta^2}+Sc\,f(\eta;p)\frac{\partial\phi(\eta;p)}{\partial\eta}-Sc\frac{\partial f(\eta;p)}{\partial\eta}\phi(\eta;p).$$

Taking $p=0$ and $p=1$, we have

$$f(\eta;0)=f_0(\eta),\qquad f(\eta;1)=f(\eta),$$
$$\theta(\eta;0)=\theta_0(\eta),\qquad \theta(\eta;1)=\theta(\eta),$$
$$\phi(\eta;0)=\phi_0(\eta),\qquad \phi(\eta;1)=\phi(\eta),$$

when $p$ increases from 0 to 1 then $f(\eta;p),\theta(\eta;p)$ and $\phi(\eta;p)$ vary from initial approximations to the final solutions. Expanding $f(\eta;p),\theta(\eta;p)$ and $\phi(\eta;p)$ in Taylor's series w.r.to $p$, we have

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)\,p^m, \tag{16}$$
$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)\,p^m, \tag{17}$$
$$\phi(\eta)=\phi_0(\eta)+\sum_{m=1}^{\infty}\phi_m(\eta)\,p^m, \tag{18}$$

where

$$f_m(\eta)=\frac{1}{m!}\left.\frac{\partial^m f(\eta;p)}{\partial p^m}\right|_{p=0},$$
$$\theta_m(\eta)=\frac{1}{m!}\left.\frac{\partial^m \theta(\eta;p)}{\partial p^m}\right|_{p=0},$$
$$\phi_m(\eta)=\frac{1}{m!}\left.\frac{\partial^m \phi(\eta;p)}{\partial p^m}\right|_{p=0.}$$

If the initial approximations, auxiliary linear operators and non-zero auxiliary parameters are chosen in such a way that the series (16) to (18) are convergent at $p=1,$ then

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta),$$
$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta),$$
$$\phi(\eta)=\phi_0(\eta)+\sum_{m=1}^{\infty}\phi_m(\eta).$$

**$m^{th}$ order deformation equations**

Differentiating equations (12) to (14) $m$ times w.r.to $p$ then setting $p=0$ and finally dividing with $m!$, we get,

$$L_1\left(f_m(\eta)-\chi_m f_{m-1}(\eta)\right)=\hbar_1 R_m^f(\eta), \tag{19}$$
$$L_2\left(\theta_m(\eta)-\chi_m \theta_{m-1}(\eta)\right)=\hbar_2 R_m^\theta(\eta), \tag{20}$$
$$L_3\left(\phi_m(\eta)-\chi_m \phi_{m-1}(\eta)\right)=\hbar_3 R_m^\phi(\eta) \tag{21}$$

with the following boundary conditions

$$\begin{aligned} &f_m(0)=0, \qquad f_m'(0)=0, \qquad f_m'(\infty)=0,\\ &\theta_m(0)=0, \qquad\qquad\qquad \theta_m(\infty)=0,\\ &\phi_m(0)=0, \qquad\qquad\qquad \phi_m(\infty)=0, \end{aligned} \tag{22}$$

where

$$R_m^f(\eta)= f_{m-1}^{'''}+\sum_{i=1}^{m-1} f_{m-1-i}\, f_i^{''}-\sum_{i=1}^{m-1} f_{m-1-i}^{'}\, f_i^{''}-M\, f_{m-1}^{'}+Gr\,\theta_{m-1}+Gc\,\phi_{m-1}-K\, f_{m-1}^{'},$$

$$R_m^\theta(\eta)=\theta_{m-1}^{''}+\Pr\sum_{i=1}^{m-1} f_{m-1-i}\,\theta_i^{'}-\Pr\sum_{i=1}^{m-1} f_{m-1-i}^{'}\,\theta_i+Ec\sum_{i=1}^{m-1} f_{m-1-i}^{''}\, f_i^{''},$$

$$R_m^\phi(\eta)=\phi_{m-1}^{''}+Sc\sum_{i=1}^{m-1} f_{m-1-i}\,\phi_i^{'}-Sc\sum_{i=1}^{m-1} f_{m-1-i}^{'}\,\phi_i,$$

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases}$$

Now to find $f_m(\eta)$, $\theta_m(\eta)$ and $\phi_m(\eta)$ for $m\ge 1$, solve the equations (19) to (21) using MATHEMATICA by employing the boundary conditions (22).

**Convergence of HAM**

As pointed by Liao [12], the convergence of solution series depends upon the choice of the initial approximations, the auxiliary linear operators and on the non-zero auxiliary parameters. Once if the initial approximations and the linear operators have been selected, the convergence of the solution series will depend upon the non-zero auxiliary parameter. Proper values of the parameters $\hbar_1,\hbar_2$ and $\hbar_3$ can be found by so-called $\hbar$-curves. According to Fig. 1, the convergent region of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ is [-2.0, 0.0]. In this paper we choose $\hbar_1=\hbar_2=\hbar_3=-1.0.$ Convergence of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for different orders of approximation is given in Table 1. This table shows that the convergence is achieved at 30th order approximation.

## II. Results and Discussions

The effects of various parameters such as magnetic parameter $M$, buoyancy parameters $Gr$ and $Gc$, suction parameter $F_w$, permeability parameter $K$, Schmidt number $Sc$ and Eckert number $Ec$ on velocity, temperature and concentration fields are shown graphically in Figs. 2-22. In this study Prandtl number $\Pr$ is considered to be 0.72 (air), Schmidt number $Sc=0.24, 0.68, 0.78, 2.72$. To ensure the accuracy of the present results, comparison is made with the existing results in the literature and is given in Table 2.

Figs. 2 to 4 show the effect of magnetic parameter on velocity, temperature and concentration profiles. Introduction of transverse magnetic field to an electrically conducting fluid develops a drag due to Lorentz force which tends to resist the fluid flow and thus reducing its velocity and to increase its temperature and concentration.

The effects of $Gr$ and $Gc$ on velocity, temperature and concentration fields are illustrated in Figs 5 to 10. As shown, temperature and the concentration are decreasing with increase in, $Gr$ and $Gc$ but the velocity increases as $Gr$ and $Gc$ increase.

Figs. 11 to 13 depict the effect of suction parameter. As the suction parameter $F_w$ increases $f'$, $\theta$ and $\phi$ are also increase.

Figs. 14 to 16 show the dimensionless velocity, temperature and concentration profiles for different values of permeability parameter $K$. It can be seen that the velocity profiles decrease with the increase of permeability parameter $K$ and also noticed that the temperature and concentration profiles increase with the increase of permeability parameter $K$.

Figs. 17 to 19 show the dimensionless velocity, temperature and concentration profiles for different values of Schmidt number $Sc$. It can be seen that the velocity profiles decrease with the increase of Schmidt number $Sc$. Further, it is observed that the temperature monotonically increases with the increase of Schmidt number $Sc$. It is seen that the concentration decreases as Schmidt number $Sc$ increases.

Figs. 20 to 22 depict the behaviour of velocity, temperature and concentration profiles for different values of Eckert number. From the figures it is clear that there is a slight increase in velocity and a slight decrease in concentration profiles with an increase in $Ec$. It is also observed that temperature increases with $Ec$.

## III. Conclusions

Analytical study has been performed to observe the physical behavior of the velocity, temperature and concentration profiles. Results are presented graphically and analyzed. By comparing the present results with previous work, it is found that there in a good agreement.

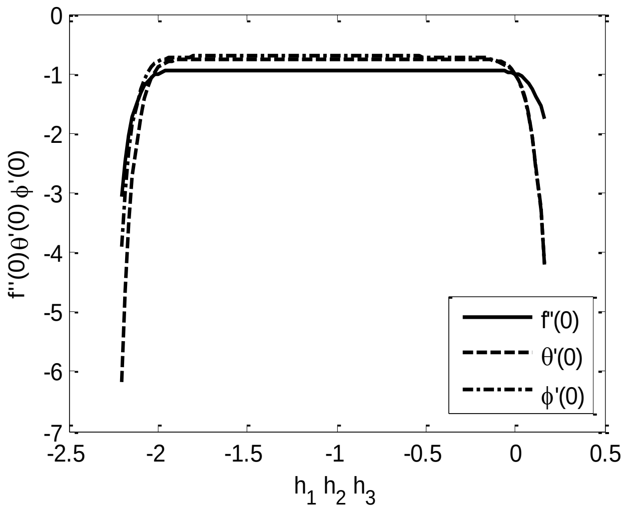


Fig. 1: $\hbar$ -curves for $f''(0), \theta'(0)$ and $\phi'(0)$ at 20th order approximations when $M = 0.1, Gr = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

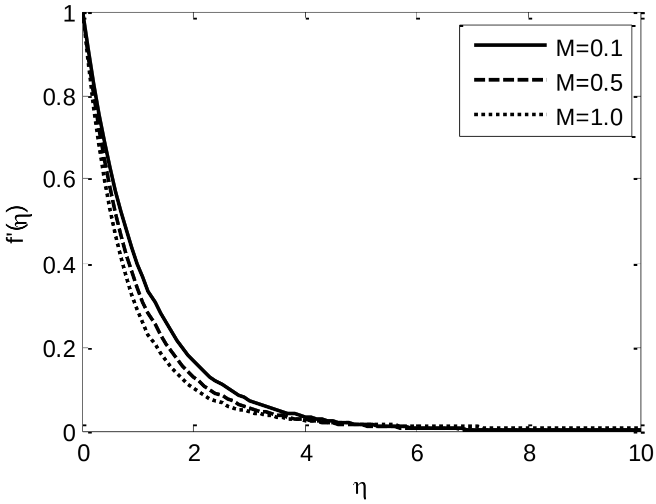


Fig. 2: Variation of $M$ on $f'$ when $Gr = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

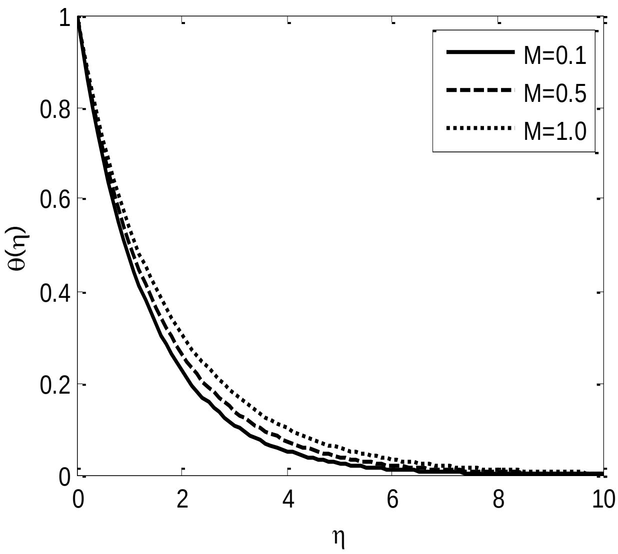


Fig. 3: Variation of $M$ on $\theta$ when $Gr = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

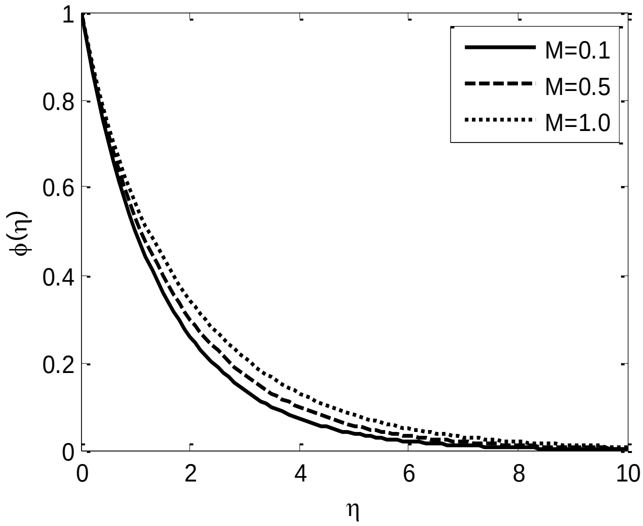


Fig. 4: Variation of $M$ on $\phi$ when $Gr = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

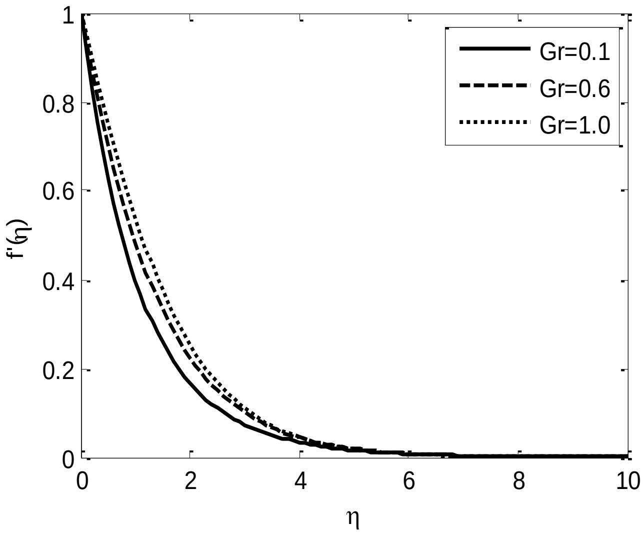


Fig. 5: Variation of $Gr$ on $f'$ when $M = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

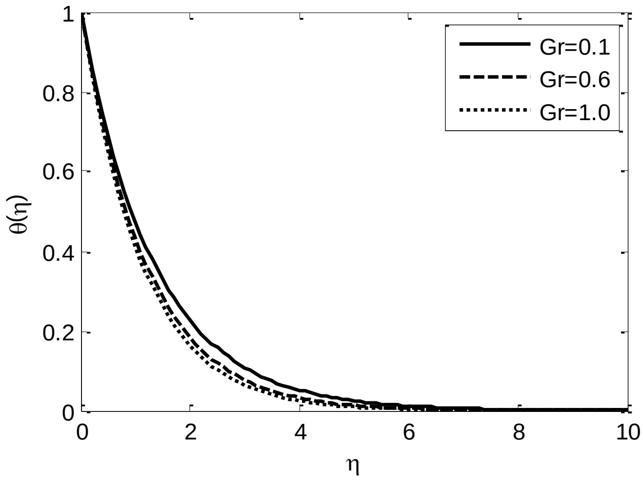


Fig. 6: Variation of $Gr$ on $\theta$ when $M = 0.1, Gc = 0.1, K = 0.1, F_w = 0.1, Ec = 0.1, Sc = 0.62, \Pr = 0.72.$

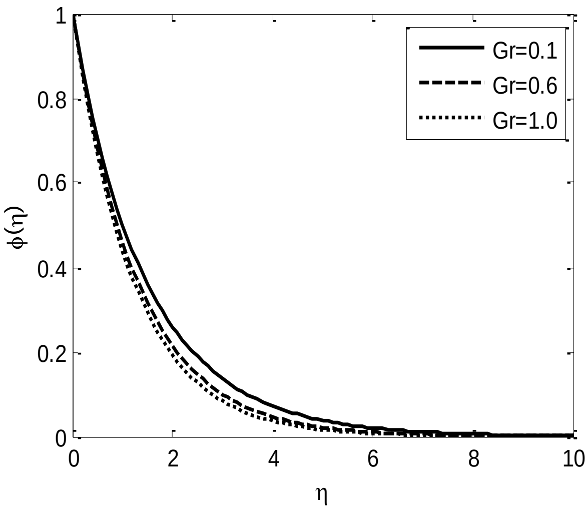


Fig. 7: Variation of $Gr$ on $\phi$ when $M=0.1, Gc=0.1, K=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

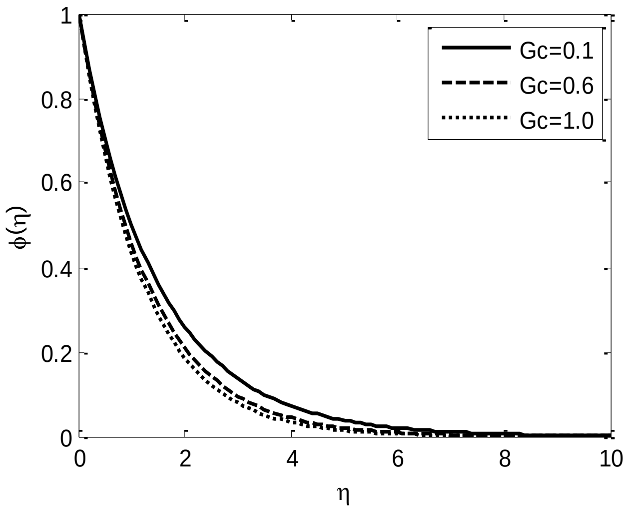


Fig. 10: Variation of $Gc$ on $\phi$ when $M=0.1, Gr=0.1, K=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

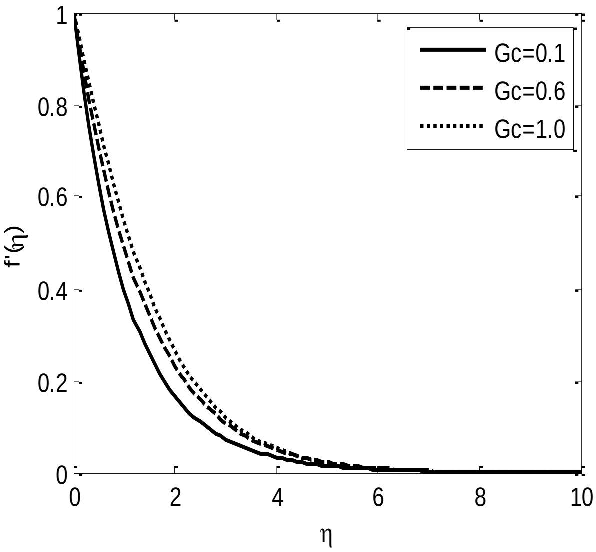


Fig. 8: Variation of $Gc$ on $f'$ when $M=0.1, Gr=0.1, K=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

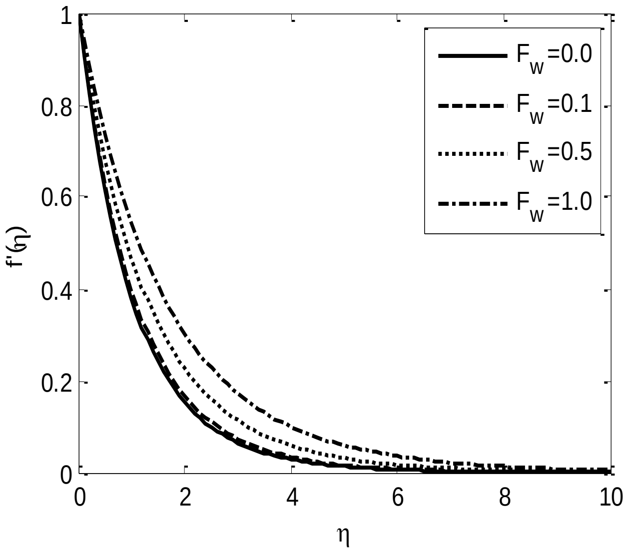


Fig. 11: Variation of $F_w$ on $f'$ when $M=0.1, Gr=0.1, Gc=0.1, K=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

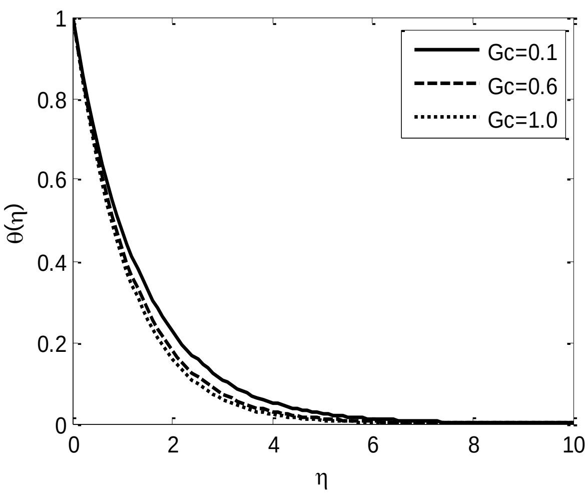


Fig. 9: Variation of $Gc$ on $\theta$ when $M=0.1, Gr=0.1, K=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

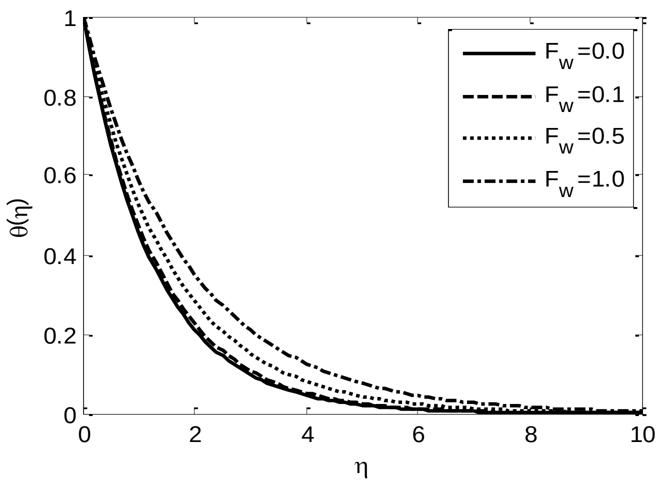


Fig. 12: Variation of $F_w$ on $\theta$ when $M=0.1, Gr=0.1, Gc=0.1, K=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

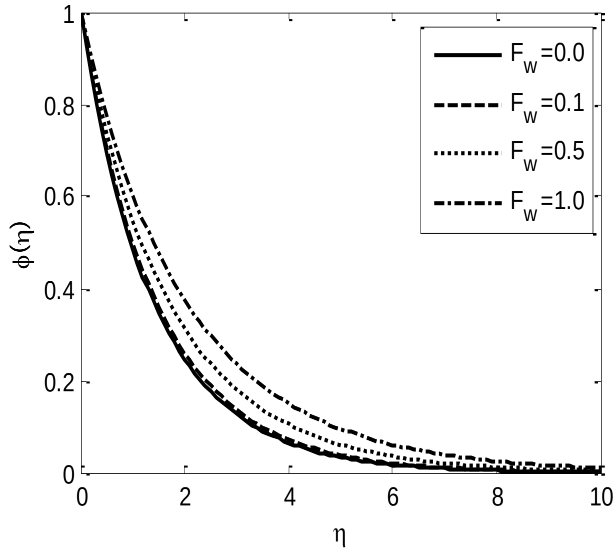


Fig. 13: Variation of $F_w$ on $\phi$ when $M=0.1, Gr=0.1, Gc=0.1, K=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

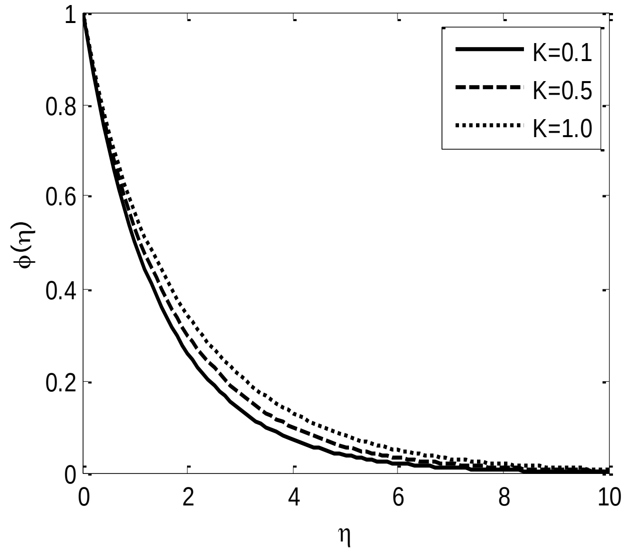


Fig. 16: Variation of $K$ on $\phi$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

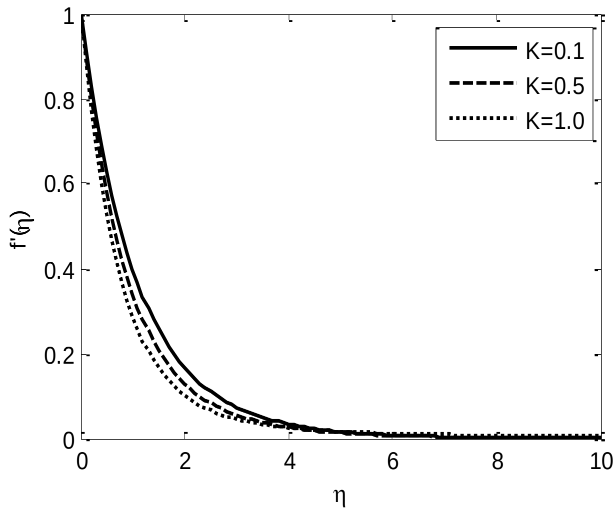


Fig. 14: Variation of $K$ on $f'$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

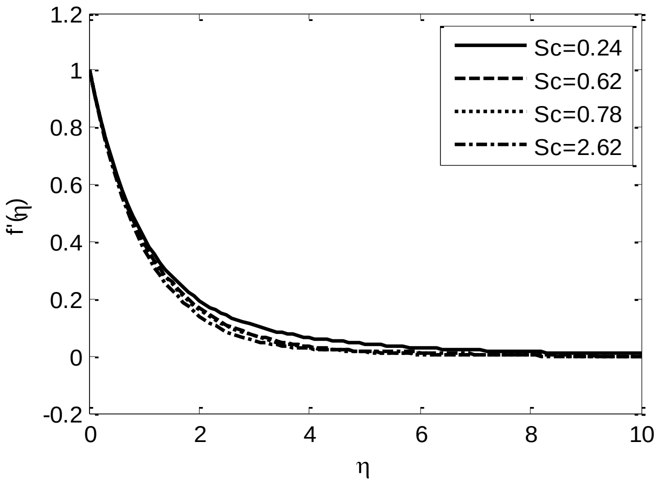


Fig. 17: Variation of $Sc$ on $f'$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Ec=0.1, \Pr=0.72.$

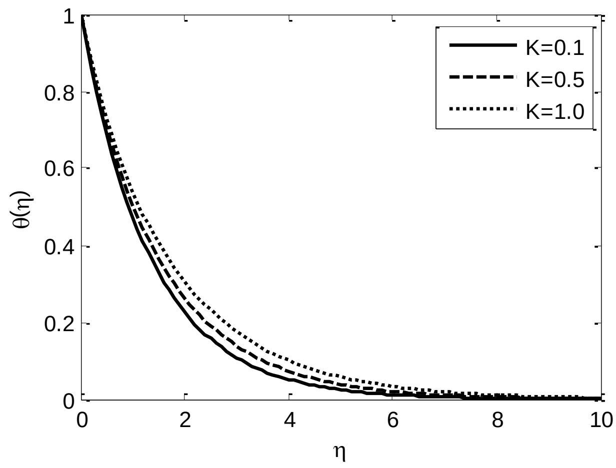


Fig. 15: Variation of $K$ on $\theta$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, Ec=0.1, Sc=0.62, \Pr=0.72.$

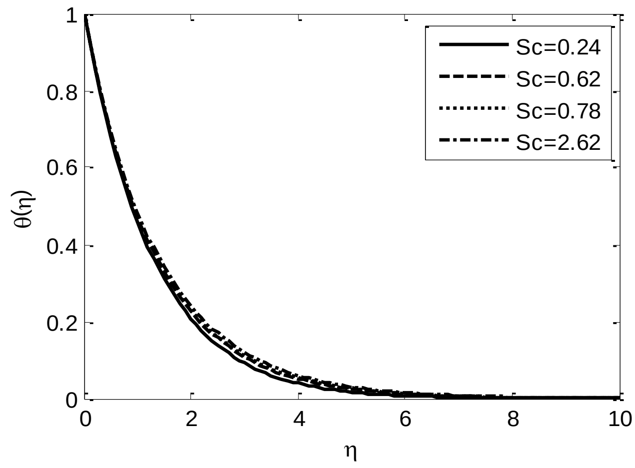


Fig. 18: Variation of $Sc$ on $\theta$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Ec=0.1, \Pr=0.72.$

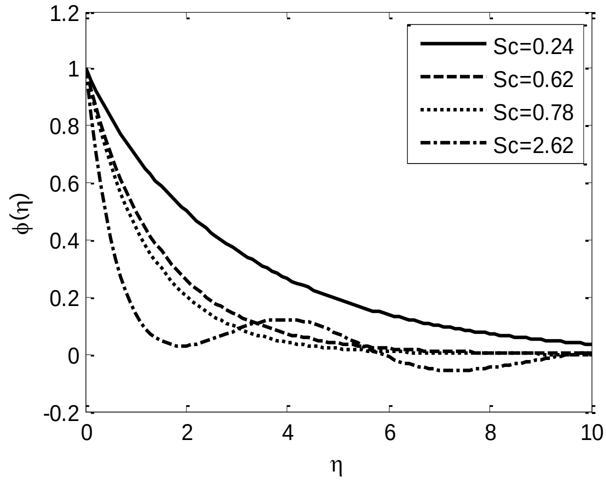


Fig. 19: Variation of $Sc$ on $\phi$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Ec=0.1, \Pr=0.72.$

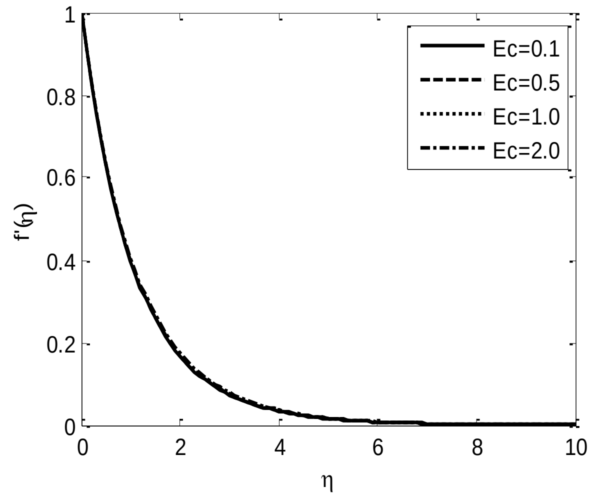


Fig. 20: Variation of $Ec$ on $f'$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Sc=0.62, \Pr=0.72.$

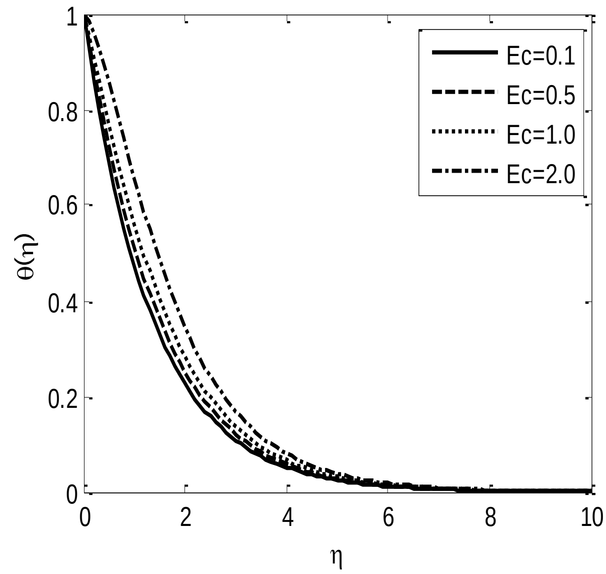


Fig. 21: Variation of $Ec$ on $\theta$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Sc=0.62, \Pr=0.72.$

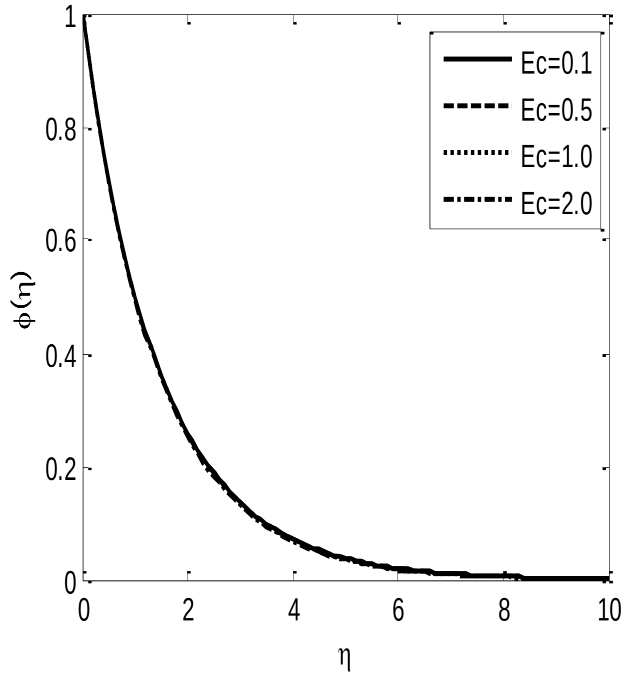


Fig. 22: Variation of $Ec$ on $\phi$ when $M=0.1, Gr=0.1, Gc=0.1, F_w=0.1, K=0.1, Sc=0.62, \Pr=0.72.$

**Table 1:** Convergence of the HAM solutions for different order of approximations when

$M=0.1, Gr=0.1, Gc=0.2, F_w=0.1, Sc=0.78, K=0.1, \Pr=0.72, Ec=0.1.$

| Order | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 5 | 0.940911 | 0.750398 | 0.82345 |
| 10 | 0.940177 | 0.74813 | 0.822822 |
| 15 | 0.940134 | 0.747914 | 0.822905 |
| 20 | .94013 | 0.747877 | 0.822931 |
| 25 | .94013 | 0.747868 | 0.822937 |
| 30 | 0.940129 | 0.747866 | 0.822938 |
| 35 | 0.940129 | 0.747866 | 0.822939 |
| 40 | 0.940129 | 0.747866 | 0.822939 |

**Table 2:** Comparison of the results of $-f''(0), -\theta'(0)\, and -\phi'(0)$ with the existing results of Ibrahim and Makinde [10] when $\Pr = 0.72, K = Ec = 0.0.$

| $Gr$ | $Gc$ | $M$ | $F_w$ | $Sc$ | Ibrahim [10] | | | HAM Results | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.62 | 0.888971 | 0.7965511 | 0.7253292 | 0.888967 | 0.7965538 | 0.7253252 |
| 0.5 | 0.1 | 0.1 | 0.1 | 0.62 | 0.695974 | 0.8379008 | 0.7658018 | 0.695973 | 0.8379015 | 0.7658006 |
| 1.0 | 0.1 | 0.1 | 0.1 | 0.62 | 0.475058 | 0.8752835 | 0.8020042 | 0.475071 | 0.8753137 | 0.8020275 |
| 0.1 | 0.5 | 0.1 | 0.1 | 0.62 | 0.686927 | 0.8421370 | 0.7701717 | 0.686926 | 0.8421389 | 0.7701723 |
| 0.1 | 1.0 | 0.1 | 0.1 | 0.62 | 0.457723 | 0.8818619 | 0.8087332 | 0.457704 | 0.8818575 | 0.8087296 |
| 0.1 | 0.1 | 1.0 | 0.1 | 0.62 | 1.264488 | 0.7089150 | 0.6400051 | 1.264514 | 0.7088625 | 0.6400368 |
| 0.1 | 0.1 | 0.1 | 1.0 | 0.62 | 0.570663 | 0.5601256 | 0.5271504 | 0.570658 | 0.5601314 | 0.5271426 |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.78 | 0.893454 | 0.7936791 | 0.8339779 | 0.893453 | 0.7936779 | 0.8339784 |
| 0.1 | 0.1 | 0.1 | 0.1 | 2.62 | 0.912307 | 0.7847840 | 1.6504511 | 0.928472 | 0.7931287 | 1.9265042 |

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