Mathematical analysis of a new sensitive-to-shock automotive crash sensor

Davood Kalantari, Mohsen Mahdiani

***Abstract*— T In this study, mathematical analysis of a simple and new mechanical sensitive-to-shock automotive crash sensor is presented. The system includes a cantilever beam with a current-carrying wire. Due to the high acceleration generated in a crash, the beam breaks and cut the electrical circuit off. With cutting off the electricity, secondary activations can occur, including activation of the air bag system, unlocking doors and disabling the alarm system. The proposed system is lightweight, low-cost and seems to have acceptable accuracy. Analysis is based on the mathematical equations particularly the Laplace transform method. The results showed that if a hard plastic cantilever beam with height of 8 cm and cross-sectional area of 3×0.3 cm2 is selected, the system will work well in accidents with speed greater than 40 km/hr.**

***Index Terms*— mathematical model, shock, safety, airbag, Laplace transform, cantilever beam**

I. INTRODUCTION

Nowadays, automotive safety is a key factor in the selection of the desired car by customers. With development of electronic systems and also the different types of central locking and alarm systems, sometimes after accidents electronic system disorders, then lock the doors and the windows of the car yielding traps of the passengers inside the car. On the other hand, a seatbelt is not capable to prevent an occupant hitting the steering wheel or the instrument panel with head and body in collisions with solid objects at speeds of over 40 km/h due to belt slack, belt stretch and the delayed effect of the belt retractor. To avoid hitting the head with steering wheel and instrument panel and reducing the level of injuries, airbag systems are developed [1]. To ensure the precision and reliable operation of the airbag, it is necessary to design a robust crash algorithm. Currently, several companies are working to achieve an optimal airbags in collisions by diversifying the type and location of crash-related sensors. Nevertheless, several problems must still be confronted. For instance, when a vehicle operates off road or when the sensor inside the airbag control unit (ACU) receives a powerful shock, the vehicle's airbags may inadvertently deploy, although no collision has occurred, because a crash like signal is delivered to the ACU [2].

Mon (2007) proposed an improved algorithm design methodology of vehicle airbag deployment decision. In this study, the adaptive-network-based fuzzy inference system (ANFIS) is used to train the suitable fuzzy membership functions and fuzzy rules based on crash data for improving the performance of the 'two stage fuzzy algorithm' [3].

In general, air bag sensors are classified to mechanical and electrical. Here especially it is only focused on the mechanical sensors. There are different types of mechanical sensors that need electrical and magnetic phenomena to run [4].

1) Sensor with damper: An example of this sensor is shown in Fig. 1a. The operation of this sensor depends on the ball movement inside the tube. In normal speeds and regular breaking conditions or even in minor collisions, the ball inside the tube is constant due to the magnetic force behind the ball. But in crashes, ball moves into the other end of the sensor and bends two golden plates and close an electrical circuit. When the ball moves, indoor air of sensor moves into tube and will expand the air bag.

2) Another type is spring-mass sensor that needs 0.01 seconds for activation of an electronic circuit and 0.03 seconds to activate the air bag, see Fig. 1b. In speed of 50 km/hr the required time is 0.04 seconds for activation of the airbag and between 0.08 to 0.1 seconds for the full opening. However this time is not constant for different accidents.

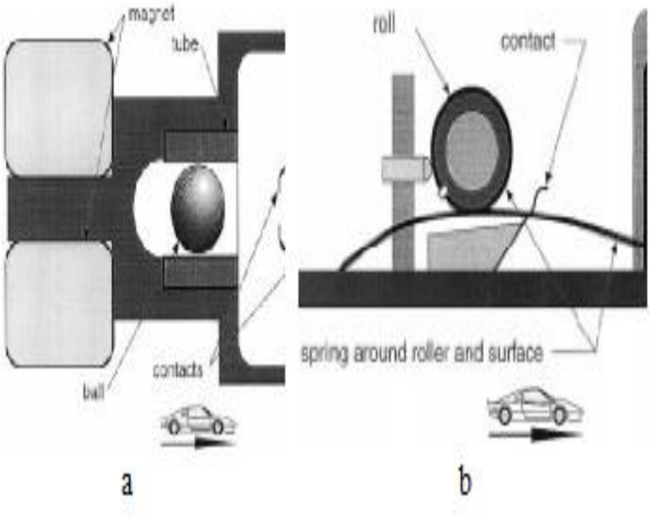

Figure 1: Two type of airbag activation sensors, a) tube sensor with damper, and b) spring-mass sensor [4].

There are instructions that can calculate the maximum forward displacement of the driver before opening the airbag, which is schematically shown in Fig. 2. One of these instructions that can be applied in practice is distance of 0.125 m before full opening of the airbag. In this method the vehicle at speed of 40-50 km/hr collides with a rigid body during the 0.04 seconds. This time includes 0.01 sec for identification and 0.03 sec for opening the airbag.

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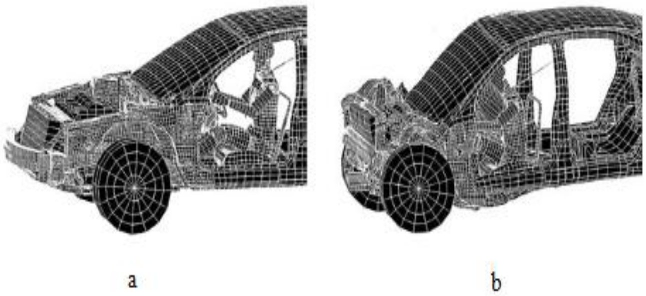

Figure 2: Schematically illustration of a car and driver at the a) instant of an accident, and b) after accident.

Xingqiao et al (2013) investigated the dependence of dummy head injury mitigation on the side curtain airbag and occupant distance under a side impact of a Dodge Neon. Their results indicated that the influence of moving deformable barrier (MDB) strike velocity was the strongest influence parameter on both cases for the head injury criteria (HIC36) and the peak head acceleration, followed by the initial airbag inlet temperature [4].

Due to the high acceleration generated in a crash, a new sensitive-to-shock automotive crash sensor is proposed in this study as a triggering device for cutting off the electricity, activation of the air bag system, unlocking doors and disabling the alarm system. The proposed system includes a cantilever beam with a current-carrying wire. The beam supposed to breaks and cut the circuit off at speeds greater than 40 km/hr. The Analysis is based on the mathematical equations particularly the Laplace transform method.

II. Material and methods

A sketch of the proposed mechanical sensitive-to-shock sensor in illustrated in Fig. 3. This sensor can be matched with car in normal speeds and designed to immediately breaks when the car hits with a rigid body at speeds of 40 km/hr or higher. Simultaneously the current-carrying wire is ruptured with breaking the sensor. According to the previous researches, the time of 0.04 sec and the velocity of 40 km/hr are considered as key parameters for activation of the airbag in this study.

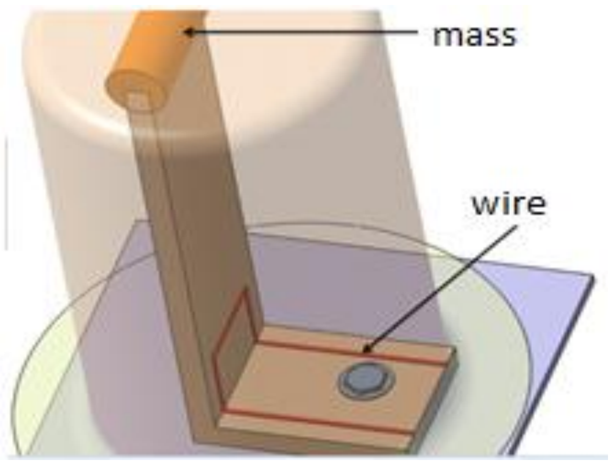

Figure 3: Designed mechanical sensor with an internal wire.

The analysis in this study will be performed for the sensor illustrated in Fig. 3 with properties listed in table 1.

Table 1: Mechanical properties of the considered plastic for manufacturing the shock sensor [6].

Mechanical property	Unit	Value
Modulus of Elasticity	MPa	3000 - 3500
Shear Modulus of elasticity	MPa	700 - 800
Ultimate Tensile strength	MPa	32 - 60
Ultimate bending strength	MPa	50 - 100
Ultimate shear strength	J/cm	0.27 - 5

The assumed beam is a cantilever beam with free vibration and a negligible damping coefficient (C ≈ 0) which is forced by a sinusoidal shock pulse. Vibrational equation of this system can be represented in the form of

$$m\ddot{x} + kx = f(t) \tag{1}$$

The amount of force being exerted on the car during the crash is shown in Fig. 4a [7].

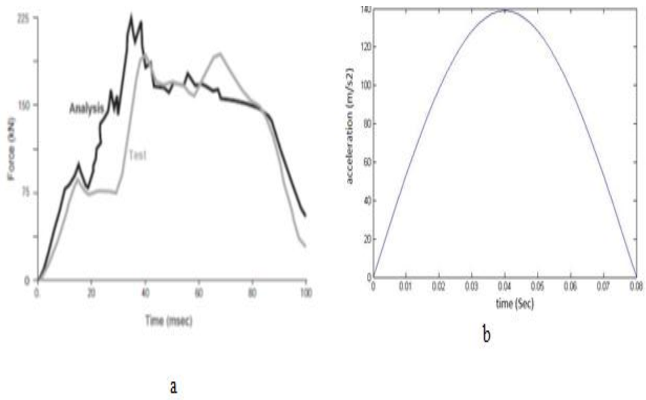

Figure 4: Variation of the force exerted on the car during the crash (a) from [7], simplified model of acceleration during the accident (b).

Based on the results shown in Fig. 4a, variation of the accident force is vanishes within the 100 ms after accident. The maximum acceleration of accident happens at around 0.04 seconds. It means that the accident force increase with a sinus function and again decrease to 0 after 0.08 seconds. In Fig. 4b, a simplified model of acceleration is shown which will be considered in the present study. Taking into account the results presented in Fig. 4a and b, the designed air bag must be opened at the time of the peak force. In other words the designed beam must be broken in 0.04 seconds. According to the Fig.4b, the force applied to the beam is expressed as follows.

$$F(t) = \begin{cases} F_0 \sin \omega t & 0 \le t \le 0.08 \\ 0 & t > 0.08 \end{cases} \qquad 2)$$

Since the speed of beam is the same as the car speed, the initial conditions are $x(0) = \dot{x}(0) = 0$. In this condition, the vibrational equation for the 1D-system reads

$$m\ddot{x} + kx = F_0 \sin \omega t \{u(t) - u(t - t_0)\} \qquad 3)$$

Where u(t) is the unit step function and u(t-t_0) is the unit step function with a time delay equal to t_0. Applying the Laplace transform to the above equation yields

$$m(S^2 X(s) - S\, x(0) - \dot{x}(0)) + kX(S) = F_0 \left\{\frac{\omega}{s^2 + \omega^2} + e^{-st_0} \frac{\omega}{s^2 + \omega^2}\right\} \qquad 4)$$

The amount of tip displacement of the beam in Laplace domain is equal to:

$$X(s) = \frac{F_0 \omega (1 + e^{-st_0})}{(s^2 + \omega^2)(ms^2 + k)} = \frac{F_0 \omega (1 + e^{-st_0})}{m\left(s^2 + \frac{k}{m}\right)(s^2 + \omega^2)} \qquad 5)$$

Where $\omega_n = \sqrt{k/m}$ is the natural vibrational frequency of the beam. By separation of the fractions, the following expression is reached.

$$X(s) = \frac{F_0}{m}\left[\frac{\omega}{(\omega^2 - {\omega_n}^2)(s^2 + {\omega_n}^2)} + \frac{-\omega}{(\omega^2 - {\omega_n}^2)(s^2 + \omega^2)}\right](1 + e^{-st_0}) \qquad 6)$$

After simplifying the above equation, we obtain

$$X(s) = \frac{F_0}{m\omega_n\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)}\left\{\left[\frac{\omega}{(s^2 + \omega^2)} - \frac{\omega_n}{(s^2 + {\omega_n}^2)}\left(\frac{\omega}{\omega_n}\right)\right] + \left[\frac{\omega}{(s^2 + \omega^2)} e^{-st_0} - \frac{\omega_n}{(s^2 + {\omega_n}^2)}\left(\frac{\omega}{\omega_n}\right) e^{-st_0}\right]\right\} \qquad 7)$$

By applying the inverse Laplace transform, displacement of the beam tip in the time space is obtained in the form of

$$x(t) = \frac{\delta_{st}}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)}\left\{\left[\sin \omega t - \left(\frac{\omega}{\omega_n}\right) \sin \omega_n t\right] + \left[\sin \omega t - \left(\frac{\omega}{\omega_n}\right) \sin \omega_n (t - t_0)\right] u(t - t_0)\right\} \qquad 8)$$

Where $\delta_{st} = F_0/k$ is called static displacement. Note that the above equation in $\omega/\omega_n = 1$ cannot be solved and it needs the Hopital rule. In these circumstances, i.e., for the $\omega/\omega_n=1$, the value of x(t) is calculated from the following equation.

$$\frac{X(t)}{\delta_{st}} = \frac{1}{2}(\sin \omega_n t - t\, \omega_n \cos \omega_n t) \qquad 9)$$

III. RESULTS AND DISCUSSION

Maximum response of the sensor tip $(x(t)/\delta_{st})_{max}$ as a function of dimensionless time t_0/τ_n, is shown in Fig. 5 using the Eqns. 8 and 9.

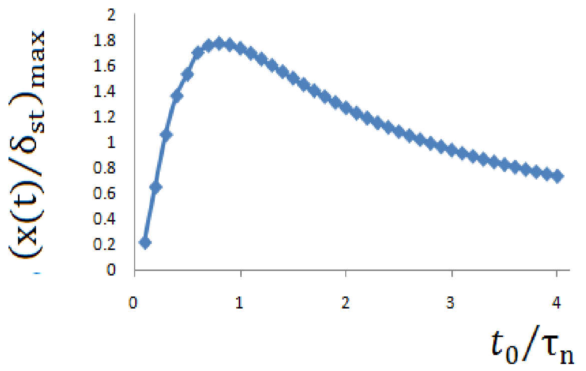

Figure 5: Maximum displacement of the sensor tip $(x(t)/\delta_{st})_{max}$ as a function of dimensionless time t_0/τ_n, for the system response.

As can be seen in Fig. 5, when t_0/τ_n equals 0.8, the value of $(x(t)/\delta_{st})_{max}$ is maximum; therefore the amplification factor of the shock $\lambda = (x(t)/\delta_{st})_{max}$ can be considered equals to 1.77 based on the results shown in this figure. Taking this value, the calculations will be started using λ=1.77. The maximum vibrational force generated in the beam will be:

$$F_{max} = \lambda \cdot ma = \lambda\left(m\frac{dv}{t}\right) = \lambda m\frac{v-0}{t_0} = 1.77 \times 0.2 \times \frac{(40/3.6)}{0.08} = 49.16\, N$$

Maximum stress in beam due to the shock of the collision is equal to:

$$\sigma_{max} = \frac{M_{max} \times C}{I} = \frac{F_{max} l \cdot C}{I} = \frac{49.16 \times 0.08C}{I}$$

If this calculated value for stress is greater than the ultimate stresses of the plastic, the mechanical sensor will be broken and consequently the automotive safety system will be triggered.

$$\frac{c}{I} = \frac{c}{\frac{1}{12}b(2c)^3} = 8.4817 \times 10^6 \Rightarrow bc^2 = 0.1179$$

Assuming the parameters given in Table 1 and width of the beam equals to 3 cm, one obtains

$$c = 1.98 \cong 2\, mm \Rightarrow t = 4\, mm$$

$$I = \frac{bt^3}{12} = 0.16 \times 10^{-6}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3IE}{ml^3}} = 3750$$

$$\frac{t_0}{\tau_n} = \frac{0.08}{2\pi/3750} = 47.74$$

Since the assumed value for t_0/τ_n is different from the above value; therefore we solved the problem again with trial and error, e.g., with the new value of λ = 1.1. The final values for the dimensions of the proposed beam was obtained equal to

$$l = 8\text{ cm}, b = 3\text{ cm}, t = 3\text{ mm}$$

IV. CONCLUSION

In this study a simple mechanical sensor is designed for triggering the airbag of the automotive or turning off the electrical units of the car after crash. The obtained results show that if a cantilever beam with the width, thickness and length of 3 cm, 3 mm, and 8 cm, respectively, be made of a hard plastic and car speed exceeds 40 km/hr, as soon as the car crash happens, the beam will be broken and current will be disconnect and gasoline alarm will stop as well.

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