

# A Theoretical Model for the Design of a Labyrinth Seal Gland

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**Abstract—** The analysis of a rectangular labyrinth seals having several configurations is carried out in this study. The laminar, compressible and axisymmetric flow in a rectangular labyrinth seal is treated and the seal performance is obtained. The analysis is based on the assumptions that adiabatic conditions exist and that the ratio of the seal height to the shaft radius is small as always the case in turbomachines. The theoretical work is capable of handling single cavity in order to compute the pressure ratio, loss in stagnation pressure, change in density, temperature ratio and the variation of inlet to exit Mach number across the cavity. These quantities are obtained at several values of leakage flow Mach number to be used in estimating the required number of cavities to get a minimum allowable leakage rate. For a subsonic leakage flow that is not in choked condition, it is concluded in this work that the performance relations of a single cavity can be used to design a multi-cavity seal to meet an allowable leakage rate.

**Index Terms** ---- Labyrinth seals - Non-contact seal - Turbomachines - Compressors - Computational fluid mechanics - Mathematical models of fluid flow.

## I. INTRODUCTION

Effective sealing in turbomachines is required in order to minimize the leakage of high-energy working fluid, thus reducing the adverse effects of seal leakage on overall engine performance. Despite the development of new techniques in sealing technology, labyrinth seals will continue to be the most widely used sealing element in high-speed turbomachines. Simplicity and compactness, reliability and temperature resistance are some of its major advantages. In general, increasing the fluid resistance through the labyrinth seal can reduce leakage. This can be accomplished either by reducing the clearance size, the development of a new geometrical seal configuration and the use of blowing through the seal gland. Many investigators tackled the leakage problem through labyrinth seals for turbomachinery. Some of the early design methods available in the literature are purely analytical [1-3] and others are analytical-experimental [4-7]. Benvenuti et. al. made a comparative study of these methods and they found that such treatments provided results with considerable differences

and contradictions. The first reliable prediction method was due to Stoff [8] who reported a theoretical and experimental study of incompressible turbulent flow through a labyrinth seal of the straight-through grooved shaft type. He was able to show how much the flow pattern in the labyrinth seal is affected by the ratio of the axial leakage velocity to the shaft peripheral speed. It was found by Rhode et. al. [9] that labyrinth seal solutions using the difference scheme of Stoff [8] may suffer from the so-called false diffusion numerical error depending on the flow field conditions. They however, presented an approach to overcome the difficulties encountered when using the scheme of Stoff. They also adopted for a typical cavity configuration a simple stair step approximation to the curved solid walls. El-Gamal et. al. [10-12] were able to examine the pressure drop and flow pattern for a number of geometrical configurations of seals. Their results showed how important the effect of shaft rotation is on the performance of the seal. Yizhang and Feng [13] applied the continuity equation, energy equation, Bernoulli's equation and the equation of state to the vortex cavity to obtain a closed set of equations. They solved these equations simultaneously and numerically to determine the velocity, pressure and temperature in each vortex cavity. Computational fluid dynamics (CFD) is increasingly used in analyzing flows inside labyrinth seals. An advantage of using CFD is its capability to analyze a large number of design configuration parameters in a relatively short period of time. Therefore, with the development of commercial codes, the use of CFD analysis has been increasing rapidly in recent years as suggested in the references [14-18]. Tong and Kyu [19] analyzed the influence of configuration and clearance on the leakage behavior of straight and steeped labyrinth seals with various clearances. Both computational fluid dynamic (CFD) and an analytical tool were used to predict the leakage flow through the straight and steeped labyrinth seals. They were compared the predicted results with experimental data [14]. The CFD predicted sufficiently well the leakage flows of different seal configurations and for various clearances, especially for the stepped seal. The analytical model provided acceptable prediction in the case of straight seal, but did not provide satisfactory predictions for the stepped seal. They stated that a straight seal with more teeth seems to be a suitable design. Zhao et al [20] studied the effects that cavity dimensions, numbers, and locations have on the leakage flow in straight-through labyrinth seals. The Reynolds averaged Navier-Stokes equations along with a standard  $k-\omega$  turbulence model were solved and the flow is assumed to be axisymmetric. They found that the cavity dimensions and cavity number have significant effects on the leakage as well as on the flow pattern in the seal. The aim of the present work is to introduce a theoretical model to be used in estimating the

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required number of cavities to get a minimum allowable leakage rate.

## II. ANALYSIS

The geometrical configuration of the labyrinth seal selected and the coordinate system used in the analysis are shown in Fig. 1. In the absence of entry swirl, the dimensionless equations governing the steady laminar compressible axisymmetric ( $\frac{\partial}{\partial \theta} = 0$ ) flow in the labyrinth seal may be expressed in the following form,

$$v \frac{\partial \bar{u}}{\partial y} + \gamma w \frac{\partial \bar{u}}{\partial z} + m \bar{u} v = \frac{\bar{\mu}}{R_e \rho} \left[ \frac{\partial^2 \bar{u}}{\partial y^2} + m \frac{\partial \bar{u}}{\partial y} + \gamma^2 \frac{\partial^2 \bar{u}}{\partial z^2} \right] \quad (1)$$

$$v \frac{\partial \bar{v}}{\partial y} + \gamma w \frac{\partial \bar{v}}{\partial z} - m \bar{u} v = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} + \frac{\bar{\mu}}{R_e \rho} \left[ \frac{\partial^2 \bar{v}}{\partial y^2} + m \frac{\partial \bar{v}}{\partial y} + \gamma^2 \frac{\partial^2 \bar{v}}{\partial z^2} \right] + \frac{\bar{\mu}}{3 R_e \rho} \frac{\partial}{\partial y} \left[ \frac{\partial \bar{v}}{\partial y} + m \bar{v} + \gamma \frac{\partial \bar{w}}{\partial z} \right] \quad (2)$$

$$v \frac{\partial \bar{w}}{\partial y} + \gamma w \frac{\partial \bar{w}}{\partial z} = -\frac{\gamma}{\rho} \frac{\partial \bar{p}}{\partial z} + \frac{\bar{\mu}}{R_e \rho} \left[ \frac{\partial^2 \bar{w}}{\partial y^2} + m \frac{\partial \bar{w}}{\partial y} + \gamma^2 \frac{\partial^2 \bar{w}}{\partial z^2} \right] + \frac{\gamma \bar{y}}{3 R_e \rho} \frac{\partial}{\partial z} \left[ \frac{\partial \bar{v}}{\partial y} + m \bar{v} + \gamma \frac{\partial \bar{w}}{\partial z} \right] \quad (3)$$

$$m \bar{\rho} v + \bar{\rho} \frac{\partial \bar{v}}{\partial y} + \bar{v} \frac{\partial \bar{\rho}}{\partial y} + \gamma \left( \bar{\rho} \frac{\partial \bar{w}}{\partial z} + \bar{w} \frac{\partial \bar{\rho}}{\partial z} \right) = 0 \quad (4)$$

$$v \frac{\partial \bar{T}}{\partial y} + \gamma w \frac{\partial \bar{T}}{\partial z} = \frac{E}{\rho} \left[ v \frac{\partial \bar{\rho}}{\partial y} + \gamma w \frac{\partial \bar{\rho}}{\partial z} \right] + \frac{E}{R_e \rho} \bar{\Lambda} + \frac{1}{Pr R_e \rho} \left[ \frac{\partial^2 \bar{T}}{\partial y^2} + m \frac{\partial \bar{T}}{\partial y} + \gamma^2 \frac{\partial^2 \bar{T}}{\partial z^2} \right] \quad (5)$$

$$\bar{p} = \frac{1}{E} \left( \frac{\gamma_g - 1}{\gamma_g} \right) \bar{\rho} \bar{T} \quad (6)$$

And,

$$\begin{aligned} \bar{\Lambda} = & \bar{\mu} \left[ 2 \left( \frac{\partial \bar{v}}{\partial y} \right)^2 + 2 \gamma^2 \left( \frac{\partial \bar{w}}{\partial z} \right)^2 + \gamma^2 \left( \frac{\partial \bar{u}}{\partial z} \right)^2 + \gamma^2 \left( \frac{\partial \bar{v}}{\partial z} \right)^2 \right. \\ & + \left. \left( \frac{\partial \bar{w}}{\partial y} \right)^2 + 2 \gamma \frac{\partial \bar{v}}{\partial z} \frac{\partial \bar{w}}{\partial y} - 2 m \bar{u} \frac{\partial \bar{u}}{\partial y} + \left( \frac{\partial \bar{u}}{\partial y} \right)^2 - \frac{2}{3} \left\{ \left( \frac{\partial \bar{v}}{\partial y} \right)^2 \right. \right. \\ & \left. \left. + 2 m \left( v \frac{\partial \bar{v}}{\partial y} + \gamma w \frac{\partial \bar{w}}{\partial z} \right) + \gamma^2 \left( \frac{\partial \bar{w}}{\partial z} \right)^2 + 2 \gamma \frac{\partial \bar{v}}{\partial y} \frac{\partial \bar{w}}{\partial z} \right\} \right] \quad (7) \end{aligned}$$

$\bar{u}$ ,  $\bar{v}$  and  $\bar{w}$  are the velocity components in dimensionless form in the  $\theta$ ,  $\bar{y}$  and  $\bar{z}$  directions respectively.  $\bar{p}$ ,  $\bar{\rho}$ ,  $\bar{\mu}$  and  $\bar{\Lambda}$  are the dimensionless pressure, density, viscosity and dissipation function respectively.

$$\text{Thus } \bar{u} = \frac{u}{W_o}, \bar{v} = \frac{v}{W_o}, \bar{w} = \frac{w}{W_o}, \bar{p} = \frac{p}{\rho_o W_o^2},$$

$$\bar{\rho} = \frac{\rho}{\rho_o}, \bar{\mu} = \frac{\mu}{\mu_o}, \bar{T} = \frac{T}{T_o}$$

And

$$\bar{y} = \frac{r-R}{b}, \bar{z} = \frac{z}{a}, \gamma = \frac{b}{a}, m = \frac{b}{R},$$

$$\bar{\Lambda} = \frac{\Lambda}{\left( \frac{\mu_o W_o^2}{b^2} \right)}, W_o = \omega R$$

$$\text{With } R_e = \frac{W_o \rho_o b}{\mu_o}, E = \frac{W_o^2}{c_p T_o}, M = \frac{W_o}{\sqrt{\gamma_g R_g T_o}} \text{ and}$$

$$Pr = \frac{c_p \mu_o}{K}$$

$R_e$ ,  $E$ ,  $M$  and  $Pr$  are the Reynolds number, Eckert number, Mach number and Prandtl number respectively. Also, we can express the Eckert number in term of Mach number as,  $E = M^2(\gamma_g - 1)$ .

Let,  $\bar{v} = o(m)$ ,  $\bar{w} = o(1)$ ,  $\bar{u} = o(1)$ ,  $\bar{\mu} = 1$ ,

$$\bar{\rho} = o(1), \bar{y} = o(1) \text{ and } \bar{z} = o(1) \quad (8)$$

Assuming that the pressure is constant across the seal height and applying the order of magnitude of (8) into equations (1), (2), (3), and (4), then

$$\frac{\partial^2 \bar{u}}{\partial y^2} + \gamma^2 \frac{\partial^2 \bar{u}}{\partial z^2} = \gamma \bar{\rho} R_e \bar{w} \frac{\partial \bar{u}}{\partial z} \quad (9)$$

$$\frac{\partial \bar{p}}{\partial y} = 0 \quad (10)$$

$$\gamma \bar{\rho} \bar{w} \frac{\partial \bar{w}}{\partial z} = -\gamma \frac{\partial \bar{p}}{\partial z} + \frac{1}{R_e} \left[ \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{4}{3} \gamma^2 \frac{\partial^2 \bar{w}}{\partial z^2} \right] \quad (11)$$

$$\bar{\rho} \frac{\partial \bar{w}}{\partial z} + \bar{w} \frac{\partial \bar{\rho}}{\partial z} = 0 \quad (12)$$

Assuming adiabatic flow condition and neglecting the temperature variation across the labyrinth height,  $\bar{T} = \bar{T}(z)$ ,  $\bar{\rho} = \bar{\rho}(z)$  and equations (5) and (7) can be written as follows,

$$\gamma w \frac{\partial \bar{T}}{\partial z} = \frac{E}{\rho} \gamma w \frac{\partial \bar{p}}{\partial z} + \frac{E}{R_e \rho} \bar{\Lambda} + \frac{\gamma^2}{Pr R_e \rho} \left( \frac{\partial^2 \bar{T}}{\partial z^2} \right) \quad (13)$$

$$\bar{\Lambda} = \left( \frac{\partial \bar{w}}{\partial y} \right)^2 + \frac{4}{3} \gamma^2 \left( \frac{\partial \bar{w}}{\partial z} \right)^2 + \left( \frac{\partial \bar{u}}{\partial y} \right)^2 + \gamma^2 \left( \frac{\partial \bar{u}}{\partial z} \right)^2 \quad (14)$$

Integrating equations (13) and (14) with respect to  $\bar{y}$ , we get

$$\gamma \frac{\partial \bar{T}}{\partial z} \int \bar{w} d\bar{y} = \frac{E}{\rho} \gamma \frac{\partial \bar{p}}{\partial z} \int \bar{w} d\bar{y} + \frac{E}{R_e \rho} \int \bar{\Lambda} d\bar{y} + \frac{\gamma^2}{Pr R_e \rho} \frac{\partial^2 \bar{T}}{\partial z^2} \quad (15)$$

$$\begin{aligned} \int \bar{\Lambda} d\bar{y} = & \int \left[ \left( \frac{\partial \bar{w}}{\partial y} \right)^2 + \left( \frac{\partial \bar{u}}{\partial y} \right)^2 \right] d\bar{y} + \frac{4}{3} \gamma^2 \int \left( \frac{\partial \bar{w}}{\partial z} \right)^2 d\bar{y} \\ & + \gamma^2 \int \left( \frac{\partial \bar{u}}{\partial z} \right)^2 d\bar{y} \quad (16) \end{aligned}$$

Using equation (6) and differentiate with respect to  $\bar{z}$ , the variation of the pressure in axial direction can be obtained as,

$$\frac{d\bar{p}}{dz} = \frac{1}{E} \left( \frac{\gamma_g - 1}{\gamma_g} \right) \left( -\frac{d\bar{T}}{dz} + \bar{T} \frac{d\bar{\rho}}{dz} \right) \quad (17)$$

Integrating the continuity equation (12) with respect to  $\bar{y}$  and

using  $\int_0^1 \bar{w} d\bar{y} = G(\bar{z})$ , we get the continuity equation in the

following form,

$$\bar{\rho} G(\bar{z}) = A \quad (18)$$

Here, A is a constant to be calculated from:  $A = G(0)$

Using equations (11), (15), (17) and (18), we get

$$EA\gamma G \bar{w} \frac{\partial \bar{w}}{\partial z} = -\gamma A \left( \frac{\gamma_g - 1}{\gamma_g} \right) \left[ G \frac{d\bar{T}}{dz} - \bar{T} \frac{dG}{dz} \right] + \frac{EG^2}{Re} \left( \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{4}{3} \gamma^2 \frac{\partial^2 \bar{w}}{\partial z^2} \right) \quad (19)$$

The grooved shaft is the only case treated here and presented as an example of a class of rectangular labyrinth seals. The presence of a clearance in the labyrinth seal causes an axial leakage of the fluid, this means that the axial leakage velocity component must be taken into consideration. It is assumed that the expansion through the clearance is isentropic and that no slip condition at the walls. The boundary conditions for the grooved shaft labyrinth shape selected here are,

$$\left. \begin{aligned} \text{At } \bar{y} = 0, u_o = 1, \bar{w} = 0 \\ \bar{y} = 1, \bar{u}_o = 0, \bar{w} = 0 \\ \bar{z} = 0 \text{ or } 1 \text{ and } \bar{y} \leq 1 - \frac{c}{b}, \\ \bar{u}_o = 1, \bar{w} = 0 \text{ and } \bar{T} = \frac{T_i}{T_o} \\ \bar{z} = 0 \text{ and } 1 > \bar{y} > 1 - \frac{c}{b}, \frac{\partial \bar{u}_o}{\partial \bar{z}} = 0, \\ \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \bar{T} = \frac{T_i}{T_o} \text{ and } \frac{\partial \bar{T}}{\partial \bar{z}} = 0 \\ \bar{z} = 1 \text{ and } 1 > \bar{y} > 1 - \frac{c}{b}, \\ \frac{\partial \bar{u}_o}{\partial \bar{z}} = 0, \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \text{ and } \frac{\partial \bar{T}}{\partial \bar{z}} = 0 \end{aligned} \right\} \Rightarrow (20)$$

### III. LEAKAGE MODEL

The present model may be employed to determine the total number of labyrinth cavities for a wide variety of designs. It does, however, require results from single-cavity computations. Hence, the prediction of leakage for a new design involves two step procedure. First, the single-cavity solution is used to determine the inlet to outlet cavity pressure ratio, temperature ratio and Mach number for the given seal configuration. This is done at several cavity inlet Mach numbers and at a fixed values of inlet stagnation pressure and stagnation temperature. Second, the inlet-to-outlet cavity pressure ratio, temperature ratio and subsequently the cavity outlet static pressure and static temperature are evaluated from the polynomial expressions at the known inlet Mach number. The cavity outlet density is evaluated from the ideal gas law. The cavity outlet velocity and subsequently the corresponding Mach number is

calculated from the mass conservation equation. Since the outlet conditions for a given cavity are the inlet conditions for the next downstream, the same sequence is repeated for each cavity. At last cavity, the exit pressure must be less than or equal downstream pressure to estimate the total number of cavities performed the seal gland. The leakage model was developed as a simple computer program to which the user specifies the desired mass leakage rate, the upstream stagnation conditions, the downstream pressure, shaft diameter, shaft speed and gland diameter.

### IV. RESULTS AND DISCUSSION

The governing equations (9), (15), (16) and (19) with the boundary conditions for the flow in grooved shaft rectangular labyrinth seals (20) are suited to numerical solution by the finite difference technique using the Gauss-Seidel under-relaxation iterative method. The domain of integration is divided into  $40 \times 40$  square grids each of mesh size  $h = 0.025$ . The results are obtained for a single grooved shaft cavity and for different values of aspect ratio  $\gamma$ . It is found that the variations in results for  $\gamma = O(1)$  are less than 1% and may be neglected. Thus the results for the steady flow in grooved shaft labyrinth seals having  $\gamma = 1$  only are presented.

The results thus obtained are used to study the variation of various parameters and conditions of flow at the inlet and exit of a single straight through grooved shaft labyrinth cavity. The performance curves and relations of a single cavity are obtained for different values of the speed of rotation of the shaft. Solving equation (16) gives the temperature distribution inside the cavity, thus the Mach number at exit can be calculated as

$$M_e = \sqrt{\left( \frac{2}{\gamma_g - 1} \right) \left( \frac{T_e}{T_o} - 1 \right)}$$

Fig. 2 shows the variation of the exit Mach number  $M_e$  with the inlet Mach number  $M_i$  for different values of the rotational Mach number  $M_r$ ,  $M_r = \frac{W_o}{\sqrt{\gamma_g R_g T_o}}$ . For a

certain value of the inlet Mach number, it is found that increasing the rotational Mach number leads to decreasing the value of the exit Mach number.

Fig. 3 presents the relation between the pressure ratio  $\frac{p_e}{p_i}$  and the inlet Mach number  $M_i$  for different values of rotational Mach number  $M_r$ . The figure shows a large pressure drop at rotational Mach number  $M_r = 0.125$ , increasing the speed of the shaft the rotational Mach number increases and the pressure drop across the seal decreases. This means that increasing the speed of the shaft decreases the acceleration of the flow in the axial direction.

In the design of a cavity of a labyrinth seal, the value of the downstream pressure is significant. The variation of the ratio

of the stagnation pressure at inlet to static pressure at exit  $\frac{p_{oi}}{p_e}$  with the inlet Mach number  $M_i$  is shown in Fig. 4.

Also, Fig. 5 shows the variation of the static temperature ratio  $\frac{T_e}{T_i}$  with the inlet Mach number  $M_i$  for different

values of rotational Mach number  $M_r$ . It is found that increasing the speed of the shaft decreases the change in the static temperature. Also, the maximum inlet Mach number the maximum decrease in static temperature across the cavity. The leakage rate through a single cavity seal depends on the inlet conditions, which are stagnation pressure and stagnation temperature,  $p_{oi}$  and  $T_{oi}$  and the value of the exit static pressure  $p_e$ . Introducing the mass (leakage) flow

parameter as  $\omega_{p_{oi}T_{oi}} = \frac{\dot{m}\sqrt{RT_{oi}}}{p_{oi}A_c}$ , we get a useful relation

between the leakage per unit area and stagnation temperature and stagnation pressure at inlet. Fig. 6 shows the variation of

leakage flow parameter  $\omega_{p_{oi}T_{oi}}$  with pressure ratio  $\frac{p_e}{p_i}$  across the seal cavity for various rotational Mach numbers. The results show that increasing the rotational Mach number in general leads to an increase in the leakage rate.

For multi cavity seal,  $p_{ou}$  and  $T_{ou}$  are the upstream stagnation pressure and upstream stagnation temperature respectively. Also,  $p_u$  and  $p_d$  are the upstream and downstream static pressures at inlet and exit of a multi cavity seal respectively. Defining the leakage rate parameter

through a multi cavity seal as  $\omega_{p_{ou}T_{ou}} = \frac{\dot{m}\sqrt{RT_{ou}}}{p_{ou}A_c}$ , Fig. 7

shows the relation between the leakage rate parameter  $\omega_{p_{ou}T_{ou}}$  and the pressure ratio  $\frac{p_u}{p_d}$  for different number of

cavities that formed a multi cavity seal. The results show that the leakage rate decreases as the number of cavity increases. Fig. 8 shows the effect of number of cavities on the leakage rate parameter for different values of pressure ratio of upstream stagnation pressure to downstream pressure. It is seen that the larger the number of cavities the smaller the

leakage rate parameter at any pressure ratio  $\frac{p_{ou}}{p_d}$ . However

there is an asymptotic value of number of cavities at which leakage rate parameter  $\omega_{p_{ou}T_{ou}}$  becomes very small.

### I. CASE STUDY

Consider a labyrinth seal of air compressor with the following conditions:

Upstream stagnation pressure = 140 kPa

Pressure ratio  $\frac{p_{ou}}{p_d} = 1.4$

Up stream stagnation temperature = 400 K

a) Shaft diameter = 80 mm

Shaft speed = 200 rps

Gland diameter = 88 mm

Allowable leakage rate = 0.001 kg/s

It is required to make a preliminary design to estimate the required number of cavities of the labyrinth seal gland.

### Design procedure:

#### 1- Inlet Conditions of the first cavity:

Assuming isentropic expansion through the inlet clearance and using the mass conservation equation we obtain the inlet Mach number, inlet static pressure and inlet static temperature.

#### 2- Polynomial expressions

The resulting inlet to outlet single cavity pressure ratio, temperature ratio and exit Mach number are inserted into the leakage model using polynomial representation versus inlet Mach number. The polynomial expression for the single cavity performance curves may be written as,

$$M_e = a_0 + a_1 M_i + a_2 M_i^2$$

$$\left(\frac{p_e}{p_i}\right) = b_0 + b_1 M_i + b_2 M_i^2 + b_3 M_i^3 + b_4 M_i^4 + b_5 M_i^5 + b_6 M_i^6 + b_7 M_i^7$$

$$\left(\frac{T_e}{T_i}\right) = e_0 + e_1 M_i + e_2 M_i^2 + e_3 M_i^3$$

Where,

The coefficients of inlet/exit Mach number relation are,

$$a_0 = 0.000636163, \quad a_1 = 1.12124, \quad a_2 = 0.122316$$

The coefficients of inlet/exit pressure ratio relation are,

$$b_0 = 1.00027, \quad b_1 = -2.60013, \quad b_2 = 19.7262, \quad b_3 = -78.5611$$

$$b_4 = 175.898, \quad b_5 = -224.228, \quad b_6 = 151.756, \quad b_7 = -42.3422$$

The coefficients of inlet/exit temperature ratio relation are,

$$e_0 = 0.999979, \quad e_1 = -0.00275745, \quad e_2 = -0.0491327,$$

$$\text{and } e_3 = -0.0375619$$

#### 3- Exit conditions of the first cavity

Using polynomial expressions, the exit Mach number of the

first cavity  $M_1$ , the pressure ratio  $\left(\frac{p_1}{p_i}\right)$  and temperature

ratio  $\left(\frac{T_1}{T_i}\right)$  can be calculated. Also, the exit static pressure

( $p_1$ ) and static temperature ( $T_1$ ) may be estimated.

#### 4- Estimation of the total number of cavities

$$(1) \quad \text{Total number of cavities} = 11$$

Estimated mass flow rate = 0.0008112 kg/s

The pressure distribution along the multi cavity seal obtained and presented in Fig. 9. Also, Fig. 9 shows the pressure distribution along the labyrinth seal for different values of allowable leakage rate, it is found that increasing the number of cavities decreases the leakage rate

V. CONCLUSIONS

A simple approach to the laminar compressible flow problem through labyrinth seals of rectangular shapes has been developed. The theoretical model enabled the effect of shaft rotation on the performance of the seal to be included. It is found that increasing the speed of the shaft decreases the pressure drop across the seal. For a single cavity and at several leakage flow Mach number, the pressure ratio, temperature ratio and the variation of inlet to exit Mach number across the seal can be obtained. These quantities are used to estimate the required number of cavities to get a minimum allowable leakage rate. For a subsonic leakage flow that is not in choked condition, it is concluded that the performance relations of a single cavity can be used to design multi-cavity seal to meet an allowable leakage rate.

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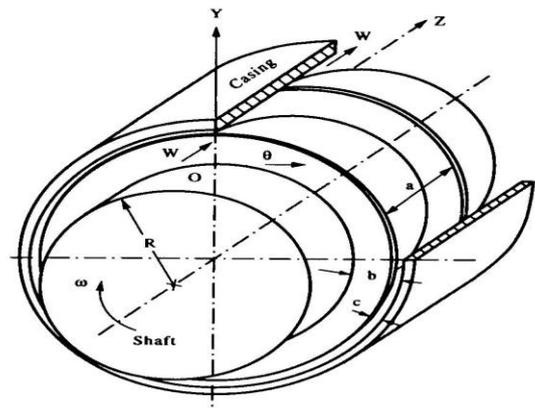


Fig. 1. Labyrinth seal coordinate system.

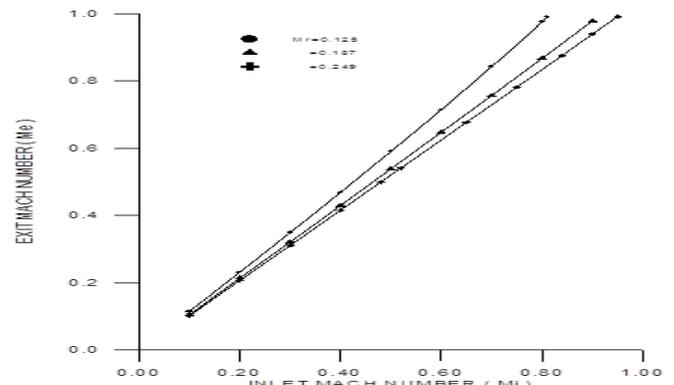


Fig. 2. Variation of seal inlet to exit Mach number.

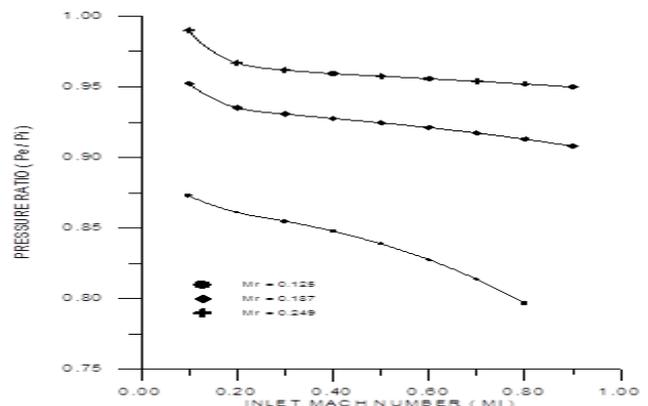


Fig. 3. Variation of static pressure ratio across the labyrinth cavity with inlet Mach number.

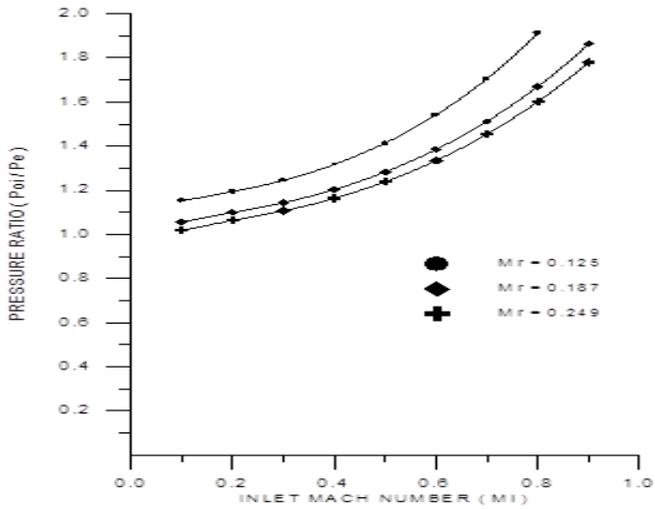


Fig. 4 Pressure ratio  $p_{oi}/p_e$  against the inlet Mach number

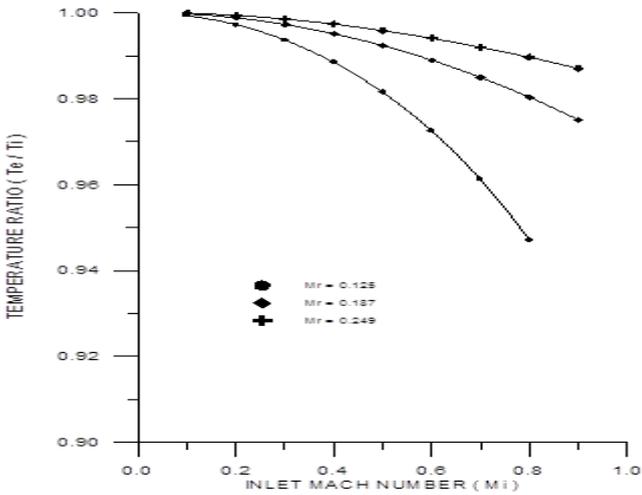


Fig. 5. Variation of temperature ratio  $\frac{T_e}{T_i}$  with inlet Mach number.

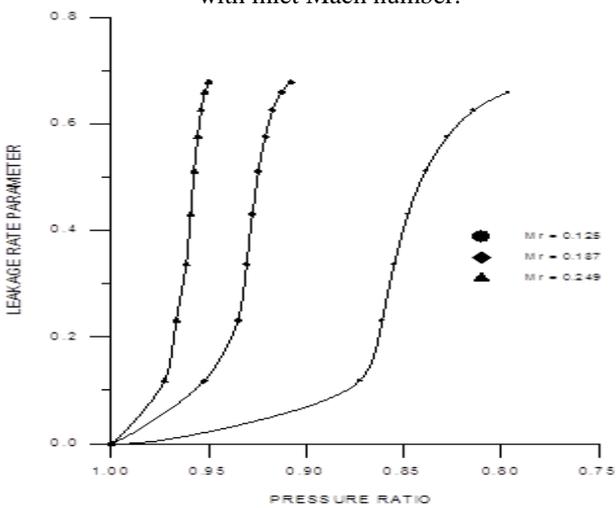


Fig. 6. Leakage rate parameter  $\omega_{pouTou}$  against pressure ratio  $\frac{P_e}{P_i}$ .

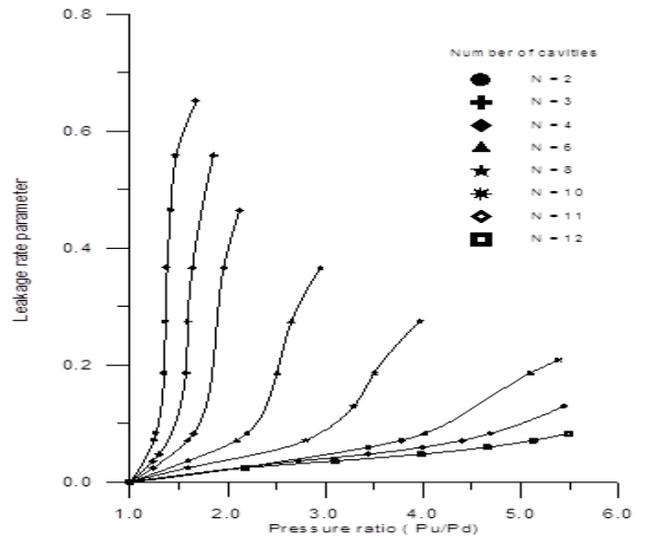


Fig. 7. Leakage rate parameter  $\omega_{pouTou}$  against pressure ratio  $\frac{P_u}{P_d}$

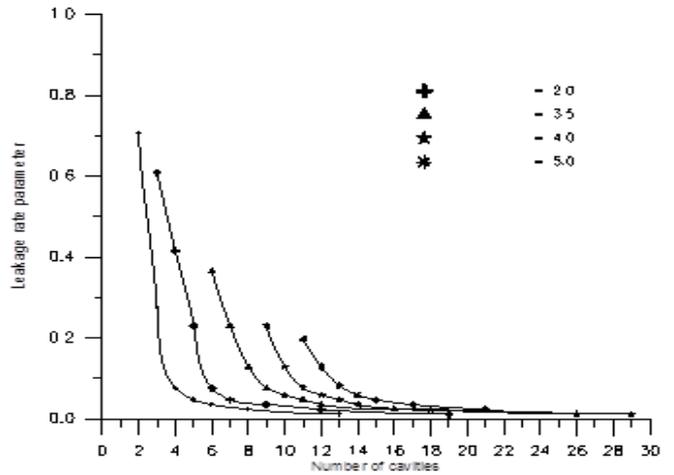


Fig. 8. Leakage rate parameter  $\omega_{pouTou}$  against number of cavities formed a multi-cavity seal for different values of  $\frac{P_{ou}}{P_d}$ .

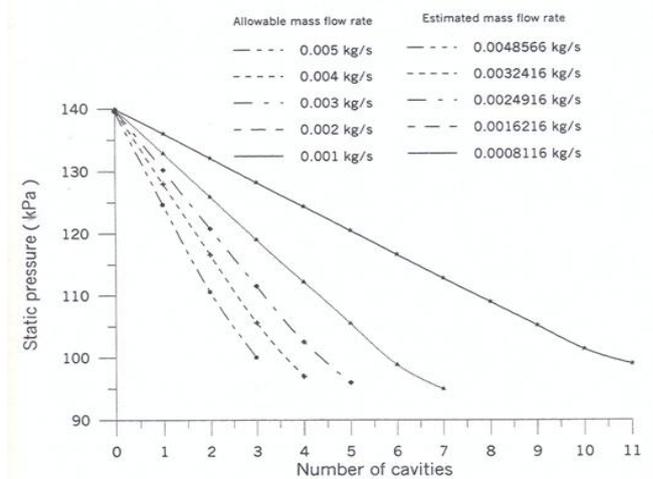


Fig. 9. Pressure distribution along the multi-cavity labyrinth seal.